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ASCENDING DOMINATION DECOMPOSITION OF SOME GRAPHS

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Abstract:

An Ascending Domination Decomposition (ADD) of a graph G is a collection $\psi = \{G_1, G_2, ..., G_n\}$ of subgraphs of G such that, each G_i is connected, every edge of G is in exactly one G_i and domination number of each G_i is i, $1 \le i \le n$. In this paper, we present some families of graphs especially splitting graphs that admit ADD.

Key Words: Domination, Decomposition, Ascending Domination Decomposition, Splitting graph.

1. Introduction:

By a graph $G(V, E)$, we mean a non-trivial, finite, simple, undirected and connected graph. The order of a graph is denoted by n. For basic definitions and notations, we refer [4]. A *full vertex* v is a vertex which is adjacent to every other vertices in G. That is, if v is a full vertex, then $d(v) = n-1$. The *corona* $G_1 \circ G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 which has p_1 vertices and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2. The graph $G \circ K_1$ is denoted by G^+. The *cartesian product* of two graphs G and H is denoted by $G \times H$.

Let $H_{n,n}$ be the *highly irregular bipartite* graph with vertex set $\{v_1, v_2, ..., v_n \ ; \ u_1, u_2, ..., u_n\}$ and the edge set $\{v_i u_j \ / \ 1 \le i \le n, \ n-i+1 \le j \le n\}$. For example, the graph $H_{5,5}$ is shown in Figure 1.

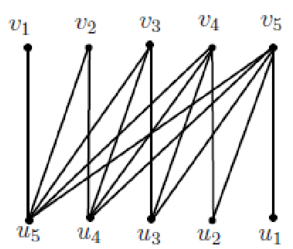

Figure 1

The *Bistar* graph $B_{n,n}$ is the graph obtained by joining the center vertices of 2 copies of the star $K_{1,n}$ by means of an edge. For example, the graph $B_{4,4}$ is shown in Figure 2.

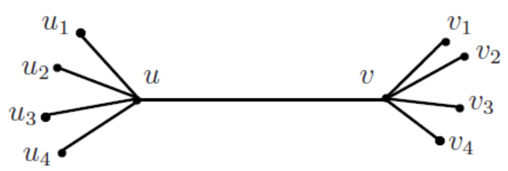

Figure 2

Let I_n denote the graph with vertex set $\{v_1, v_2, ..., v_n\}$ and edge set $\{v_{n+1-i} v_j / \ 1 \le i \le \lfloor n/2 \rfloor, \ i \le j \le n-i\}$ [3]. For example, the graph I_6 is shown in Figure 3.

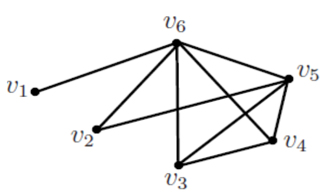

Figure 3

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The graph $K_{1,m} \times P_2$ is called the $m-book$ graph and is denoted by B_m. For example, the graph B_3 is shown in Figure 4.

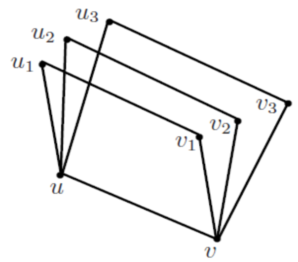

Figure 4

The *Jelly fish* graph $J(m,n)$ is obtained from a 4-cycle $xuyvx$ by joining x and y with an edge and appending m pendant vertices to u and n pendant vertices to v.

For example, the jelly fish graph $J(4,6)$ is shown in Figure 5.

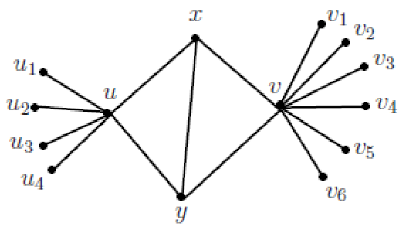

Figure 5

The concept of splitting graph $S(G)$ was introduced by Sampath Kumar and Walikar [8]. The graph $S(G)$ obtained from G, by adding a new vertex w for every vertex $v \in V$ and joining w to all vertices adjacent to v in G, is called the *splitting graph* of G.

For example, a graph G and its splitting graph $S(G)$ are shown in Figure 6.

