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COVARIANCE OF CUBIC SETS

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Abstract:

For given cubic set $\mathcal{A}$, the set $\mathcal{C}(\mathcal{A})$ of all cubic sets $\mathcal{C}(\mathcal{A}) = \{\mathcal{A}^* | \mathcal{A}\mathcal{A}^* = \mathcal{A}^*\mathcal{A}\}$ is the covariance of cubic set of $\mathcal{A}$. The relation between the covariance cubic sets of union, intersection and complement are discussed.

Index Terms: Cubic Set, Internal Cubic Set & External Cubic Set.

Introduction:

In 1965, Zadeh [5] made an extension of the concept of a fuzzy set by an interval-valued fuzzy set. In traditional fuzzy logic, the experts degree of certainity in different statements were given by numbers from the interval [0,1]. It is often difficult for an expert to exactly quantify his or her certainity. Therefore, instead of a real number, it is more adequate to represent this degree of certainity by an interval or even by a fuzzy set. Interval-valued fuzzy sets have been actively used in real-life applications. Cubic sets were discussed by Jun.et.al [3]. Some characterizations on operations on soft sets, Cubic ideals of γ -semigroups were by Chinnadurai.et.al [1, 2].

In this paper, using a fuzzy set, interval-valued fuzzy set and * cubic set, we introduce a new notion called a (interval, external) covariance cubic set and investigate several related properties.

Definition: 1.1 Let X be a nonempty set. A fuzzy set $\mathcal{A}$ in X is characteristic by its membership function (generalized characteristic function).

$$\mu_A : X \to [0,1]$$

and μ_A(x) is interpreted as its degree of membership elements x in a fuzzy set A for each x$\in X$. i.e. [0,1] is the subset of nonnegative real numbers whose supremum is finite.

Definition: 1.2 Let X be any set. A mapping $\overline{A} : X \to D[0,1]$ is called an interval- valued fuzzy subset (briefly, i-v fuzzy subset) of X, where D[0,1] denotes the family of all closed subintervals of [0, 1] and $\overline{A}$ (x) = [A^-(x), A^+(x)] for all x$\in X$, where A^- and A^+ are fuzzy subsets of X such that $A^-(x) \leq A^+(x)$ for all x$\in X$.

Definition: 1.3 Let X be a nonempty set. A cubic set $\mathcal{A}$ and $\mathcal{B}$ is a structure of the form

$$\mathcal{A} = \{< x, A(x), \lambda(x)>| x \in X\} \text{ and } \mathcal{B} = \{< x, B(x), \mu(x)>| x \in X\}$$

Then by define * property

$$\mathcal{A}^* = \{< x, A(x), \mu(x)>| x \in X\} \text{ and } \mathcal{B}^* = \{< x, B(x), \lambda(x)>| x \in X\}$$

in which A and B is an interval valued fuzzy set (IVF) in X and λ and μ is a fuzzy set in X.

A cubic set $\mathcal{A}^* = \{< x, A(x), \mu(x)>| x \in X\}$ is simply denoted by $\mathcal{A}^*$ = <A, μ> and $\mathcal{B}^* = \{< x, B(x), \lambda(x)>| x \in X\}$ is simply denoted by $\mathcal{B}^*$= <B, λ>.

Definition: 1.4 Let X be a nonempty set. A cubic set $\mathcal{A}^*$ = <A, μ> and $\mathcal{B}^*$ = <B, λ> in X said to be internal cubic set (briefly, ICS) if $A^-(x) \leq \mu(x) \leq A^+(x)$ and $B^-(x) \leq \lambda(x) \leq B^+(x)$ for all $x \in X$.

Definition: 1.5 Let X be a nonempty set. A cubic set $\mathcal{A}^*$ = <A, μ> and $\mathcal{B}^*$ = <B, λ> in X said to be external cubic set (briefly, ECS) if $\mu(x) \notin (A^-(x), A^+(x))$ and $\lambda(x) \notin (B^-(x), B^+(x))$ for all $x \in X$.

Definition: 1.6 For a family $\{A_i \,|\, i \in \wedge\}$ of IVF sets in X where $\wedge$ is an index set, the union G = $\bigcup_{i\in\wedge} A_i$ and the intersection F = $\bigcap_{i\in\wedge} A_i$ are defined as follows:

$$G(x) = (\bigcup_{i\in\wedge} A_i)(x) = r\sup_{i\in\wedge} A_i(x) \text{ and,}$$

$$F(x) = (\bigcap_{i\in\wedge} A_i)(x) = r\inf_{i\in\wedge} A_i(x) \text{ for all } x \in X.$$

Definition: 1.7 Let X be a nonempty set. Let $\mathcal{A}$ = < A, λ > and $\mathcal{B}$ = <B, μ > be two cubic sets of X. Then $\mathcal{A}^*$ = <A, μ > and $\mathcal{B}^*$ = <B, λ> be also cubic sets. The class of all cubic sets commutes with $\mathcal{A}^*$,

$$\mathcal{C}(\mathcal{A}) = \{\mathcal{A}^* \,|\, \mathcal{A}\mathcal{A}^* = \mathcal{A}^*\mathcal{A}\}$$

is called covariance cubic set.

