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# SUN-PAN'S IDENTITIES VIA Z-TRANSFORM

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**Abstract:**

Generating Functions and Transforms has been put in to great use for proving several interesting and new results spanning connections between Number Theory and other branches of mathematics. One among such useful transforms is Z - Transform often considered as discrete version of Laplace Transforms. Z - Transforms is an important mathematical tool often applied in Digital Signal Processing and Control Theory to convert discrete time signals to complex frequency domain. The concept of Binomial coefficients which occur as coefficients in the standard power series expansion of two particular terms, has been very useful tool in handling several combinatorial problems. In this paper, we use the Z-transform to prove two interesting identities of Sun-Pan involving binomial coefficients.

**Key Words:** Sun-Pan's Identities, Binomial Coefficients, Z-Transform.

## 1. Introduction:

Sun-Pan [1] showed the identities:

$$A_n := \sum_{k=0}^{n} (-1)^k k^2 \binom{n-k+2}{3} = 0, \tag{1}$$

$$B_n := \sum_{k=0}^{n} (-1)^k k^4 \left[ 12 \binom{n-k+3}{5} + \binom{n-k+2}{3} \right] = 0, \tag{2}$$

For $n = 1, 3, 5, \ldots$ Here we shall employ the following properties of Z-transform [2, 3] to prove (1) and (2):

$$\begin{aligned} &Z\{(-1)^n n^2\} = \frac{z\,(1-z)}{(1+z)^3} \\ &Z\{(-1)^n n^4\} = \frac{z\,(1-z)\,(z^2-10\,z+1)}{(1+z)^5} \\ &Z\left\{\binom{n+3}{5}\right\} = \frac{1}{20}\, Z\left\{n\,(n-1)\binom{n+3}{3}\right\} = \frac{z^4}{(z-1)^6} \\ &Z\left\{\binom{n+2}{3}\right\} = \frac{1}{3}\, Z\left\{n\binom{n+2}{2}\right\} = \frac{z^3}{(z-1)^4} \end{aligned} \tag{3}$$

## 2. Proofs of (1) and (2):

From the definition of $A_n$ and the values (3):

$$Z\{A_n\} = Z\{(-1)^n n^2\}\, Z\left\{\binom{n+2}{3}\right\} = \frac{z^4}{(1-z^2)^3} = -\frac{1}{z^2}\left(1-\frac{1}{z^2}\right)^{-3} = -\sum_{k=0}^{\infty} \frac{\binom{k+2}{2}}{z^{2(k+1)}}, \tag{4}$$

Where only participate even powers of $\frac{1}{z}$, therefore (1) is immediate:

$$A_n = \begin{cases} 0, & n = 1, 3, 5, \ldots \\ -\binom{m+1}{2}, & n = 2m = 2, 4, 6, \ldots \end{cases} \tag{5}$$

Similarly, from the definition of $B_n$ and the expressions (3):

$$Z\{B_n\} = \frac{z^4(z^2-10\,z+1)\,(z^2+10\,z+1)}{(1-z^2)^5} = -\frac{1}{z^2}\left(1-\frac{1}{z^2}\right)^{-5}\left(1-\frac{98}{z^2}+\frac{1}{z^4}\right), \tag{6}$$

Involving only even powers of $z^{-1}$, then (2) is immediate, that is, $B_n = 0,\ n = 1, 3, 5, \ldots$

Our approach shows that the Z-transform technique is very useful to prove combinatorial identities.

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