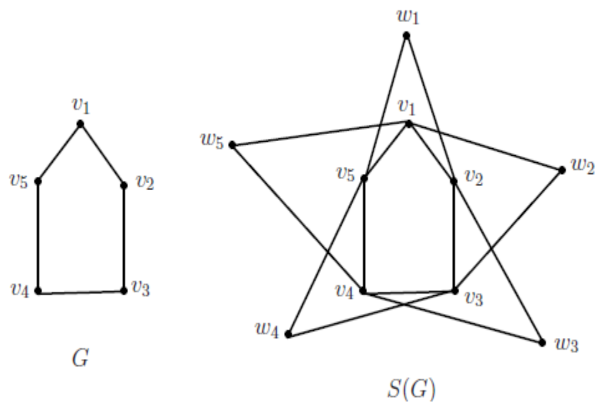

Figure 6

For further results on splitting graphs one can refer $[1,2,8]$.

The mathematical study of domination theory in graphs started around 1960. In a graph, a *dominating set* is a subset S of the vertices such that every vertex is either in S or adjacent to a vertex in S. The *domination number* $\gamma(G)$, is the minimum cardinality among all dominating sets of G. The concept of domination number was introduced by C.Berge in 1958. The notation $\gamma(G)$ was first used by E.J.Cockayne and S.T.Hedetniemi $[5]$ for the domination number of a graph which subsequently became the accepted notation.

The origin of the study of graph decomposition emerged in 19^{th} century. Graph decomposition problems rank among the most prominent areas of research in graph theory and combinatorics. It has numerous applications in various fields such as networking, block designs, bio informatics, coding theory and other fields. For further results on decomposition one can refer $[6]$.

A *decomposition* of a graph G is a collection ψ of edge disjoint subgraphs $G_1, G_2, ..., G_n$ of G such that every edge of G is in exactly one G_i. An *Ascending Domination Decomposition* (ADD) of a graph G is a collection ψ =

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$\{G_1, G_2, ..., G_n\}$ of subgraphs of G such that

i) Each G_i is connected

ii) Every edge of G is in exactly one G_i

iii) $\gamma(G_i)$ = i, for each i, $1 \le i \le n$.

For example, a graph G with ADD is shown in Figure 7.

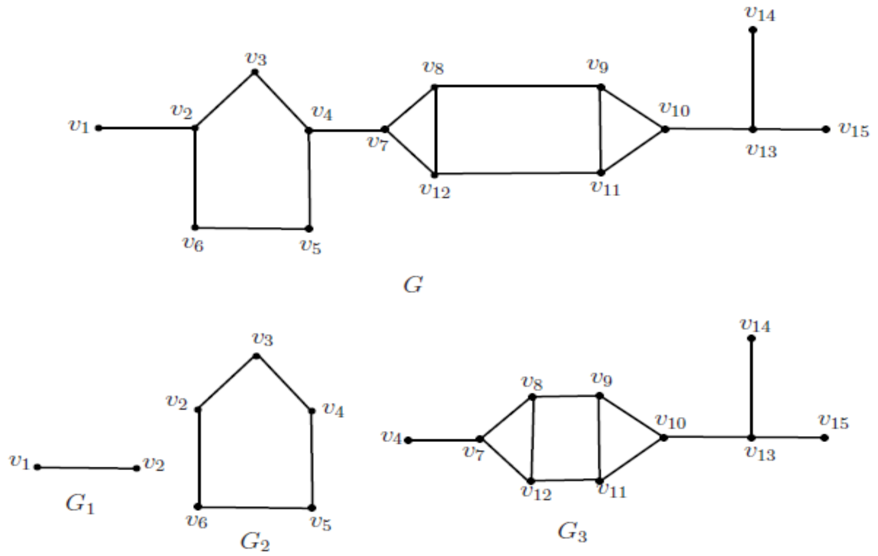


Figure 7

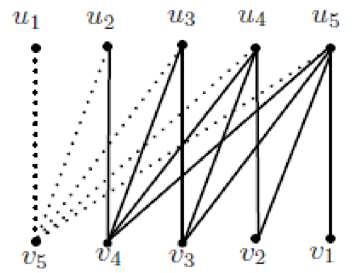
In [7], it has been proved that K_n, W_n, $K_{1,n}$, $K_{m,n}$, P_n, C_n and the corona graphs P_p^+, C_p^+ and $K_{1,p}^+$ admit ADD. In this paper, we prove that $H_{n,n}$, bistar, book graph, jelly fish graph and Petersen graph admit ADD. We also prove that the splitting graph of $H_{n,n}$, $B_{n,n}$, $K_{m,n}$, I_n, K_n, book graph and Petersen graph admit ADD.