Example: 1.8 Let X be a nonempty set. A cubic set $\mathcal{A}$ and $\mathcal{B}$ is defined as

Case 1; $\lambda < \mu$

$\mathcal{A}$ = {[0.3, 0.5], 0.4} and $\mathcal{B}$ = {[0.4, 0.6], 0.5} then $\mathcal{A}^*$ = {[0.3, 0.5], 0.5} and $\mathcal{B}^*$ = {[0.4, 0.6], 0.4}. Hence,

$$\mathcal{A}\mathcal{A}^* = \{[0.3, 0.5], 0.5\}$$
$$\mathcal{A}^*\mathcal{A} = \{[0.3, 0.5], 0.5\}$$

Hence, $\mathcal{C}(\mathcal{A}) = \{\mathcal{A}^* \mid \mathcal{A}\mathcal{A}^* = \mathcal{A}^*\mathcal{A}\}$.

Case 2; $\lambda > \mu$

$\mathcal{A}$ = {[0.6, 0.8], 0.7} and $\mathcal{B}$ = {[0.4, 0.5], 0.5} then $\mathcal{A}^*$ = {[0.6, 0.8], 0.5} and $\mathcal{B}^*$ = {[0.4, 0.5], 0.7}. Hence,

$$\mathcal{A}\mathcal{A}^* = \{[0.6, 0.8], 0.7\}$$
$$\mathcal{A}^*\mathcal{A} = \{[0.6, 0.8], 0.7\}$$

Hence, $\mathcal{C}(\mathcal{A}) = \{\mathcal{A}^* \mid \mathcal{A}\mathcal{A}^* = \mathcal{A}^*\mathcal{A}\}$.

Definition: 1.9 Let X be a nonempty set. Let $\mathcal{A}$ = < A, λ > and $\mathcal{B}$ = <B, μ > be two cubic sets of X. Then $\mathcal{A}^*$ = <A, μ > and $\mathcal{B}$* = <B, λ> be also cubic sets. The class of all cubic sets commutes with $\mathcal{A}^*$,

$$\mathcal{C}(\mathcal{A})^c = \{\mathcal{A}^* \mid (\mathcal{A}\mathcal{A}^*)^c = (\mathcal{A}^*\mathcal{A})^c\}$$

is called complement of covariance cubic set.

Example: 1.10 Let X be a nonempty set. A cubic set $\mathcal{A}$ and $\mathcal{B}$ is defined as $\mathcal{A}$ = {[0.3, 0.5], 0.4} and $\mathcal{B}$ = {[0.4, 0.6], 0.5} then * property as $\mathcal{A}^*$ = {[0.3, 0.5], 0.5} and $\mathcal{B}^*$ = {[0.4, 0.6], 0.4}. Then,

$$\mathcal{A}\mathcal{A}^* = \{[0.3, 0.5], 0.5\},\ (\mathcal{A}\mathcal{A}^*)^c = \{[0.5, 0.7], 0.5\} \text{ and}$$
$$\mathcal{A}^*\mathcal{A} = \{[0.3, 0.5], 0.5\},\ (\mathcal{A}^*\mathcal{A})^c = \{[0.5, 0.7], 0.5\}$$

Hence, $\mathcal{C}(\mathcal{A})^c = \{\mathcal{A}^* \mid (\mathcal{A}\mathcal{A}^*)^c = (\mathcal{A}^*\mathcal{A})^c\}$.

Remark: 1.11 $\mathcal{C}(\mathcal{A}) \neq \mathcal{C}(\mathcal{A}^c)$ for any $\mathcal{A}$. For example, Let X be a nonempty set. A cubic set $\mathcal{A}$ and $\mathcal{B}$ is defined as $\mathcal{A}$ = {[0.3, 0.5], 0.4} and $\mathcal{B}$ = {[0.4, 0.6], 0.5} then * property as $\mathcal{A}^*$ = {[0.3, 0.5], 0.5} and $\mathcal{B}^*$ = {[0.4, 0.6], 0.4}. Then,

$$\mathcal{C}(\mathcal{A}) = \{[0.3, 0.5], 0.5\},$$
$$\mathcal{C}(\mathcal{A})^c = \{[0.5, 0.7], 0.5\}.$$

Hence, $\mathcal{C}(\mathcal{A}) \neq \mathcal{C}(\mathcal{A})^c$.

Definition: 1.12 Let $\mathcal{A}$ = < A, λ> and $\mathcal{B}$ = < B, μ> be cubic sets. Then $\mathcal{A}$* = <A, μ> and $\mathcal{B}$* = <B, λ> is defined to be a cubic sets $\mathcal{A}^{*c}$={< x, A^c(x), 1- μ(x)>| $x \in X$ } and $\mathcal{B}^{*c}$={< x, B^c(x), 1- λ(x)>| $x \in X$ } where $A^c(x) = [1- A^+(x), 1- A^-(x)]$ and $B^c(x) = [1- B^+(x), 1- B^-(x)]$.

Result: 1.13 For any $\mathcal{A}$, $(\mathcal{C}(\mathcal{A}))^c = \mathcal{C}(\mathcal{A})^c$.

Theorem: 1.14 Let $\mathcal{A}$ and $\mathcal{B}$ be two cubic sets in X. Then $\mathcal{A}$* and $\mathcal{B}$* be also two cubic sets in X. If its satisfying the following conditions:

(1) $\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{B})$ iff $\mathcal{C}(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{B})$.
(2) $\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{A})$ iff $\mathcal{C}(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{B})$.
(3) $\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{A})$ iff $\mathcal{C}(\mathcal{A}) \supseteq \mathcal{C}(\mathcal{B})$.
(4) $\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{B})$ iff $\mathcal{C}(\mathcal{A}) \supseteq \mathcal{C}(\mathcal{B})$.

Proof:

(1) Let $x \in \mathcal{C}(\mathcal{A})$. Then $x \in \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{B})$.

Hence $\mathcal{C}(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{B})$.

Conversely, let $(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{B})$.

Let $x \in \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})$.

