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SOME PROPERTIES OF CUBIC SOFT SETS

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Abstract:

The notion of cubic soft sets P-union, P-intersection, are introduced and their related properties are investigated. We discussed about the commutative and absorption property along with examples and also discussed the internal and external cubic soft sets.

Key Words: Cubic Soft Set, Properties of Cubic Soft Sets & Internal and External Cubic Soft Sets.

1. Introduction:

In order to deal with many complicated problems in the fields of engineering, social science, economic, medical science, agriculture, etc., involving uncertainties, classical methods are found to be inadequate in recent time. Molodtsov [4] pointed out that the important existing theories viz. Probability theory, fuzzy set theory etc, which can be considered as mathematical tools for dealing with uncertainties have their own difficulties. Maji *et. al*[3] worked on some new operations in soft set theory. Chinnadurai and anu [1] discussed some characterization on operations in soft sets.

2. Preliminaries:

Definition: 2.1

A pair $(\tilde{F}, I)$ is called cubic soft set over X if and only if $\tilde{F}$ is a mapping of I$(\subseteq E)$ into the set of all cubic sets in X, i.e., $\tilde{F}: I \to CP(X)$ where I is any subset of parameter's set E, X is an initial universe set and $CP(X)$ is the collection of all cubic sets in X. Here we denote and define cubic soft set as $(\tilde{F}, I) = \tilde{F}(e_i) = \mathcal{A} = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >; x \in X\} e_i \in I$ in this set corresponding to each $e_i \in I$, $\mathcal{A} = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >; x \in X\}$ is a cubic set in X in which $A_{e_i}(x)$ is an interval valued fuzzy set (briefly, an IVF set) and $\lambda_{e_i}(x)$ is a fuzzy set.

Definition: 2.2

A cubic soft set $(\tilde{F}, I)$ is said to be an internal cubic soft set (ICSS), if for all $e_i \in I$ $(\subseteq E)$ (E is set of parameters) $\tilde{F}(e_i) = \mathcal{A}$ is so that $A^-_{e_i}(x) \le \lambda_{e_i}(x) \le A^+_{e_i}(x). \quad \forall e_i \in I$ and $\forall x \in X$.

Definition: 2.3

A cubic soft set $(\tilde{F}, I)$ is said to be an external cubic soft set (ECSS), if for all $e_i \in I$ $(\subseteq E)$ (E is set of parameters) $\tilde{F}(e_i) = \mathcal{A}$ is so that $\lambda_{e_i}(x) \notin (A^-_{e_i}(x), A^+_{e_i}(x)). \quad \forall e_i \in I$ and $\forall x \in X$.

Definition: 2.4

Let $(\tilde{F}, I)$ and $(\tilde{G}, J)$ be two cubic soft sets in X, where I and J are any subsets of parameter's set E. Then, we define P-union $(\tilde{F}, I) \cup_P (\tilde{G}, J) = (\tilde{H}, C)$ where $C = I \cup J$

$$
\begin{aligned}
\tilde{H}(e_i) &= \tilde{F}(e_i) && if \quad e_i \in I - J \\
&= \tilde{G}(e_i) && if \quad e_i \in J - I \\
&= \tilde{F}(e_i) \vee_P \tilde{G}(e_i) && if \quad e_i \in I \cap J.
\end{aligned}
$$

Where $\tilde{F}(e_i) \vee_P \tilde{G}(e_i)$ is defined as,

$$\tilde{F}(e_i) \vee_P \tilde{G}(e_i) = \{< x, r\max\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x) \vee \mu_{e_i}(x)) >: x \in X\}, e_i \in I \cap J.$$

Definition: 2.4

Let $(\tilde{F}, I)$ and $(\tilde{G}, J)$ be two cubic soft sets in X, where I and J are any subsets of parameter's set E. Then, we define P-intersection $(\tilde{F}, I) \cap_P (\tilde{G}, J) = (\tilde{H}, C)$ where $C = I \cap J$

$$
\begin{aligned}
\tilde{H}(e_i) &= \tilde{F}(e_i) && if \quad e_i \in I - J \\
&= \tilde{G}(e_i) && if \quad e_i \in J - I \\
&= \tilde{F}(e_i) \wedge_P \tilde{G}(e_i) && if \quad e_i \in I \cap J.
\end{aligned}
$$

Where $\tilde{F}(e_i)\wedge_P \tilde{G}(e_i)$ is defined as,

$$\tilde{F}(e_i)\wedge_P \tilde{G}(e_i)=\{< x, r\min\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x)\wedge \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$$