2. Main Results:

We first prove that the graph $H_{n,n}$ admits ADD.

Theorem 2.1: *For every* $n \ge 3$, $H_{n,n}$ *admits ADD.*

Proof: Let X = $U \cup V$ be the bipartition of $H_{n,n}$ with $|U| = n = |V|$. Let U = $\{u_1, u_2, ..., u_n\}$ and V = $\{v_1, v_2, ..., v_n\}$. Let G_1 be an edge induced subgraph induced by the edge set $E = \{v_n u_i / 1 \le i \le n\}$. Then, $G_1 \cong K_{1,n}$ and $\gamma(G_1) = 1$. Also $G_2 = H_{n,n} - \{u_1, v_n\} \cong H_{n-1,n-1}$. Clearly $\gamma(G_2)$ = 2 with a $\gamma - set$ $\{u_n, v_{n-1}\}$. Hence ψ = $\{G_1, G_2\}$ is an ADD for $H_{n,n}$. For example, the graph $H_{5,5}$ with ADD graphs G_1 and G_2 are shown in Figure 8



$H_{5,5}$

Here the edges of the graph G_1 *are drawn with dotted lines.*

Figure 8

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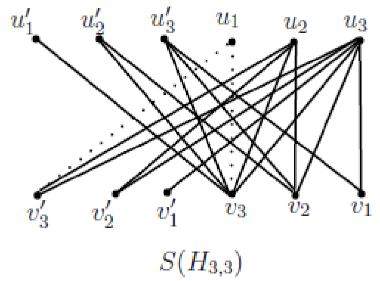
Theorem 2.2: *For every* $n \geq 2$, *the splitting graph of* $H_{n,n}$ *admits ADD.*

Proof: Let $G \cong S(H_{n,n})$ be the splitting graph of $H_{n,n}$. Let X = U ∪ V be the bipartition of $S(H_{n,n})$ with $|U| = 2n = |V|$, such that U = $\{u_1, u_2, ..., u_n, u_1', u_2', ..., u_n'\}$ and V = $\{v_1, v_2, ..., v_n, v_1', v_2', ..., v_n'\}$ with u_i', v_i' are the corresponding newly added vertices for u_i and v_i, $1 \leq i \leq n$ respectively. Let G_1 be the edge induced subgraph induced by the edges incident with u_1. Then, $G_1 \cong K_{1,2}$ and $\gamma(G_1) = 1$. Also $G_2 = S(H_{n,n}) - u_1$. Clearly $\gamma(G_2) = 2$ with a $\gamma - set$ $\{u_n, v_n\}$. Hence $\psi = \{G_1, G_2\}$ is an ADD for $S(H_{n,n})$.

For example, the graph $S(H_{3,3})$ with ADD graphs G_1 and G_2 are given in Figure 9.



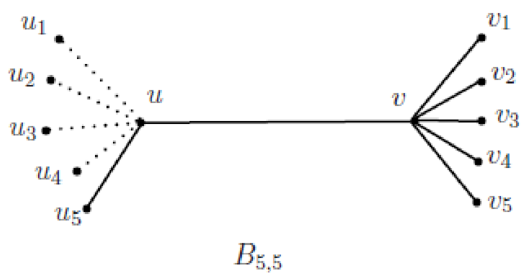
Here the edges of the graph G_1 are drawn with dotted lines.

Figure 9

Theorem 2.3: *The bistar graph* $B_{n,n}$ *admits ADD, where* $n \geq 2$.