Since, $(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{B})$, $x \in \mathcal{C}(\mathcal{A}) \Rightarrow x \in \mathcal{C}(\mathcal{B})$.

Thus $x \in \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) \Rightarrow x \in \mathcal{C}(\mathcal{B})$.

Hence $\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) \subseteq \mathcal{C}(\mathcal{B})$.

$\mathcal{C}(\mathcal{B}) \subseteq \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})$ is always true.

Therefore $\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{B})$.

(2) Let $x \in \mathcal{C}(\mathcal{B})$. Then $x \in \mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{A})$.

Hence $\mathcal{C}(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{B})$.

Conversely, let $(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{B})$.

Let $x \in \mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})$.

Since, $(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{B})$, $x \in \mathcal{C}(\mathcal{A}) \Rightarrow x \in \mathcal{C}(\mathcal{B})$.

Thus $x \in \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) \Rightarrow x \in \mathcal{C}(\mathcal{A})$.

Hence $\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}) \subseteq \mathcal{C}(\mathcal{A})$.

$\mathcal{C}(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})$ is always true.

Therefore $\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{A})$.

(3) Let $x \in \mathcal{C}(\mathcal{B})$. Then $x \in \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{A})$.

Hence $\mathcal{C}(\mathcal{A}) \supseteq \mathcal{C}(\mathcal{B})$.

Conversely, let $(\mathcal{A}) \supseteq \mathcal{C}(\mathcal{B})$.

Let x $\in \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})$.

Since, $(\mathcal{A}) \supseteq \mathcal{C}(\mathcal{B})$, x $\in \mathcal{C}(\mathcal{A}) \Rightarrow$ x $\in \mathcal{C}(\mathcal{B})$.

Thus x $\in \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) \Rightarrow$ x $\in \mathcal{C}(\mathcal{A})$.

Hence $\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) \subseteq \mathcal{C}(\mathcal{A})$.

$\mathcal{C}(\mathcal{A}) \subseteq \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})$ is always true.

Therefore $\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{A})$.

(4) Let x $\in \mathcal{C}(\mathcal{A})$. Then x $\in \mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{B})$.

Hence $\mathcal{C}(\mathcal{A}) \supseteq \mathcal{C}(\mathcal{B})$.

Conversely, let $(\mathcal{A}) \supseteq \mathcal{C}(\mathcal{B})$.

Let x $\in \mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})$.

Since, $(\mathcal{A}) \supseteq \mathcal{C}(\mathcal{B})$, x $\in \mathcal{C}(\mathcal{A}) \Rightarrow$ x $\in \mathcal{C}(\mathcal{B})$.

Thus x $\in \mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}) \Rightarrow$ x $\in \mathcal{C}(\mathcal{A})$.

Hence $\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}) \subseteq \mathcal{C}(\mathcal{B})$.

$\mathcal{C}(\mathcal{B}) \supseteq \mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})$ is always true.

Therefore $\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}) = \mathcal{C}(\mathcal{B})$.

Hence proved.

Theorem: 1.15 Let $\mathcal{A}$ and $\mathcal{B}$ be two cubic sets in X. Then $\mathcal{A}*$ and $\mathcal{B}*$ be also two cubic sets in X. If its satisfying the following conditions:

(1) $\mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c = \mathcal{C}(\mathcal{A})^c$ iff $\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{B})^c$.

(2) $\mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c = \mathcal{C}(\mathcal{B})^c$ iff $\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{B})^c$.

(3) $\mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c = \mathcal{C}(\mathcal{B})^c$ iff $\mathcal{C}(\mathcal{A})^c \supseteq \mathcal{C}(\mathcal{B})^c$.

(4) $\mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c = \mathcal{C}(\mathcal{A})^c$ iff $\mathcal{C}(\mathcal{A})^c \supseteq \mathcal{C}(\mathcal{B})^c$.

Proof: (1) Let x $\in \mathcal{C}(\mathcal{B})^c$. Then x $\in \mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c = \mathcal{C}(\mathcal{A})^c$.

Hence $\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{B})^c$.

Conversely, let $\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{B})^c$.

Let x $\in \mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c$.

Since, $\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{B})^c$, x $\in \mathcal{C}(\mathcal{A})^c \Rightarrow$ x $\in \mathcal{C}(\mathcal{B})^c$.

Thus x $\in \mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c \Rightarrow$ x $\in \mathcal{C}(\mathcal{A})^c$.

Hence $\mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c \subseteq \mathcal{C}(\mathcal{A})^c$.

$\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c$ is always true.

Therefore $\mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c = (\mathcal{A})^c$.

(2) Let x $\in \mathcal{C}(\mathcal{A})^c$. Then x $\in \mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c = \mathcal{C}(\mathcal{B})^c$.

Hence $\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{B})^c$.

Conversely, let $\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{B})^c$.

Let x $\in \mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c$.

Since, $\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{B})^c$, x $\in \mathcal{C}(\mathcal{A})^c \Rightarrow$ x $\in \mathcal{C}(\mathcal{B})^c$.

Thus x $\in \mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c \Rightarrow$ x $\in \mathcal{C}(\mathcal{B})^c$.

Hence $\mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c \subseteq \mathcal{C}(\mathcal{B})^c$.

$\mathcal{C}(\mathcal{B})^c \subseteq \mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c$ is always true.

Therefore $\mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c = (\mathcal{B})^c$.

(3) Let x $\in \mathcal{C}(\mathcal{A})^c$. Then x $\in \mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c = \mathcal{C}(\mathcal{B})^c$.

