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PRIME LABELING OF SPLIT GRAPH OF PATH GRAPH PN

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Abstract:

In the present work we have discussed relative prime of split graph of the path graph P_n when n is odd or even. We have derived an algorithm which admits prime labeling to the split graph of the path graph P_n.

Key Words: Graph Labeling, Prime Labeling, Path Graph & Split Graph of a Graph G.

1. Introduction:

We consider only simple, finite, undirected connected and non-trivial graph $G = (V, E)$ with the vertex set V and the edge set E. The number of elements of V, denoted as $|V|$ is called the order of the graph G while the number of elements of E denoted as $|E|$ is called the size of the graph G, $spl(P_n)$ denotes the split graph of the path graph P_n . The notion of a prime labeling originated with Entringer and was introduced in a paper by Tout, Dabbouchy and Howalla [3]. Entringer conjectured that all trees have a prime labeling. Haxell, Pikhurko and taraz [10] proved that all large trees are prime graph. Among the classes of trees known to have prime labelings are paths, stars, caterpillars, complete binary trees, spiders, olive trees, palm trees and others. We will give brief summary of definitions and other information which are useful for the present task. For various graph theoretic notations and terminology we follow Gross and Yellen [7] and Bondy S. Murthy [1] whereas for number theory we follow D. M. Burton [2].

Definition 1.1: For a graph $G = (V, E)$ a function having domain V, E (or) $V \cup E$ is said to be a graph labeling of G. If the domain is V, E (or) $V \cup E$ then the corresponding labeling is said to be a vertex labeling, an edge labeling (or) a total labeling.

Definition 1.2: A prime labeling of a graph G of order n is an injective function $f: V \to \{1,2, \dots. n\}$ such that for every pair of adjacent vertices u and v, $\gcd\big(f(u), f(v)\big) = 1$. The graph which admits prime labeling is called a prime graph.

Definition 1.3: For $n \geq 2$, an n-path (or simply path graph) denoted P_n, is a connected graph consisting of two vertices with degree 1 and $n-2$ vertices of degree 2. A path graph P_n with n vertices has $n-1$ edges.

Definition 1.4: For a graph G, the split graph which is denoted by $spl(G)$ is obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G.

2. Main Results:

2.1 Algorithm for prime labeling of split graph of path graph P_n

Step 1: Let P_n be the given path graph of n vertices and let $v_1, v_2, \dots v_n$ be their vertices and let $v_1', v_2', \dots v_n'$ be the new vertices corresponding to each vertices $v_1, v_2, \dots v_n$ respectively. Let $G = spl(P_n)$ be the split graph of P_n

Step 2: Obviously $|V(G)| = 2n$ and $|E(G)| = 3n - 3$. Therefore, define a function $f: V(G) \to \{1,2, \dots 2n\}$ injectively as follows

$$f(v_i) = 2i - 1 \text{ for } i = 1,2 \dots n$$
$$f(v_i') = 2i \text{ for } i = 1,2 \dots n$$

Step 3: Enumerate the different types of edges in G in which we have to check the relative prime of end points of each type of edges. In G, there are 3 type edges, $v_i\, v_{i+1}$, $v_i\, v_{i+1}'$, $v_i' v_{i+1}$ for $i = 1,2 \dots n-1$. Out of these 3 type of edges, we need to check the relative prime of edges of type $v_i\, v_{i+1}'$ for $(i = 1,2 \dots n-1)$ only.

Step 4: Checking the relative prime of each pair of vertices

In this step, we need to check the relative prime of only the pair of vertices v_i and v_{i+1}' of the labeled graph G obtained in step 2.

