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A MATHEMATICAL MODEL FOR THE EFFECT OF OXYTOCIN USING FUZZY GENERALIZED RAYLEIGH DISTRIBUTION

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Abstract:

In this article we originate the fuzzy generalized Rayleigh distribution to analyze the effect of oxytocin. We deliver ample representation of the fuzzy properties of the generalized Rayleigh distribution along with the consequence of 5u doses of Oxytocin by measuring the expected value of cardiac output for lower and upper alpha cuts. The results show that the expected values for lower alpha cut increases and the expected values of upper alpha cut were decreases.

Key Words: Oxytocin, Generalized Rayleigh Distribution, Fuzzy Mean& Fuzzy Variance

1. Introduction:

In numerous realistic disciplines such as medicine, engineering and finance, amongst others, modeling and investigating lifespan data is essential. Quite a few lifetime distributions have been used to model such kind of data. The excellence of the techniques used in a statistical analysis rest on extremely on the presumed probability model or distribution. As a consequence of this, substantial effort has been spent in the progress of huge classes of standard probability distributions along with relevant statistical methodologies. Nevertheless, there still remain various significant complications where the real data does not follow any of the classical or standard probability models.

The Rayleigh distribution [12] is the special case of Weibull distribution, is widely used to model events data. The one parameter Rayleigh distributional properties, estimation and characterizations have been studied by many authors, such as Johnson, Kotz and Balakrishnan [5], Dey and Das [2], Dey [1]. In recent times Surles and Padgett [14] investigational the two-parameter generalized Rayleigh distribution be able to used pretty excellently in modeling strength and general lifetime data. Kundu and Raqab [6] used diverse approaches to assess the parameters of the generalized Rayleigh on simple data. Acceptance sampling based on life time data discussed by Tzong-Ru Tsai and Shuo-Jye Wu [17]. In 2016, Dey et al. [3] derived interval and point estimates of the scale and location parameters of a two parameter Rayleigh distribution using progressive Type-II censored samples. Recenty (2017) Murithiet. al [10] estimate the parameters of the two parameter of Rayleigh distribution based on Type II Censored data.

Oxytocin is the leading choice medication for improving uterine narrowing after delivery. Thereare oxytocin receptors in the uterus, and receptors have also been placed in mammary, endothelial, and central nervous tissue as well. The effect of oxytocin on endothelial receptors produces a calcium dependent vasodilator effect via stimulation of the nitric oxide pathway [15]. The substantial haemodynamic effect of oxytocin 5u i.v. in healthy pregnant patients during spinal anaesthesia for Caesarean section hasbeen published [7], [9], [11], [13], [16].

In this article we present a new generalization of the Rayleigh distribution (GRD) called the fuzzy generalized Rayleigh distribution (FGRD) and we study the hemodynamic effects of 5u i.v. bolus doses of oxytocinby finding the mean and variance of fuzzy generalized Rayleigh distribution.

2. Notations:

β – Shape parameter
χ – Scale parameter
$\overline{\beta}(\alpha)$ – Alpha cut of shape parameter
$\overline{\chi}(\alpha)$ – Alpha cut of scale parameter
$E(X)$ – Expected value of X
$V(X)$ – Variance value of X
$\overline{E}(X)$ – Fuzzy expected value of X
$\overline{V}(X)$ – Fuzzy variance value of X

3. Generalized Rayleigh Distribution:

A random variable X follows the GRD has probability density function (p.d.f.) $f(x:\beta,\chi)=\frac{2}{\Gamma(\beta+1)\chi^{\beta+1}}x^{2\beta+1}e^{\frac{x^2}{\beta}}, x>0$, where $\beta\geq 0$ is the shape parameter and $\chi>0$ is the scale parameter. When $\beta=0$ and set $\chi=2b^2$ the GRD reduce to Rayleigh distribution with scale parameter b. The cumulative distribution function (c.d.f.) of X is given by

$$F(x;\beta,\chi)=1-\sum_{i=0}^{\beta}\left(\frac{\left(x^2/\chi\right)^i e^{\left(-x^2/\chi\right)}}{i!}\right).$$