Proof: Let $V(B_{n,n}) = \{u, v; u_1, u_2, ..., u_n; v_1, v_2, ..., v_n\}$ and $E(B_{n,n}) = \{uv, uu_i, vv_i \ / \ 1 \leq i \leq n\}$ be the vertex set and edge set of $B_{n,n}$. Let G_1 be the edge induced subgraph induced by the edge set $\{uu_i \ / \ 1 \leq i \leq n-1\}$. Then, $G_1 \cong K_{1,n-1}$ and $\gamma(G_1) = 1$. Also $G_2 = B_{n,n} - \{u_1, u_2, ..., u_{n-1}\}$. Clearly $\gamma(G_2) = 2$ with a $\gamma - set$ $\{u, v\}$. Hence $\psi = \{G_1, G_2\}$ is an ADD for $B_{n,n}$.

As an illustration, the graph $B_{5,5}$ with ADD graphs G_1 and G_2 are shown in Figure 10



Here the edges of the graph G_1 are drawn with dotted lines.

Figure 10

Theorem 2.4: *The splitting graph of* $B_{n,n}$ *admits ADD, where* $n \geq 1$.

Proof: Let $G \cong S(B_{n,n})$ be the splitting graph of $B_{n,n}$. Let $V(B_{n,n}) = \{u, v, u_i, v_i \ / \ 1 \leq i \leq n\}$. Now $|V(G)| = 4n + 4$. Let $E(B_{n,n}) = \{uv, uu_i, vv_i \ / \ 1 \leq i \leq n\}$. Let u_i', v_i', $1 \leq i \leq n$ be the newly added vertices corresponding to u_i and v_i respectively in $S(G)$. Let G_1 be the edge induced subgraph induced by the edges incident with u'. Then, $G_1 \cong K_{1,n+1}$ and $\gamma(G_1) = 1$. Also $G_2 = S(B_{n,n}) - u'$. Clearly $\gamma(G_2) = 2$ with a $\gamma - set$ $\{u, v\}$. Hence $\psi = \{G_1, G_2\}$ is an ADD for $S(B_{n,n})$.

As an illustration, the graph $S(B_{2,2})$ with ADD graphs G_1 and G_2 are given in Figure 11.

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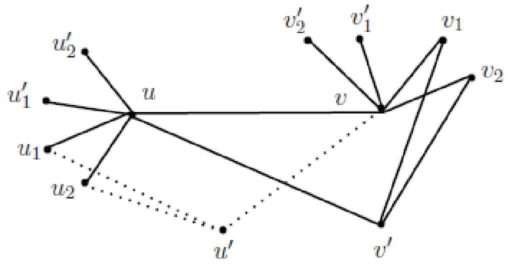


$S(B_{2,2})$

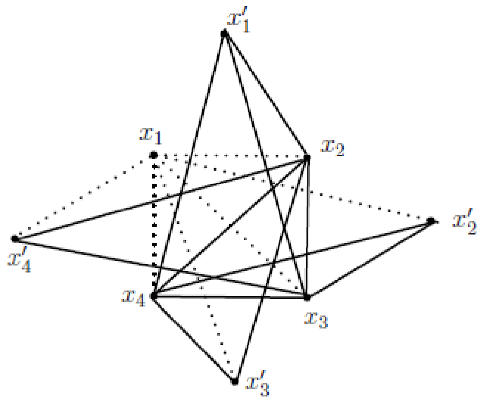
Here the edges of the graph G_1 are drawn with dotted lines.

Figure 11

Theorem 2.5: *For every $n \geq 3$, the splitting graph of K_n admits ADD.*

Proof: Let $G \cong S(K_n)$ be the splitting graph of K_n. Let $V(K_n) = \{x_1, x_2, ..., x_n\}$. Now $|V(G)| = 2n$. Let $E(K_n)$ = $\{x_i x_j \ / \ i \neq j,\ 1 \leq i, j \leq n\}$. Let x_i', $1 \leq i \leq n$ be the newly added vertices corresponding to x_i in $S(K_n)$. Let G_1 be the edge induced subgraph induced by the edges incident with x_1. Then, $G_1 \cong K_{1,2(n-1)}$ and $\gamma(G_1)$ = 1. Also G_2 = $S(K_n) - x_1$. Clearly $\gamma(G_2)$ = 2 with a $\gamma - set$ $\{x_i, x_i'\}$ for any i, $2 \leq i \leq n$. Hence ψ = $\{G_1, G_2\}$ is an ADD for $S(K_n)$. ■

For example, the graph $S(K_4)$ with ADD graphs G_1 and G_2 are illustrated in Figure 12.