Hence $\mathcal{C}(\mathcal{A})^c \supseteq \mathcal{C}(\mathcal{B})^c$.

Conversely, let $\mathcal{C}(\mathcal{A})^c \supseteq \mathcal{C}(\mathcal{B})^c$.

Let x $\in \mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c$.

Since, $\mathcal{C}(\mathcal{A})^c \supseteq \mathcal{C}(\mathcal{B})^c$, x $\in \mathcal{C}(\mathcal{A})^c \Rightarrow$ x $\in \mathcal{C}(\mathcal{B})^c$.

Thus x $\in \mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c \Rightarrow$ x $\in \mathcal{C}(\mathcal{B})^c$.

Hence $\mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c \subseteq \mathcal{C}(\mathcal{B})^c$.

Therefore $\mathcal{C}(\mathcal{A})^c \cup \mathcal{C}(\mathcal{B})^c = (\mathcal{B})^c$.

(4) Let x $\in \mathcal{C}(\mathcal{B})^c$. Then x $\in \mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c = \mathcal{C}(\mathcal{A})^c$.

Hence $\mathcal{C}(\mathcal{A})^c \supseteq \mathcal{C}(\mathcal{B})^c$.

Conversely, let $\mathcal{C}(\mathcal{A})^c \supseteq \mathcal{C}(\mathcal{B})^c$.

Let x $\in \mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c$.

Since, $\mathcal{C}(\mathcal{A})^c \supseteq \mathcal{C}(\mathcal{B})^c$, x $\in \mathcal{C}(\mathcal{A})^c \Rightarrow$ x $\in \mathcal{C}(\mathcal{B})^c$.

Thus x $\in \mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c \Rightarrow$ x $\in \mathcal{C}(\mathcal{A})^c$.

Hence $\mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c \subseteq \mathcal{C}(\mathcal{A})^c$.

$\mathcal{C}(\mathcal{A})^c \subseteq \mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c$ is always true.

Therefore $\mathcal{C}(\mathcal{A})^c \cap \mathcal{C}(\mathcal{B})^c = (\mathcal{A})^c$.

Hence proved.

Remark: 1.16 Union of covariance cubic set is a cubic set. Intersection of covariance cubic set is a cubic set.

Example: 1.17 Let X be a nonempty set. A cubic set $\mathcal{A}$ and $\mathcal{B}$ is defined as $\mathcal{A}$ = {[0.3, 0.5], 0.4} and $\mathcal{B}$ = {[0.5, 0.7], 0.9} then $\mathcal{A}$* = {[0.3, 0.5], 0.9} and $\mathcal{B}$* = {[0.5, 0.7], 0.4}. Hence $\mathcal{C}(\mathcal{A})$ = {[0.3, 0.5], 0.9} and $\mathcal{C}(\mathcal{A})^c$ = {[0.5, 0.7],0.1}. Therefore, $\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{A})^c$ = {[0.5, 0.7],0.9} and $\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{A})^c$ = {[0.3, 0.5],0.1}.

Theorem: 1.18 Demorgan's Law

For any three covariance of cubic sets $\mathcal{C}(\mathcal{A}), \mathcal{C}(\mathcal{B})$ and $\mathcal{C}(\mathcal{C})$ we have

(1) $\mathcal{C}(\mathcal{A}) \cup (\mathcal{C}(\mathcal{B}) \cap \mathcal{C}(\mathcal{C})) = (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})) \cap (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{C}))$.

(2) $\mathcal{C}(\mathcal{A}) \cap (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C})) = (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})) \cup (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{C}))$.

Proof: (1) Let x $\in \mathcal{C}(\mathcal{A}) \cup (\mathcal{C}(\mathcal{B}) \cap \mathcal{C}(\mathcal{C}))$.

Then x $\in \mathcal{C}(\mathcal{A})$ or x $\in (\mathcal{C}(\mathcal{B}) \cap \mathcal{C}(\mathcal{C}))$.

$\Rightarrow$ x $\in \mathcal{C}(\mathcal{A})$ or (x $\in \mathcal{C}(\mathcal{B})$ and x $\in \mathcal{C}(\mathcal{C})$).

$\Rightarrow$ (x $\in \mathcal{C}(\mathcal{A})$ or x $\in \mathcal{C}(\mathcal{B})$) and (x $\in \mathcal{C}(\mathcal{A})$ or x $\in \mathcal{C}(\mathcal{C})$).

$\Rightarrow$ x $\in (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}))$ and x $\in (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{C}))$.

$\Rightarrow$ x $\in (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})) \cap (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{C}))$.

Hence $\mathcal{C}(\mathcal{A}) \cup (\mathcal{C}(\mathcal{B}) \cap \mathcal{C}(\mathcal{C})) \subseteq (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})) \cap (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{C}))$.

Now let x $\in (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})) \cap (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{C}))$.

Then x $\in (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B}))$ and x $\in (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{C}))$.

$\Rightarrow$ (x $\in \mathcal{C}(\mathcal{A})$ or x $\in \mathcal{C}(\mathcal{B})$) and (x $\in \mathcal{C}(\mathcal{A})$ or x $\in \mathcal{C}(\mathcal{C})$).

$\Rightarrow$ x $\in \mathcal{C}(\mathcal{A})$ or (x $\in \mathcal{C}(\mathcal{B})$ and x $\in \mathcal{C}(\mathcal{C})$).

$\Rightarrow$ x $\in \mathcal{C}(\mathcal{A})$ or x $\in (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C})$

$\Rightarrow$ x $\in \mathcal{C}(\mathcal{A}) \cup (\mathcal{C}(\mathcal{B}) \cap \mathcal{C}(\mathcal{C}))$.