Definition: 2.3

The complement of a cubic soft set, $(\tilde{F},I)=\tilde{F}(e_i)=\mathcal{A}=\{< x, A_{e_i}(x), \lambda_{e_i}(x)>; x\in X\} e_i\in I$ is denoted by $(\tilde{F},I)^c$ and defined as $(\tilde{F},I)^c=(\tilde{F}^c,\neg I)$, where $\tilde{F}^c:\neg I\to CP(X)$ and

$$\tilde{F}^c(e_i)=(\tilde{F}(\neg e_i))^c, \quad forall\ e_i\notin I$$
$$=(\tilde{F}(e_i))^c, \quad (as\ \neg(\neg e_i)=e_i)$$
$$(\tilde{F},I)^c=(\tilde{F}(e_i))^c=\mathcal{A}^c=\{< x, A^c{}_{e_i}(x), \lambda^c{}_{e_i}(x)>; x\in X\} e_i\in I$$

3. Main Results:

Properties: 3.1

(i) $(\tilde{F},I)\cup_P(\tilde{G},J)=(\tilde{G},J)\cup_P(\tilde{F},I)$

(ii) $(\tilde{F},I)\cap_P(\tilde{G},J)=(\tilde{G},J)\cap_P(\tilde{F},I)$

Proof:

(i) $(\tilde{F},I)\cup_P(\tilde{G},J)=(\tilde{H},C)$ where $C=I\cup J$

$$\tilde{H}(e_i)=\tilde{F}(e_i) \quad if \quad e_i\in I-J$$
$$=\tilde{G}(e_i) \quad if \quad e_i\in J-I$$
$$=\tilde{F}(e_i)\vee_P \tilde{G}(e_i) \quad if \quad e_i\in I\cap J.$$

Where, $\tilde{F}(e_i)\vee_P \tilde{G}(e_i)=\{< x, r\max\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x)\vee \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$

$(\tilde{G},I)\cup_P(\tilde{F},J)=(\tilde{K},D)$ where $D=I\cup J$

$$\tilde{H}(e_i)=\tilde{G}(e_i) \quad if \quad e_i\in J-I$$
$$=\tilde{F}(e_i) \quad if \quad e_i\in I-J$$
$$=\tilde{G}(e_i)\vee_P \tilde{F}(e_i) \quad if \quad e_i\in J\cap I.$$

Where, $\tilde{G}(e_i)\vee_P \tilde{F}(e_i)=\{< x, r\max\{B_{e_i}(x), A_{e_i}(x)\}, (\mu_{e_i}(x)\vee \lambda_{e_i}(x))>: x\in X\}, e_i\in J\cap I.$

$$=\{< x, r\max\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x)\vee \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$$
$$=\tilde{F}(e_i)\vee_P \tilde{G}(e_i)$$

Hence Proved.

(ii) $(\tilde{F},I)\cap_P(\tilde{G},J)=(\tilde{H},C)$ where $C=I\cap J$

$$\tilde{H}(e_i)=\tilde{F}(e_i) \quad if \quad e_i\in I-J$$
$$=\tilde{G}(e_i) \quad if \quad e_i\in J-I$$
$$=\tilde{F}(e_i)\wedge_P \tilde{G}(e_i) \quad if \quad e_i\in I\cap J.$$

Where, $\tilde{F}(e_i)\wedge_P \tilde{G}(e_i)=\{< x, r\min\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x)\vee \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$

$(\tilde{G},I)\cup_P(\tilde{F},J)=(\tilde{K},D)$ where $D=I\cup J$

$$\tilde{H}(e_i)=\tilde{G}(e_i) \quad if \quad e_i\in J-I$$
$$=\tilde{F}(e_i) \quad if \quad e_i\in I-J$$
$$=\tilde{G}(e_i)\wedge_P \tilde{F}(e_i) \quad if \quad e_i\in J\cap I.$$

Where, $\tilde{G}(e_i)\wedge_P \tilde{F}(e_i)=\{< x, r\min\{B_{e_i}(x), A_{e_i}(x)\}, (\mu_{e_i}(x)\wedge \lambda_{e_i}(x))>: x\in X\}, e_i\in J\cap I.$

$$=\{< x, r\min\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x)\wedge \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$$
$$=\tilde{F}(e_i)\wedge_P \tilde{G}(e_i)$$

Hence Proved.