$S(K_4)$

Here the edges of the graph G_1 are drawn with dotted lines.

Figure 12

Theorem 2.6: *For every $m, n \geq 2$, the splitting graph of $K_{m,n}$ admits ADD.*

Proof: Let $G \cong S(K_{m,n})$ be the splitting graph of $K_{m,n}$. Let V(G) = X ∪ Y be the bipartition of G, with $|X| = 2m$ and $|Y|$ = 2n, such that X = $\{x_1, x_2, ..., x_m, x_1', x_2', ..., x_m'\}$ and Y = $\{y_1, y_2, ..., y_n, y_1', y_2', ..., y_n'\}$. Let x_i', y_i', $1 \leq i \leq n$ be the newly added vertices corresponding to x_i and y_i respectively in $S(K_{m,n})$. Let G_1 be the edge induced subgraph induced by the edges incident with x_1. Then, $G_1 \cong K_{1,2n}$ and $\gamma(G_1) = 1$. Also G_2 = $S(K_{m,n}) - x_1$. Clearly $\gamma(G_2)$ = 2 with a $\gamma - set$ $\{x_i, y_j\}$ for any i, j, $2 \leq i \leq m, 1 \leq j \leq n$. Hence ψ = $\{G_1, G_2\}$ is an ADD for $S(K_{m,n})$.

As an illustration, the graph $S(K_{2,3})$ with ADD graphs G_1 and G_2 are given in Figure 13.

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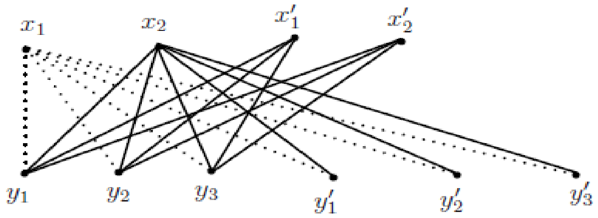


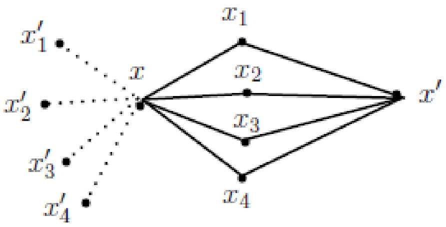
$S(K_{2,3})$

Here the edges of the graph G_1 *are drawn with dotted lines.*

Figure 13

Corollary 2.7: *For every* $n \geq 2$, *the splitting graph of a star, that is* $S(K_{1,n})$ *admits ADD.*

The ADD of the graph $S(K_{1,4})$ is shown in Figure 14.



$S(K_{1,4})$

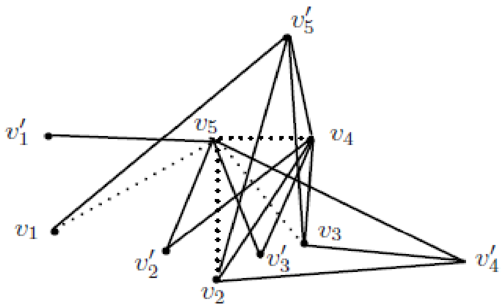
Here the edges of the graph G_1 *are drawn with dotted lines.*

Figure 14

Theorem 2.8: *For every* $n \geq 3$, *the splitting graph of* I_n *admits ADD.*

Proof: Let $G \cong S(I_n)$ be the splitting graph of I_n. Let $V(I_n) = \{v_1, v_2, ..., v_n\}$ and $E(I_n) = \{v_{n+1-i}\, v_j \ / \ 1 \leq i \leq \lfloor n/2 \rfloor,\ i \leq j \leq n-i\}$. Let v_i', $1 \leq i \leq n$ be the newly added vertices corresponding to v_i in $S(I_n)$. Now $|V(G)| = 2n$. Let G_1 be the edge induced subgraph induced by the edge set $\{v_n v_i \ ,\ 1 \leq i \leq n-1\}$. Then, $G_1 \cong K_{1,n-1}$ and $\gamma(G_1) = 1$. Also $G_2 = S(I_n) - E(G_1)$. Clearly $\gamma(G_2) = 2$ with a $\gamma - set$ $\{v_n, v_n'\}$. Hence $\psi = \{G_1, G_2\}$ is an ADD for $S(I_n)$.