Hence x $\in (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})) \cap (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{C})) \subseteq \mathcal{C}(\mathcal{A}) \cup (\mathcal{C}(\mathcal{B}) \cap \mathcal{C}(\mathcal{C}))$.

Therefore $\mathcal{C}(\mathcal{A}) \cup (\mathcal{C}(\mathcal{B}) \cap \mathcal{C}(\mathcal{C})) = (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{B})) \cap (\mathcal{C}(\mathcal{A}) \cup \mathcal{C}(\mathcal{C}))$.

(2) Let x $\in \mathcal{C}(\mathcal{A}) \cap (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C}))$.

Then x $\in \mathcal{C}(\mathcal{A})$ or x $\in (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C}))$.

$\Rightarrow$ x $\in \mathcal{C}(\mathcal{A})$ and (x $\in \mathcal{C}(\mathcal{B})$ or x $\in \mathcal{C}(\mathcal{C})$).

$\Rightarrow$ (x $\in \mathcal{C}(\mathcal{A})$ and x $\in \mathcal{C}(\mathcal{B})$) or (x $\in \mathcal{C}(\mathcal{A})$ and x $\in \mathcal{C}(\mathcal{C})$).

$\Rightarrow$ x $\in (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}))$ or x $\in (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{C}))$.

$\Rightarrow$ x $\in (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})) \cup (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{C}))$.

Hence $\mathcal{C}(\mathcal{A}) \cap (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C})) \subseteq (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})) \cup (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{C}))$.

Now let x $\in (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})) \cup (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{C}))$.

Then x $\in (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B}))$ and x $\in (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{C}))$.

$\Rightarrow$ (x $\in \mathcal{C}(\mathcal{A})$ and x $\in \mathcal{C}(\mathcal{B})$) or (x $\in \mathcal{C}(\mathcal{A})$ and x $\in \mathcal{C}(\mathcal{C})$).

$\Rightarrow$ x $\in \mathcal{C}(\mathcal{A})$ and (x $\in \mathcal{C}(\mathcal{B})$ or x $\in \mathcal{C}(\mathcal{C})$).

$\Rightarrow$ x $\in \mathcal{C}(\mathcal{A})$ and x $\in (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C})$

$\Rightarrow$ x $\in \mathcal{C}(\mathcal{A}) \cap (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C}))$.

Hence x $\in (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})) \cup (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{C})) \subseteq \mathcal{C}(\mathcal{A}) \cap (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C}))$.

Therefore $\mathcal{C}(\mathcal{A}) \cap (\mathcal{C}(\mathcal{B}) \cup \mathcal{C}(\mathcal{C})) = (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{B})) \cup (\mathcal{C}(\mathcal{A}) \cap \mathcal{C}(\mathcal{C}))$.

Hence proved.

Definition: 1.19 Given cubic set $\mathcal{A}$, this exists cubic set $\mathcal{B}$ such that $(\mathcal{B}\mathcal{A}\mathcal{B}^c)^c \neq \mathcal{B}\mathcal{A}\mathcal{B}^c$, then $\mathcal{A}$ is called met covariance under $\mathcal{C}(\mathcal{A})$. i.e,

$$\mathcal{C}(\mathcal{A}) = \{ \mathcal{B} \mid (\mathcal{B}\mathcal{A}\mathcal{B}^c)^c \neq \mathcal{B}\mathcal{A}\mathcal{B}^c \}$$

Example: 1.20 Let $\mathcal{A}$= {[0.3,0.5],0.4} and $\mathcal{B}$ = {[0.4,0.6],0.5} be a cubic sets. Let $\mathcal{A}$ = {[0.3,0.5],0.5} and $\mathcal{B}^*$ = {[0.4,0.6],0.4} be also a cubic sets. Then $\mathcal{A}$ = {[0.5,0.7],0.6}, $\mathcal{B}^c$ = {[0.4,0.6],0.5}, $\mathcal{A}^c$ = {[0.5,0.7],0.5} and $\mathcal{B}^{*c}$ = {[0.4,0.6],0.6} be a cubic sets. Therefore,

$$(\mathcal{B}\mathcal{A}\mathcal{B}^c)^c \neq \mathcal{B}\mathcal{A}\mathcal{B}^c$$

Example: 1.21 Let $\mathcal{A}$= {[0.2,0.5],0.9} and $\mathcal{B}$ = {[0.3,0.7],0.8} be a cubic sets. Let $\mathcal{A}$ = {[0.2,0.5],0.8} and $\mathcal{B}^*$ = {[0.3,0.7],0.9} be also a cubic sets. Then $\mathcal{A}$ = {[0.5,0.8],0.1}, $\mathcal{B}^c$ = {[0.3,0.7],0.2}, $\mathcal{A}^c$ = {[0.5,0.7],0.2} and $\mathcal{B}^{*c}$ = {[0.3,0.7],0.1} be a cubic sets. Therefore,

$$(\mathcal{B}\mathcal{A}\mathcal{B}^c)^c \neq \mathcal{B}\mathcal{A}\mathcal{B}^c$$

Definition: 1.22 Let $\mathcal{A}$ and $\mathcal{B}$ be a cubic sets. Also $\mathcal{A}$ and $\mathcal{B}^*$ be a cubic sets. Then we define a form of structure ,

$$G(\mathcal{A}) = \{\mathcal{B} \mid \mathcal{B} \text{ commutes with } \mathcal{A}\mathcal{A} \text{ and } \mathcal{A}\mathcal{A}\}$$

Therefore, $\mathcal{B}\mathcal{A}\mathcal{A} = \mathcal{A}\mathcal{A}\mathcal{B}$ and $\mathcal{B}\mathcal{A}\mathcal{A} = \mathcal{A}\mathcal{A}\mathcal{B}$.