The GRD has the r^th^ moment,

$$E(X^r)=\int_{-\infty}^{\infty}x^r f(x)\,dx$$

$$=\int_{-\infty}^{\infty}x^r\frac{2}{\Gamma(\beta+1)\chi^{\beta+1}}x^{2\beta+1}e^{\frac{x^2}{\beta}}\,dx=\frac{\Gamma(\beta+r/2+1)}{\Gamma(\beta+1)}\chi^{r/2}\quad r=1,2,...$$

Therefore the mean and variance of GRD is given by

$$F(x;\beta,\chi)=\int_{-\infty}^{x}f(x)dx$$

$$=\Gamma_{I,\beta+1}\left(\frac{x^2}{\chi}\right),$$

Where $\Gamma_{I,\beta}(n)=\frac{1}{\Gamma(\beta)}\int_0^n x^{\beta-1}e^{-x}dx$ is known as the incomplete gamma function. If β is an integer, the c.d.f. of GRD reduces to

$E(X)=\frac{\Gamma\left(\beta+\frac{3}{2}\right)}{\Gamma(\beta+1)}\sqrt{\chi}$ and the variance is $V(X)=\left((\beta+1)-\left(\frac{\Gamma\left(\beta+\frac{3}{2}\right)}{\Gamma(\beta+1)}\right)^2\right)\chi$.

4. Fuzzy generalized Rayleigh Distribution:

Modeling, in general logic refers to the establishment of a description of a system (a plant, a process, etc.) in mathematical terms, which describes the behavior of the original organism [4]. Such a design is a mathematical representation, called a mathematical model, of the physical system. Some physical systems, specifically those complex ones, are uncompromising to model by an accurate and precise mathematical procedure or equation due to the complexity of the system structure, nonlinearity, uncertainty, randomness, etc.Therefore, approximation modeling is often necessary and practical in real world applications. Naturally, approximate modeling is always possible. But, the important questions are what kind of approximation is upright, where the logic of "goodness" has to be first defined, of course, and how to formulate such a good approximation in modeling a system such that it is mathematically rigorous and can produce satisfactory results in both theory and applications. It is clear that interval mathematics and fuzzy logic together can provide a promising alternative to mathematical modeling for many physical systems that are too vague or too complicated to be described by simple and crisp mathematical formulas or equations. When interval mathematics and fuzzy logic are employed, the interval of confidence and the fuzzy membership functions are used as approximation measures, leading to the so-called fuzzy systems modeling. From this perspective, fuzzy logic is a technique to formalize the human capacity of imprecise reasoning, approximate reasoning. Such reasoning represents the human ability to reason approximately and judge under uncertainty.

At the present time consider the GRD with fuzzy parameters $\overline{\beta},\overline{\chi}$ which are reformed with the parameters β,χ. A random variable X follows fuzzy generalized Rayleigh distribution is symbolized by $X \sim FGRD\left(x:\overline{\beta},\overline{\chi}\right)$. The p.d.f. of a random variable $X \sim FGRD\left(x:\overline{\beta},\overline{\chi}\right)$ is defined by

$$f\left(x:\overline{\beta},\overline{\chi}\right)=\left\{f(x)[\alpha],\mu_{f(x)}\,\middle|\,f(x)[\alpha]=\left[f_l(x)[\alpha],f_u(x)[\alpha]\right],\mu_{f(x)}=\alpha\right\}$$

$$f_l(x)[\alpha] = \inf.\left\{ f(x:\beta,\chi)(\alpha) \middle| \beta \in \overline{\beta}(\alpha), \varphi \in \overline{\chi}(\alpha) \right\},$$

$$f_u(x)[\alpha] = sup.\left\{ f(x:\beta,\chi)(\alpha) \middle| \beta \in \overline{\beta}(\alpha), \varphi \in \overline{\chi}(\alpha) \right\},$$

$$f(x:\overline{\beta},\overline{\chi}) = \frac{2}{\Gamma(\overline{\beta}+1)\,\overline{\chi}^{\overline{\beta}+1}} x^{2\overline{\beta}+1} e^{\frac{x^2}{\overline{\beta}}}, \quad x > 0, \overline{\beta} \in \overline{\beta}(\alpha), \overline{\chi} \in \overline{\chi}(\alpha).$$