As an illustration, the graph $S(I_5)$ with ADD graphs G_1 and G_2 are shown in Figure 15.



$S(I_5)$

Here the edges of the graph G_1 *are drawn with dotted lines.*

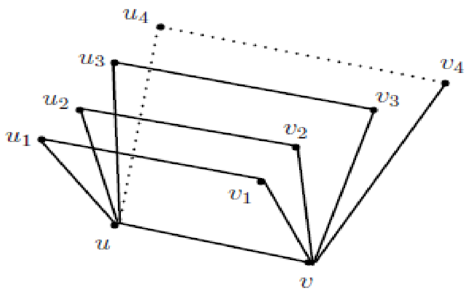
Figure 15

Theorem 2.9: *For every* $n \geq 2$, *the book graph* B_n *admits ADD.*

Proof: Let $G \cong B_n$ be the n-Book graph. Let $V(G) = \{u, v, u_i, v_i \ / \ 1 \leq i \leq n\}$ and $|V(G)| = 2n+2$. Let $E(G) = \{uv, uu_i, vv_i, u_i v_i \ / \ 1 \leq i \leq n\}$. Let G_1 be the edge induced subgraph induced by the edges incident with u_n. Then,

$G_1 \cong K_{1,2}$ and $\gamma(G_1) = 1$. Also $G_2 = G - u_n$. Clearly $\gamma(G_2) = 2$ with a $\gamma - set$ $\{u, v\}$. Hence $\psi = \{G_1, G_2\}$ is an ADD for G

For example, the book graph B_4 with ADD graphs G_1 and G_2 are given in Figure 16.



B_4

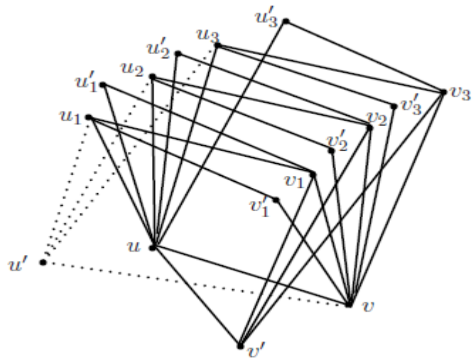
Here the edges of the graph G_1 are drawn with dotted lines.

Figure 16

Theorem 2.10: *For every $n \geq 1$, the splittting of book graph B_n admits ADD.*

Proof: Let $G \cong S(B_n)$ be the splitting of n-Book graph. Let $V(B_n) = \{u, v, u_i, v_i\}$ and $E(G) = \{uv, uu_i, vv_i, u_iv_i$ / $1 \le i \le n\}$. Let u_i', v_i', $1 \le i \le n$ be the newly added vertices corresponding to u_i and v_i respectively in $S(B_n)$. Let G_1 be the edge induced subgraph induced by the edges incident with u'. Then, $G_1 \cong K_{1,n+1}$ and $\gamma(G_1) = 1$. Also $G_2 = G - u'$. Clearly $\gamma(G_2) = 2$ with a $\gamma - set$ $\{u, v\}$. Hence $\psi = \{G_1, G_2\}$ is an ADD for G.

As an illustration, the book graph $S(B_3)$ with ADD graphs G_1 and G_2 are shown in Figure 17.



$S(B_3)$

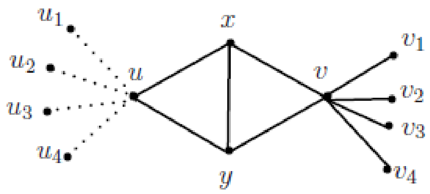
Here the edges of the graph G_1 are drawn with dotted lines.