Example: 1.23 Let $\mathcal{A}$= {[0.2,0.5],0.9} and $\mathcal{B}$ = {[0.3,0.7], 0.8} be a cubic sets. Let $\mathcal{A}$ = {[0.2,0.5], 0.8} and $\mathcal{B}^*$ = {[0.3,0.7],0.9} be also a cubic sets. Then $\mathcal{A}$ = {[0.5,0.8],0.1}, $\mathcal{B}^c$ = {[0.3,0.7],0.2}, $\mathcal{A}^c$ = {[0.5,0.7],0.2} and $\mathcal{B}^{*c}$ = {[0.3,0.7],0.1} be a cubic sets. Therefore,

$$\mathcal{B}\mathcal{A}\mathcal{A} = \mathcal{A}\mathcal{A}\mathcal{B} \Rightarrow \{[0.2,0.5],0.8\} = \{[0.2,0.5],0.8\},$$
$$\mathcal{B}\mathcal{A}\mathcal{A} = \mathcal{A}\mathcal{A}\mathcal{B} \Rightarrow \{[0.2,0.5],0.8\} = \{[0.2,0.5],0.8\}$$

Example: 1.24 Let $\mathcal{A}$= {[0.3,0.5],0.4} and $\mathcal{B}$ = {[0.4,0.6], 0.5} be a cubic sets. Let $\mathcal{A}$ = {[0.3,0.5], 0.5} and $\mathcal{B}^*$ = {[0.4,0.6],0.4} be also a cubic sets. Then $\mathcal{A}$ = {[0.5,0.7],0.6}, $\mathcal{B}^c$ = {[0.4,0.6],0.5}, $\mathcal{A}^c$ = {[0.5,0.7],0.5} and $\mathcal{B}^{*c}$ = {[0.4,0.6],0.6} be a cubic sets. Therefore,

$$\mathcal{B}\mathcal{A}\mathcal{A} = \mathcal{A}\mathcal{A}\mathcal{B} \Rightarrow \{[0.3,0.5],0.4\} = \{[0.3,0.5],0.4\},$$
$$\mathcal{B}\mathcal{A}\mathcal{A} = \mathcal{A}\mathcal{A}\mathcal{B} \Rightarrow \{[0.3,0.5],0.5\} = \{[0.3,0.5],0.4\}$$

Definition: 1.25 Let $\mathcal{A}$ and $\mathcal{B}$ be a cubic sets. Also $\mathcal{A}$ and $\mathcal{B}^*$ be a cubic sets. Then we define a form of structure,

$$G(\mathcal{A}) = \{\mathcal{B} \mid \mathcal{B} \text{ commutes with } (\mathcal{A}\mathcal{A})^c \text{ and } (\mathcal{A}\mathcal{A})^c\}.$$

Therefore, $\mathcal{B}(\mathcal{A}\mathcal{A})^c = (\mathcal{A}\mathcal{A})^c\mathcal{B}$ and $\mathcal{B}(\mathcal{A}\mathcal{A})^c = (\mathcal{A}\mathcal{A})^c\mathcal{B}$.

Example: 1.26 Let $\mathcal{A}$= {[0.2,0.5],0.9} and $\mathcal{B}$ = {[0.3,0.7], 0.8} be a cubic sets. Let $\mathcal{A}$ = {[0.2,0.5], 0.8} and $\mathcal{B}^*$ = {[0.3,0.7],0.9} be also a cubic sets. Then $\mathcal{A}$ = {[0.5,0.8],0.1}, $\mathcal{B}^c$ = {[0.3,0.7],0.2}, $\mathcal{A}^c$ = {[0.5,0.7],0.2} and $\mathcal{B}^{*c}$ = {[0.3,0.7],0.1} be a cubic sets. Therefore,

$$\mathcal{B}(\mathcal{A}\mathcal{A})^c = (\mathcal{A}\mathcal{A}^*)^c\mathcal{B} \Rightarrow \{[0.7,0.8],0.8\} = \{[0.7,0.8],0.8\},$$
$$\mathcal{B}(\mathcal{A}\mathcal{A})^c = (\mathcal{A}\mathcal{A})^c\mathcal{B} \Rightarrow \{[0.7,0.8],0.8\} = \{[0.7,0.8],0.8\}$$

Example: 1.27 Let $\mathcal{A}$= {[0.3,0.5],0.4} and $\mathcal{B}$ = {[0.4,0.6], 0.5} be a cubic sets. Let $\mathcal{A}$ = {[0.3,0.5], 0.5} and $\mathcal{B}^*$ = {[0.4,0.6],0.4} be also a cubic sets. Then $\mathcal{A}$ = {[0.5,0.7],0.6}, $\mathcal{B}^c$ = {[0.4,0.6],0.5}, $\mathcal{A}^c$ = {[0.5,0.7],0.5} and $\mathcal{B}^{*c}$ = {[0.4,0.6],0.6} be a cubic sets. Therefore,

$$\mathcal{B}(\mathcal{A}\mathcal{A})^c = (\mathcal{A}\mathcal{A}^*)^c\mathcal{B} \Rightarrow \{[0.5,0.7],0.5\} = \{[0.5,0.7],0.5\},$$
$$\mathcal{B}(\mathcal{A}\mathcal{A})^c = (\mathcal{A}\mathcal{A})^c\mathcal{B} \Rightarrow \{[0.5,0.7],0.5\} = \{[0.5,0.7],0.5\}$$

Definition: 1.28 Let X be a nonempty set. Let $\mathcal{A}$ and $\mathcal{B}$ be a cubic sets of X. Also $\mathcal{A}$ and $\mathcal{B}^*$ be a cubic sets of X. Then we define a form of structure,

(1) $\mathcal{T}(\mathcal{A}) = \{x \mid \mathcal{A}\mathcal{A}\mathcal{A}(x) = \mathcal{A}(x), x \in X\}$ if $\lambda < \mu$.