The expected value of $X \sim FGRD\left(x;\overline{\beta},\overline{\chi}\right)$ is given by

$$\overline{E}(X) = \left\{ E(X)[\alpha], \mu_{E(X)} \middle| E(X)[\alpha] = E_l(X)[\alpha], E_u(X)[\alpha], \mu_{E(X)} = \alpha \right\}$$

$$E_l(X)[\alpha] = \inf.\left\{ E(X) \middle| \beta \in \overline{\beta}(\alpha), \chi \in \overline{\chi}(\alpha) \right\},$$

$$E_u(X)[\alpha] = \sup.\left\{ E(X) \middle| \beta \in \overline{\beta}(\alpha), \chi \in \overline{\chi}(\alpha) \right\}$$

$$\overline{E}[X] = \frac{\Gamma\left(\overline{\beta}+\frac{3}{2}\right)}{\Gamma\left(\overline{\beta}+1\right)}\sqrt{\overline{\chi}}, \quad \overline{\beta} \in \overline{\beta}(\alpha), \overline{\chi} \in \overline{\chi}(\alpha).$$

The variance value of $X \sim FGGD\left(x;\overline{\lambda},\overline{\mu},\overline{\varphi}\right)$ is given by

$$\overline{V}(X) = \left\{ V(X)[\alpha], \mu_{V(X)} \middle| V(X)[\alpha] = V_l(X)[\alpha], V_u(X)[\alpha], \mu_{V(X)} = \alpha \right\}$$

$$V_l(X)[\alpha] = \inf\left\{ V(X) \middle| \beta \in \overline{\beta}(\alpha), \chi \in \overline{\chi}(\alpha) \right\},$$

$$V_u(X)[\alpha] = \sup\left\{ V(X) \middle| \beta \in \overline{\beta}(\alpha), \chi \in \overline{\chi}(\alpha) \right\}.$$

$$\overline{V}[X] = \left(\left(\overline{\beta}+1\right) - \left(\frac{\Gamma\left(\overline{\beta}+\frac{3}{2}\right)}{\Gamma\left(\overline{\beta}+1\right)} \right)^2 \right)\overline{\chi}, \quad \overline{\beta} \in \overline{\beta}(\alpha), \overline{\chi} \in \overline{\chi}(\alpha) \cdot$$

5. Results and Discussion

Consider the study taken by [8], Women were given spinal anaesthesia with isobaric bupivacaine (7 or 10 mg) and sufentanil 4 µg with a prophylactic phenylephrine infusion or a placebo infusion. An i.v. bolus of oxytocin 5 u (Syntocinon, Novartis, and Copenhagen, Denmark) was injected into a rapidly running i.v. line immediately after delivery.

All women had an arterial line inserted, and LidCOPlus (LiDCO, London, UK) was used for invasive monitoring of cardiac output (CO), and other haemodynamic effects such as stroke volume (SV), cardiac output (CO), and systemic vascular resistance (SVR). This monitor performs a beat-by-beatanalysis of the arterial pressure wave to determine CO and other haemodynamic variables which are stored in the computer.Table 1 shows the CO effect after the administration of the medication oxytocin.

Table 1: Cardiac output effect of an i.v. bolus of oxytocin 5 u

Time (Sec)	0	10	20	30	40	50	60	70	80	90	100	110	120
CO	6.5	7	7.7	8	8.3	10	11.9	12.1	11.8	10.2	9	8.4	8.1

From the experiment the parameters of GRD for 5u are $\beta = 2.6445$ and $\chi = 5.8812$. The corresponding fuzzy triangular numbers are $\left[1.8405 + 0.804\alpha,\ 2.6445,\ 3.4025 - 0.758\alpha\right]$ and $\left[5.0362 + 0.845\alpha,\ 5.8812,\ 6.6632 - 0.782\alpha\right]$. The fuzzy mean and variance values for the dose of oxytocin 5 u are presented in the Fig. 4.1 and Fig. 4.2.