Figure 17

Theorem 2.11: *For every $m, n \geq 0$, the Jelly fish graph $J_{m,n}$ admits ADD.*

Proof: Let $G \cong J_{m,n}$ where $J_{m,n}$ is the Jelly fish graph. Let V(G) = $\{x, y, u, v, u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ and $E(G) = \{xy, xu, xv, uu_i, vv_i$ / $1 \le i \le n\}$. When $m = n = 0$, then $J_{m,n} \cong K_4 \setminus e$. Take $G_1 = xy$. Therefore $\gamma(G_1) = 1$. Clearly $G_2 = J_{0,0} - xy$ and $\gamma(G_2) = 2$ with a $\gamma - set$ $\{u, v\}$. When $m, n > 0$, then G_1 be the edge induced subgraph induced by the pendant edges incident with u. Then, $G_1 \cong K_{1,m}$ and $\gamma(G_1) = 1$. Also $G_2 = G - E(G_1)$. Clearly $\gamma(G_2) = 2$ with a $\gamma - set$ $\{u, v\}$. Hence $\psi = \{G_1, G_2\}$ is an ADD for G.

For example, the graph $J_{4,4}$ with ADD graphs G_1 and G_2 are illustrated in Figure 18.

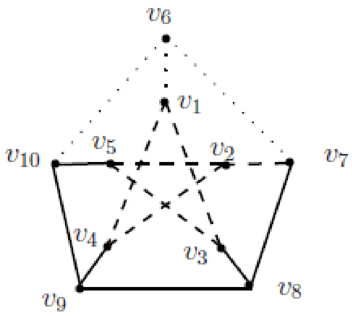


$J_{4,4}$

Here the edges of the graph G_1 *are drawn with dotted lines.*

Figure 18

One can easily note that the Petersen graph admits ADD which is shown in Figure 19.



G

Here the edges of the graph G_1 are drawn with dotted lines and that of G_2 are drawn with dashed lines.

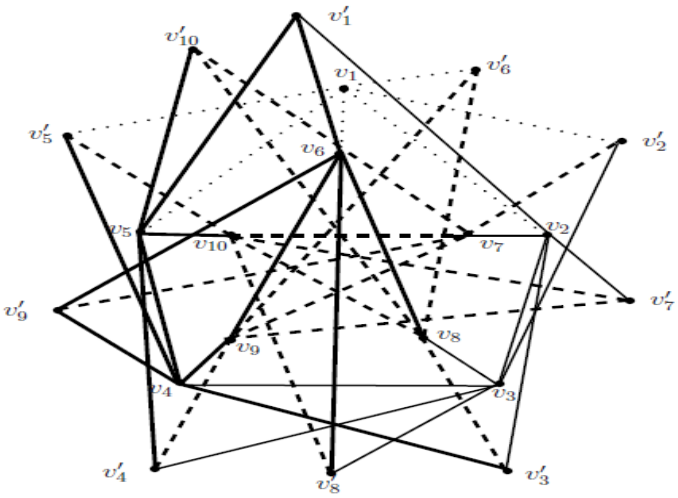
Figure 19

Also, the splitting of the Petersen graph admits ADD which is proved in next theorem.

Theorem 2.12: *S(G) admits ADD, where S(G) is the splitting graph of the Petersen graph.*

Proof: Let G be a Petersen graph and $S(G)$ be its splitting graph. let $V(S(G))$ = $\{v_1, v_2, ..., v_{10}, v_1', v_2', ..., v_{10}'\}$ such that v_i' is the corresponding newly added vertex for v_i. Let G_1 be the edge induced subgraph induced by the edges incident with v_1. Then, $G_1 \cong K_{1,6}$ and $\gamma(G_1)$ = 1. Let G_2 be the edge induced subgraph induced by the edges incident with v_2 and v_3 except the edge v_2v_1. Then, $G_2 \cong B_{4,5}$ and $\gamma(G_2)$ = 2. Let G_3 be the edge induced subgraph induced by the edges incident with v_4 , v_5 and v_6 except the edges v_4v_3, v_5v_1, v_6v_1 . Therefore $\gamma(G_3) = 3$. Clearly, $G_4 = S(G) - E(G_1, G_2, G_3)$. Then, $\gamma(G_4)$ = 4 with a $\gamma - set$ $\{v_7, v_8, v_9, v_{10}\}$. Hence ψ = $\{G_1, G_2, G_3, G_4\}$ is an ADD for $S(G)$.

As an illustration, the splitting of the Petersen graph with ADD graphs G_1, G_2, G_3 and G_4 are given in Figure 20.



$S(G)$

Here the edges of the graph G_1, G_2 and G_3 are drawn with dotted lines, slim lines and bold lines respectively

Figure 20

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