(2) $\mathcal{R}(\mathcal{A}) = \{x \mid \mathcal{A}\mathcal{A}\mathcal{A}(x) = \mathcal{A}(x), x \in X\}$ if $\lambda > \mu$.

Remark: 1.29 $\mathcal{T}(\mathcal{A}) = \mathcal{R}(\mathcal{A})$ when $\lambda = \mu$.

Example: 1.30 Let $\mathcal{A}$ and $\mathcal{B}$ be a cubic sets in X. Also $\mathcal{A}$ and $\mathcal{B}^*$ be a cubic sets in X. If A (x) = [0.3,0.5], λ (x) = 0.4, B (x) = [0.4,0.6] and μ (x) = 0.5. Then $\mathcal{T}(\mathcal{A}) = \{x \mid \mathcal{A}\mathcal{A}\mathcal{A}(x) = \mathcal{A}(x), x \in X\}$ if $\lambda < \mu$.

Example: 1.31 Let $\mathcal{A}$ and $\mathcal{B}$ be a cubic sets in X. Also $\mathcal{A}$ and $\mathcal{B}^*$ be a cubic sets in X. If A (x) = [0.2,0.5], λ (x) = 0.9, B (x) = [0.3,0.7] and μ (x) = 0.8. Then $\mathcal{R}(\mathcal{A}) = \{x \mid \mathcal{A}\mathcal{A}\mathcal{A}(x) = \mathcal{A}(x), x \in X\}$ if $\lambda > \mu$.

Definition: 1.32 Let X be a nonempty set. Let $\mathcal{A}$ and $\mathcal{B}$ be a cubic sets of X. Also $\mathcal{A}$ and $\mathcal{B}^*$ be a cubic sets of X. Then we define a form of structure,

(1) $\mathcal{T}(\mathcal{A})^c = \{x \mid (\mathcal{A}\mathcal{A}\mathcal{A})^c(x) = \mathcal{A}^c(x), x \in X\}$ if $\lambda < \mu$.

(2) $\mathcal{R}(\mathcal{A})^c = \{x \mid (\mathcal{A}\mathcal{A}\mathcal{A})^c(x) = \mathcal{A}(x), x \in X\}$ if $\lambda > \mu$.

Example: 1.33 Let $\mathcal{A}$ and $\mathcal{B}$ be a cubic sets in X. Also $\mathcal{A}$ and $\mathcal{B}^*$ be a cubic sets in X. If A (x) = [0.3,0.5], λ (x) = 0.4, B (x) = [0.4,0.6] and μ (x) = 0.5. Then $\mathcal{T}(\mathcal{A})^c = \{x \mid (\mathcal{A}\mathcal{A}\mathcal{A})^c(x) = \mathcal{A}^c(x), x \in X\}$ if $\lambda < \mu$.

Example: 1.34 Let $\mathcal{A}$ and $\mathcal{B}$ be a cubic sets in X. Also $\mathcal{A}$ and $\mathcal{B}^*$ be a cubic sets in X. If A (x) = [0.2,0.5], λ (x) = 0.9, B (x) = [0.3,0.7] and μ (x) = 0.8. Then $\mathcal{R}(\mathcal{A})^c = \{x \mid (\mathcal{A}\mathcal{A}\mathcal{A})^c(x) = \mathcal{A}(x), x \in X\}$ if $\lambda > \mu$.

Theorem: 1.35 Let $\mathcal{A}$= < A, λ> and B = < B, μ> be a cubsic set in X. Also let $\mathcal{A}^*$ = < A, μ>, $\mathcal{B}^*$ = < B, λ> be a cubic set in X . If $\mathcal{T}(\mathcal{A})$ is an ICS, then $\mathcal{T}(\mathcal{A})^c$ is an ICS.

Proof: Since $\mathcal{A}$= < A, λ> is an ICS in X, we have $A^-(x) \le \lambda(x) \le A^+(x)$ for all $x \in X$. This implies that $1-A^+(x) \le 1-\lambda(x) \le 1-A^-(x)$. Since $\mathcal{A}^*$ = < A, μ> is an ICS in X, we have $A^-(x) \le \mu(x) \le A^+(x)$ for all $x \in X$. This implies that $1-A^+(x) \le 1-\mu(x) \le 1-A^-(x)$. By using the definition $\mathcal{T}(\mathcal{A})^c = \{x \mid (\mathcal{A}\mathcal{A}\mathcal{A})^c(x) = \mathcal{A}^c(x), x \in X\}$. Hence $\mathcal{T}(\mathcal{A})^c$ is an ICS in X.

Theorem: 1.36 Let $\mathcal{A}$ = < A, λ> and $\mathcal{A}$= < B, μ> be a cubic set in X. Also let $\mathcal{A}^*$ = < A, μ>, $\mathcal{B}^*$ = < B, λ> be a cubic set in X . If $\mathcal{T}(\mathcal{A})$ is an ECS, then $\mathcal{T}(\mathcal{A})^c$ is an ECS.