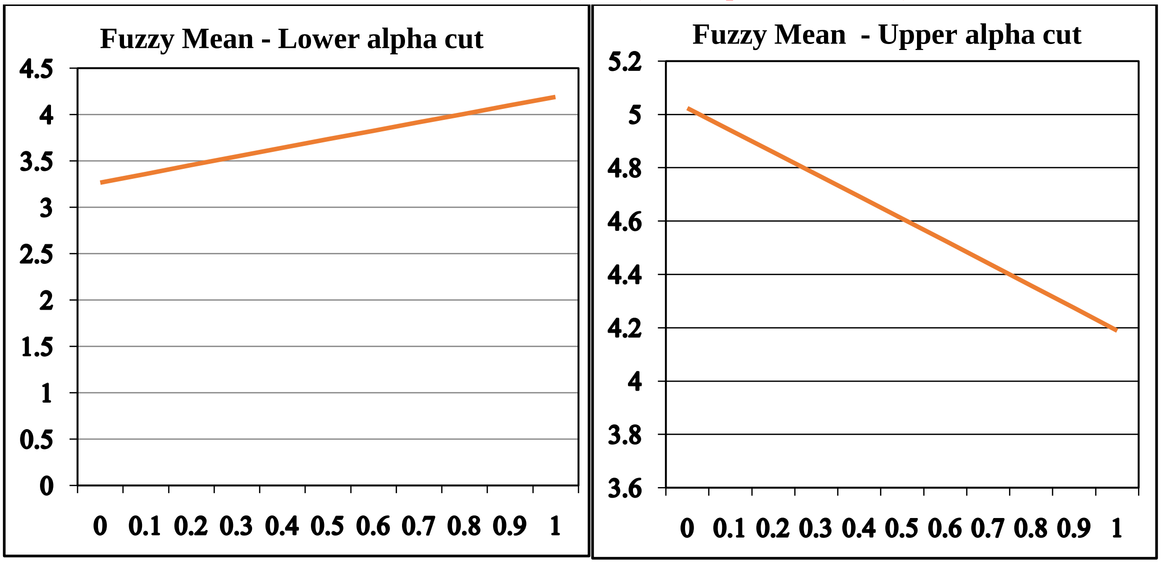

Figure 1: Fuzzy Mean values of oxytocin 5u for upper and lower alpha cuts

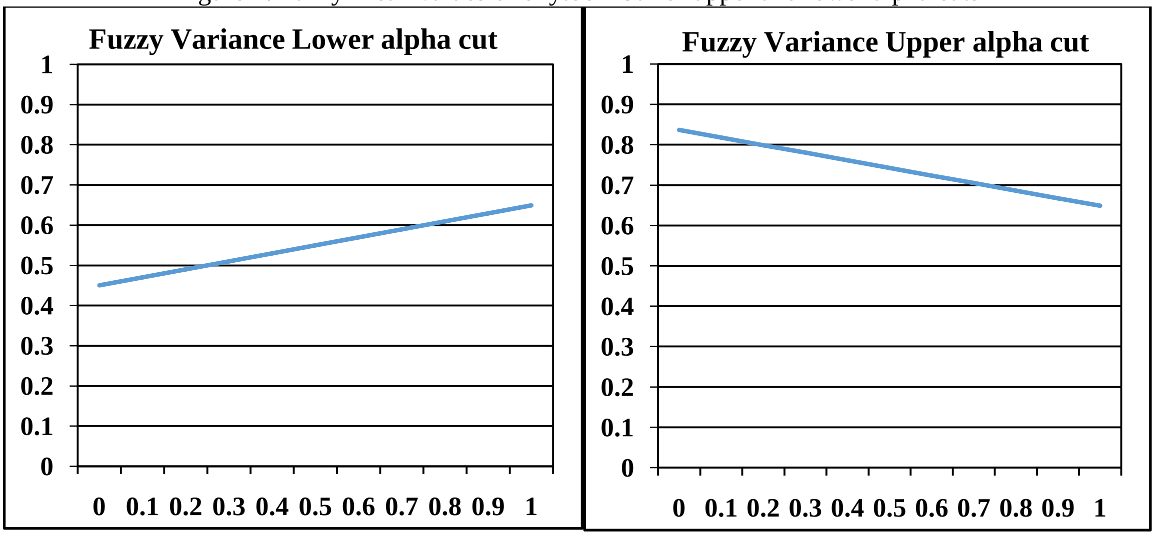

Figure 2: Fuzzy Variance values of oxytocin 5u for upper and lower alpha cuts

6. Conclusion:

A mathematical model using FGRD was successfully established. Using FGRD the effects of cardiac output was calculated by finding the mean and variance values of FGRD. The results shows that an bolus of oxytocin 5 u produced prominent haemodynamic changes, and the mean values and variance values are increasing for the lower alpha cuts and decreasing for upper alpha cuts.