Example: 3.2

Let X= $\{P_1,P_2,P_3,P_4\}$ be initial universe, I= $\{e_1,e_2\}$ and J= $\{e_1,e_2,e_3\}$ are any two subsets of parameters set E= $\{e_1,e_2,e_3\}$. Let $(\tilde{F},I)$ be cubic soft set define as,

P	$\tilde{F}(e_1)=\mathcal{A}_1=< A_{e_1}(p), \lambda_{e_1}(p)>$	$\tilde{F}(e_2)=\mathcal{A}_2=< A_{e_2}(p), \lambda_{e_2}(p)>$
$\mathbf{P_1}$	[0.4,0.5], 0.3	[0.3,0.6], 0.7
$\mathbf{P_2}$	[0.3,0.6], 0.8	[0.7,1], 0.8

P₃	[0.5,0.8], 0.7	[0.3,0.6], 0.6
P₄	[0.6,0.9], 0.5	[0.1,0.4], 0.9

$$(\tilde{G},I)=\tilde{G}(e_i)=\mathfrak{B}=\{<x,B_{e_i}(p),\mu_{e_i}(p)>: p\in X\}e_i\in J, i=1,2,3$$

P	$\tilde{G}(e_1)=\mathfrak{B}_1= <B_{e_1}(p),\mu_{e_1}(p)>$	$\tilde{G}(e_2)=\mathfrak{B}_2$ $<B_{e_2}(p),\mu_{e_2}(p)>$	$\tilde{G}(e_3)=\mathfrak{B}_3= <B_{e_3}(p),\mu_{e_3}(p)>$
P_1	[0.3,0.6], 0.8	[0.2,0.5], 0.7	[0.6,0.9], 0.7
P_2	[0.4,0.7], 0.8	[0.3,0.6], 0.8	[0.3,0.6], 0.5
P_3	[0.5,0.8], 0.4	[0.4,0.7], 0.9	[0.1,0.4], 0.9
P_4	[0.6,0.9], 0.7	[0.5,0.8], 0.6	[0.4,0.7], 0.6

(i) $(\tilde{F},I)\cup_P(\tilde{G},J)=(\tilde{G},J)\cup_P(\tilde{F},I)$

L.H.S, $(\tilde{F},I)\cup_P(\tilde{G},J)$

W.K.T,

$$\tilde{F}(e_i)\vee_P\tilde{G}(e_i)=\{<x, r\max\{A_{e_i}(x),B_{e_i}(x)\},(\lambda_{e_i}(x)\vee\mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$$

P	$\tilde{F}(e_1)\vee_P\tilde{G}(e_1)=\{<x, r\max\{A_{e_i}(p),B_{e_i}(p)\},(\lambda_{e_1}(p)\vee\mu_{e_1}(p))>: p\in X\}$
P_1	[0.4,0.7], 0.8
P_2	[0.4,0.7], 0.8
P_3	[0.5,0.8], 0.7
P_4	[0.6,0.9], 0.7

P	$\tilde{F}(e_2)\vee_P\tilde{G}(e_2)=\{<x, r\max\{A_{e_2}(p),B_{e_{2i}}(p)\},(\lambda_{e_2}(p)\vee\mu_{e_2}(p))>: p\in X\}$
P_1	[0.3,0.6], 0.7
P_2	[0.7,1], 0.8
P_3	[0.4,0.7], 0.9
P_4	[0.5,0.8], 0.9

P	$\tilde{G}(e_3)=\mathfrak{B}_3= <B_{e_3}(p),\mu_{e_3}(p)>$
P_1	[0.6,0.9], 0.7
P_2	[0.3,0.6], 0.5
P_3	[0.1,0.4], 0.9
P_4	[0.4,0.7], 0.6

R.H.S,

$$(\tilde{G},I)\cup_P(\tilde{F},J)$$

W.K.T,

$$\tilde{G}(e_i)\vee_P\tilde{F}(e_i)=\{<x, r\max\{B_{e_i}(x),A_{e_i}(x)\},(\mu_{e_i}(x)\vee\lambda_{e_i}(x))>: x\in X\}, e_i\in J\cap I.$$

P	$\tilde{G}(e_1)\vee_P\tilde{F}(e_1)=\{<x, r\max\{B_{e_i}(p),A_{e_i}(p)\},(\mu_{e_1}(p)\vee\lambda_{e_1}(p))>: p\in X\}$
P_1	[0.4,0.7], 0.8
P_2	[0.4,0.7], 0.8
P_3	[0.5,0.8], 0.7
P_4	[0.6,0.9], 0.7