Proof: Since $\mathcal{A}$= < A, λ> is an ECS in X, we have $\lambda(x) \notin (A^-(x), A^+(x))$ for all $x \in X$. This implies that $1-\lambda(x) \notin (1-A^+(x), 1-A^-(x))$.Since $\mathcal{A}^*$ = < A, μ> is an ECS in X, we have $\mu(x) \notin (A^-(x), A^+(x))$ for all $x \in X$. This implies that $1-\mu(x) \notin (1-A^+(x), 1-A^-(x))$. By using the definition, $\mathcal{T}(\mathcal{A})^c = \{x \mid (\mathcal{A}\mathcal{A}\mathcal{A})^c(x) = \mathcal{A}^c(x), x \in X\}$Hence $\mathcal{T}(\mathcal{A})^c$ is an ECS.

Theorem: 1.37 Let $\mathcal{A}$= < A, λ> and B = < B, μ> be a cubic set in X. Also let $\mathcal{A}$* = < A, μ>, $\mathcal{B}$* = < B, λ> be a cubic set in X . If $\mathcal{R}(\mathcal{A})$ is an ICS, then $\mathcal{R}(\mathcal{A})^c$ is an ICS.

Proof: Since $\mathcal{A}$= < A, λ> is an ICS in X, we have $A^-(x) \leq \lambda(x) \leq A^+(x)$ for all x ∈ X. This implies that $1-A^+(x) \leq 1-\lambda(x) \leq 1-A^-(x)$. Since $\mathcal{A}$* = < A, μ> is an ICS in X, we have $A^-(x) \leq \mu(x) \leq A^+(x)$ for all x ∈ X. This implies that $1-A^+(x) \leq 1-\mu(x) \leq 1-A^-(x)$. By using the definition $\mathcal{R}(\mathcal{A})^c = \{ x| (\mathcal{AA}\mathcal{A})^c(x) = \mathcal{A}(x), x \in X \}$. Hence $\mathcal{R}(\mathcal{A})^c$ is an ICS in X.

Theorem: 1.38 Let $\mathcal{A}$ = < A, λ> and $\mathcal{A}$= < B, μ> be a cubic set in X. Also let $\mathcal{A}$* = < A, μ>, $\mathcal{B}$* = < B, λ> be a cubic set in X . If $\mathcal{R}(\mathcal{A})$ is an ECS, then $\mathcal{R}(\mathcal{A})^c$ is an ECS.

Proof: Since $\mathcal{A}$= < A, λ> is an ECS in X, we have $\lambda(x) \notin (A^-(x), A^+(x))$ for all x ∈ X. This implies that $1-\lambda(x) \notin (1-A^+(x), 1-A^-(x))$. Since $\mathcal{A}$* = < A, μ> is an ECS in X, we have $\mu(x) \notin (A^-(x), A^+(x))$ for all x ∈ X. This implies that $1-\mu(x) \notin (1-A^+(x), 1-A^-(x))$. By using the definition, $\mathcal{R}(\mathcal{A})^c = \{ x| (\mathcal{AA}\mathcal{A})^c(x) = \mathcal{A}(x), x \in X \}$. Hence $\mathcal{R}(\mathcal{A})^c$ is an ECS.

Theorem: 1.39 Let $\mathcal{A}$= < A, λ> and B = < B, μ> be a cubic set in X. Also let $\mathcal{A}$* = < A, μ>, $\mathcal{B}$* = < B, λ> be a cubic set in X . Then $(\mathcal{T}(\mathcal{A})^c)^c = \mathcal{T}(\mathcal{A})$.

Proof: Let $\mathcal{T}(\mathcal{A})^c = \{ x| (\mathcal{AA}\mathcal{A})^c(x) = \mathcal{A}^c(x), x \in X \}$.

$$(\mathcal{T}(\mathcal{A})^c)^c = \{ x | ((\mathcal{AA}\mathcal{A})^c(x))^c = (\mathcal{A}^c(x))^c, x \in X \}$$
$$= \{ x |((1- A^+(x), 1- A^-(x))^c = ((1- A^+(x), 1- A^-(x))^c, x \in X \}$$
$$= \{ x | (A^-(x), A^+(x)) = (A^-(x), A^+(x)), x \in X \}$$
$$= \{ x| \mathcal{AA}\mathcal{A}(x) = \mathcal{A}(x), x \in X \}$$
$$(\mathcal{T}(\mathcal{A})^c)^c = \mathcal{T}(\mathcal{A}).$$

Theorem: 1.40 Let $\mathcal{A}$= < A, λ> and B = < B, μ> be a cubic set in X. Also let $\mathcal{A}$* = < A, μ>, $\mathcal{B}$* = < B, λ> be a cubic set in X . Then $(\mathcal{R}(\mathcal{A})^c)^c = \mathcal{R}(\mathcal{A})$.

Proof: Let $\mathcal{R}(\mathcal{A})^c = \{ x| (\mathcal{AA}\mathcal{A})^c(x) = \mathcal{A}(x), x \in X \}$

$$(\mathcal{R}(\mathcal{A})^c)^c = \{ x | ((\mathcal{AA}\mathcal{A})^c(x))^c = (\mathcal{A}(x))^c, x \in X \}$$
$$= \{ x |((1- A^+(x), 1- A^-(x))^c = ((1- A^+(x), 1- A^-(x))^c, x \in X \}$$
$$= \{ x | (A^-(x), A^+(x)) = (A^-(x), A^+(x)), x \in X \}$$
$$= \{ x| \mathcal{AA}\mathcal{A}(x) = \mathcal{A}(x), x \in X \}$$
$$(\mathcal{R}(\mathcal{A})^c)^c = \mathcal{R}(\mathcal{A}).$$

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