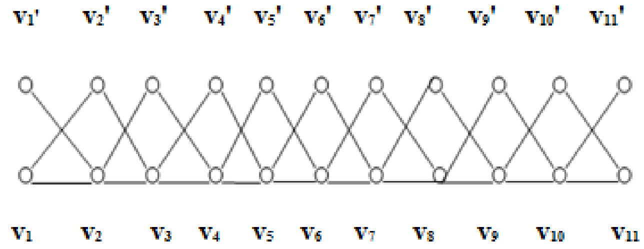
If $gcd(f(v_i), f(v_{i+1}')) = 1$ for $i = 1,2 \dots n-1$ then the graph G admits prime labeling

If $gcd(f(v_i), f(v_{i+1}')) \neq 1$ for $i = 1,2 \dots n-1$ then we have to do the following steps

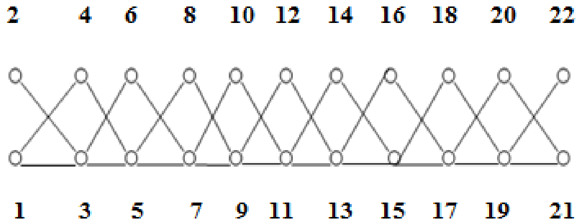
Step 5: Suppose $gcd(f(v_i), f(v_{i+1}')) \neq 1$ for some i then select all those pairs of vertices v_i and v_{i+1}' $(i = 1,2 \dots n-1)$ for which $f(v_i)$ and $f(v_{i+1}')$ are not relatively prime and encircle each pairs with in a circle. Now, interchange the labels of v_i' and v_{i+1}' (where v_{i+1}' is the encircled vertex). The procedure is repeated until all encircled vertex v_i' are exhausted. Now, the newly labeled graph admits prime labeling.

Illustrations:

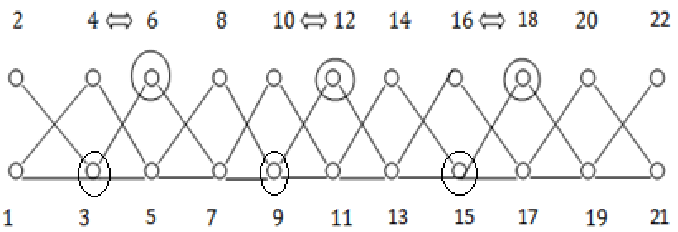
Illustration 2.1: n is odd, $spl(P_{11})$ is a prime graph.



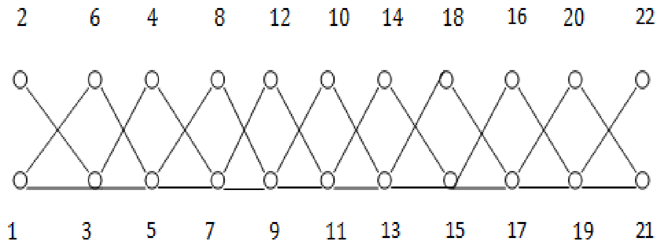
Labeling the vertices of $spl(P_{11})$ by using $f(v_i) = 2i-1$ for $i = 1,2 \ldots 11$ and $f(v_i') = 2i$ for $i = 1,2 \ldots 11$. Now we get the following labeled graph.



Checking the relative prime of each pair of vertices v_i and v_{i+1}' and mark the vertex v_i and v_{i+1}' within circles (which are not relatively prime) we get the following graph



Now, change the label of v_{i+1}' (which is encircled) with the label of v_i'. The procedure is continued for all encircled v_i' we get the following resulting labeled graph



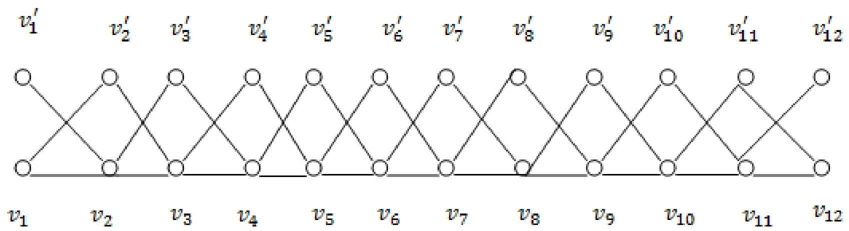
Now, in the above graph, $gcd(v_i, v_{i+1}) = 1$; for $i = 1,2 \ldots n-1$