P	$\tilde{G}(e_2)\vee_P\tilde{F}(e_2)=\{<x, r\max\{B_{e_2}(p),A_{e_{2i}}(p)\},(\mu_{e_2}(p)\vee\lambda_{e_2}(p))>: p\in X\}$
P_1	[0.3,0.6], 0.7
P_2	[0.7,1], 0.8
P_3	[0.4,0.7], 0.9
P_4	[0.5,0.8], 0.9

P	$\tilde{G}(e_3)=\mathfrak{B}_3= <B_{e_3}(p),\mu_{e_3}(p)>$
P_1	[0.6,0.9], 0.7
P_2	[0.3,0.6], 0.5
P_3	[0.1,0.4], 0.9
P_4	[0.4,0.7], 0.6

Hence L.H.S=R.H.S

(ii) $(\tilde{F},I)\cap_P(\tilde{G},J)=(\tilde{G},J)\cap_P(\tilde{F},I)$

L.H.S, $(\tilde{F},I)\cap_P(\tilde{G},J)$

W.K.T,

$$\tilde{F}(e_i)\wedge_P\tilde{G}(e_i)=\left\{<x,r\min\{A_{e_i}(x),B_{e_i}(x)\},(\lambda_{e_i}(x)\wedge\mu_{e_i}(x))>:x\in X\right\},e_i\in I\cap J.$$

P	$\tilde{F}(e_1)\wedge_P\tilde{G}(e_1)=\left\{<x,r\min\{A_{e_1}(p),B_{e_1}(p)\},(\lambda_{e_1}(p)\wedge\mu_{e_1}(p))>:p\in X\right\}$
P_1	[0.3,0.6], 0.3
P_2	[0.3,0.6], 0.6
P_3	[0.5,0.8], 0.4
P_4	[0.6,0.9], 0.5

P	$\tilde{F}(e_2)\wedge_P\tilde{G}(e_2)=\left\{<x,r\min\{A_{e_2}(p),B_{e_{2i}}(p)\},(\lambda_{e_2}(p)\wedge\mu_{e_2}(p))>:p\in X\right\}$
P_1	[0.2,0.5], 0.7
P_2	[0.3,0.6], 0.8
P_3	[0.3,0.6], 0.6
P_4	[0.1,0.4], 0.6

P	$\tilde{G}(e_3)=\mathfrak{B}_3=<B_{e_3}(p),\mu_{e_3}(p)>$
P_1	[0.6,0.9], 0.7
P_2	[0.3,0.6], 0.5
P_3	[0.1,0.4], 0.9
P_4	[0.4,0.7], 0.6

R.H.S,

$$(\tilde{G},I)\cap_P(\tilde{F},J)$$

W.K.T,

$$\tilde{G}(e_i)\wedge_P\tilde{F}(e_i)=\left\{<x,r\min\{B_{e_i}(x),A_{e_i}(x)\},(\mu_{e_i}(x)\wedge\lambda_{e_i}(x))>:x\in X\right\},e_i\in J\cap I.$$

P	$\tilde{G}(e_1)\wedge_P\tilde{F}(e_1)=\left\{<x,r\min\{B_{e_i}(p),A_{e_i}(p)\},(\mu_{e_1}(p)\wedge\lambda_{e_1}(p))>:p\in X\right\}$
P_1	[0.4,0.7], 0.8
P_2	[0.4,0.7], 0.8
P_3	[0.5,0.8], 0.7
P_4	[0.6,0.9], 0.7

P	$\tilde{G}(e_2)\wedge_P\tilde{F}(e_2)=\left\{<x,r\min\{B_{e_2}(p),A_{e_{2i}}(p)\},(\mu_{e_2}(p)\wedge\lambda_{e_2}(p))>:p\in X\right\}$
P_1	[0.3,0.6], 0.7
P_2	[0.7,1], 0.8
P_3	[0.4,0.7], 0.9
P_4	[0.5,0.8], 0.9

P	$\tilde{G}(e_3)=\mathfrak{B}_3=<B_{e_3}(p),\mu_{e_3}(p)>$
P_1	[0.6,0.9], 0.7
P_2	[0.3,0.6], 0.5
P_3	[0.1,0.4], 0.9
P_4	[0.4,0.7], 0.6