7. References:

1. Dey, S., "Comparison of Bayes Estimators of the parameter and reliability function for Rayleigh distribution under different loss functions", Malaysian Journal of Mathematical Sciences, Vol. 3, 247-264, 2009.
2. Dey, S. and Das, M.K., "A Note on Prediction Interval for a Rayleigh Distribution: Bayesian Approach", American Journal of Mathematical and Management Science, Vol. 27 (1&2), 43 – 48, 2007.
3. Dey, Tanujit, Sanku Dey, and Debasis Kundu. "On Progressively Type-II Censored Two-parameter Rayleigh Distribution, Communications in Statistics-Simulation and Computation, 45.2: 438-455, 2016
4. Guanrong Chen, Trung Tat Pham, Introduction to fuzzy sets, fuzzy logic, and fuzzy control systems, by CRC Press LLC© 2001.
5. Johnson, N.L., Kotz, S. and Balakrishnan, N., Continuous Univariate Distribution Vol. 1, 2nd Edition, John Wiley & Sons, New York, 1995.
6. Kundu, D. and Raqab, M.Z., Generalized Rayleigh distribution: different methods of estimation, Computational Statistics and Data Analysis, 49, 187-200, 2005.
7. Langesaeter E, Rosseland LA, Stubhaug A, Continuous invasive blood pressure and cardiac output monitoring during caesarean delivery: a randomized, double-blind comparison of low dose versus high

dose spinal anesthesia with intravenous phenylephrine or placebo infusion. Anesthesiology 109: 856-6, 2008.
8. Langesaeter E, Rosseland LA, Stubhaug A., Haemodynamic Effects of Repeated Doses of Oxytocin during Caesarean Delivery in Healthy ParturientsBr J Anaesth., 103(2):260-262, 2009.
9. Langesaeter E, Rosseland LA, Stubhaug A., Hemodynamic effects of oxytocin during cesarean delivery. Int J Obstet Anesth., 95:467, 2006.
10. Murithi Daniel Fundi, Edward Gachangi Njenga, Kemboi George Keitany. Estimation of Parameters of the Two-Parameter Rayleigh Distribution Based on Progressive Type-II Censoring Using Maximum Likelihood Method via the NR and the EM Algorithms. American Journal of Theoretical and Applied Statistics. Vol. 6, No. 1, pp. 1-9, 2017. doi: 10.11648/j.ajtas.20170601.11
11. Pinder AJ, Dresner M, Calow C, Shorten GD, O'Riordan J, Johnson R. Haemodynamic changes caused by oxytocin during caesarean section under spinal anaesthesia. Int J Obstet Anesth., 11:156-9, 2002.
12. Rayleigh, Lord. On the capillary phenomena of jets, Proc. R. Soc. London., Vol. 29. No. 196-199. 1879.
13. Sartain JB, Barry JJ, Howat PW, McCormack DI, Bryant M. Intravenous oxytocin bolus of 2 units is superior to 5 units during elective Caesarean section. Br J Anaesth., 101:822-6, 2008.
14. Surles, J.G. and Padgett, W.J., Inference for reliability and stress-strength for a scaled Burr type X distribution, Lifetime Data Analysis, 7, 187-200, 2001.
15. Thibonnier M, Conarty DM, Preston JA, Plesnicher CL, Dweik RA, Erzurum SC. Human vascular endothelial cells express oxytocin receptors. Endocrinology, 140:1301-9, 1999.
16. Thomas JS, Koh SH, Cooper GM. Haemodynamic effects of oxytocin given as i.v. bolus or infusion on women undergoing Caesarean section. Br J Anaesth, 98:1169, 2007.
17. Tzong-Ru Tsai &Shuo-Jye Wu, Acceptance sampling based on truncated life tests for generalized Rayleigh distribution, Journal of Applied Statistics, 33:6, 595-600,2006, DOI: 10.1080/026647606 00679700