$gcd(v_i, v_{i+1}') = 1$; for $i = 1,2 \ldots n-1$ and $gcd(v_i', v_{i+1}) = 1$; for $i = 1,2 \ldots n-1$

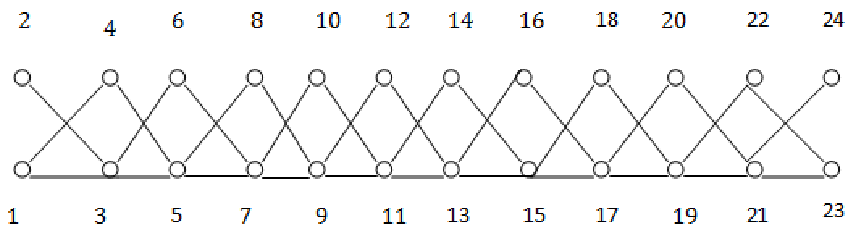
Therefore, $spl(P_{11})$ admits prime labeling.

Hence, $spl(P_{11})$ is a prime graph.

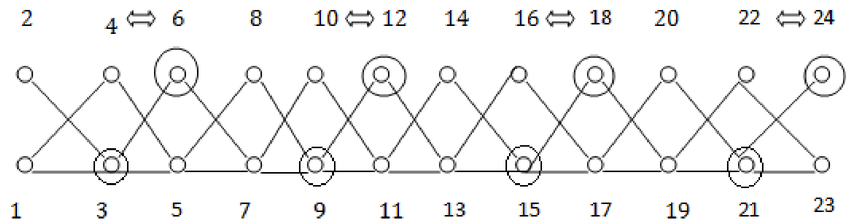
Illustration 2.2: n is even, $spl(P_{12})$ is a prime graph.



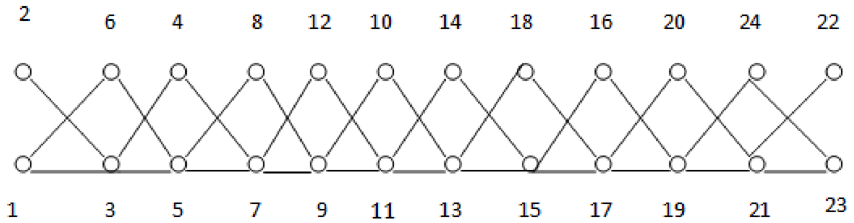
Labeling the vertices of $spl(P_{12})$ by using $f(v_i) = 2i-1$ for $i = 1,2 \ldots 12$ and $f(v_i') = 2i$ for $i = 1,2 \ldots 12$. Now we get the following labeled graph.



Checking the relative prime of each pair of vertices v_i and v'_{i+1} and mark the vertex v_i and v'_{i+1} within circles (which are not relatively prime) we get the following graph



Now, change the label of v'_{i+1} (which is encircled) with the label of v'_i. The procedure is continued for all encircled v'_i we get the following resulting labeled graph



Now, in the above graph, $gcd(v_i, v_{i+1}) = 1$; for $i = 1,2 \ldots n-1$

$gcd(v_i, v'_{i+1}) = 1$; for $i = 1,2 \ldots n-1$ and $gcd(v'_i, v_{i+1}) = 1$; for $i = 1,2 \ldots n-1$

Therefore, $spl(P_{12})$ admits prime labeling. Hence, $spl(P_{12})$ is a prime graph.

3. Conclusion:

We have presented an algorithm for prime labeling of certain class of graph such as splitting graph of path P_n and illustrate with two examples for the cases n is odd and n is even separately. In general, all the graphs are not prime,it is interesting to investigate graph families which admits prime labeling.

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