Hence L.H.S=R.H.S

Properties: 3.3

For any two cubic soft sets $(\tilde{F},I)$, and $(\tilde{G},J)$

$(i)\ (\tilde{F},I)\cup_P((\tilde{F},I)\cap_P(\tilde{G},J))=(\tilde{F},I)\quad if\ (\tilde{F},I)\supset(\tilde{G},J)$

$(ii)\ (\tilde{F},I)\cap_P((\tilde{F},I)\cup_P(\tilde{G},J))=(\tilde{F},I)\quad if\ (\tilde{F},I)\supset(\tilde{G},J)$

Proof:

$(i)\quad(\tilde{F},I)\cap_P(\tilde{G},J)=(\tilde{H},C)\quad if\ (\tilde{F},I)\supset(\tilde{G},J)$

By the definition,

$$\begin{aligned}\tilde{H}(e_i)&=\tilde{F}(e_i) && if\quad e_i\in I-J\\&=\tilde{G}(e_i) && if\quad e_i\in J-I\\&=\tilde{F}(e_i)\wedge_P\tilde{G}(e_i) && if\quad e_i\in I\cap J.\end{aligned}$$

Where, $\tilde{F}(e_i)\wedge_P \tilde{G}(e_i)=\{<x, r\min\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x)\vee \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$

$(\tilde{F},I)\cup_P((\tilde{F},I)\cap_P(\tilde{G},J))=(\tilde{F},I)\cup_P(\tilde{H},C)=(\tilde{K},D)$

By the definition,

$$\tilde{K}(e_i)=\tilde{F}(e_i) \quad if \quad e_i\in I-C$$
$$=\tilde{H}(e_i) \quad if \quad e_i\in C-I$$
$$=\tilde{F}(e_i)\vee_P \tilde{H}(e_i) \quad if \quad e_i\in I\cap C.$$

Where,

$$\tilde{F}(e_i)\vee_P(\tilde{F}(e_i)\wedge_P \tilde{G}(e_i))=\{<x, r\max\{A_{e_i}(x), r\min\{A_{e_i}(x), B_{e_i}(x)\}\}, \lambda_{e_i}\vee(\lambda_{e_i}(x)\wedge \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$$
$$=\{<x,\{A_{e_i}(x), \lambda_{e_i}(x)\}>: x\in X\}$$
$$=(\tilde{F},I)$$

Hence Proved.

(ii) $(\tilde{F},I)\cup_P(\tilde{G},J)=(\tilde{H},C) \quad if \ (\tilde{F},I)\supset(\tilde{G},J)$

By the definition,

$$\tilde{H}(e_i)=\tilde{F}(e_i) \quad if \quad e_i\in I-J$$
$$=\tilde{G}(e_i) \quad if \quad e_i\in J-I$$
$$=\tilde{F}(e_i)\vee_P \tilde{G}(e_i) \quad if \quad e_i\in I\cap J.$$

Where, $\tilde{F}(e_i)\vee_P \tilde{G}(e_i)=\{<x, r\max\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x)\vee \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$

$(\tilde{F},I)\cap_P((\tilde{F},I)\cup_P(\tilde{G},J))=(\tilde{F},I)\cap_P(\tilde{H},C)=(\tilde{K},D)$

By the definition,

$$\tilde{K}(e_i)=\tilde{F}(e_i) \quad if \quad e_i\in I-C$$
$$=\tilde{H}(e_i) \quad if \quad e_i\in C-I$$
$$=\tilde{F}(e_i)\wedge_P \tilde{H}(e_i) \quad if \quad e_i\in I\cap C.$$

Where, $\tilde{F}(e_i)\wedge_P(\tilde{F}(e_i)\vee_P \tilde{G}(e_i))=\{<x, r\min\{A_{e_i}(x), r\max\{A_{e_i}(x), B_{e_i}(x)\}\}, \lambda_{e_i}\wedge(\lambda_{e_i}(x)\vee \mu_{e_i}(x))>: x\in X\}, e_i\in I\cap J.$

$$=\{<x,\{A_{e_i}(x), \lambda_{e_i}(x)\}>: x\in X\}$$
$$=(\tilde{F},I)$$

Hence Proved.

Example: 3.4

Let X= $\{P_1,P_2,P_3,P_4\}$ be initial universe, I= $\{e_1,e_2\}$and J= $\{e_1,e_2,e_3\}$ are any two subsets of parameters set E= $\{e_1,e_2,e_3\}$. Let $(\vec{F},I)$ be cubic soft set define as,

P	$\tilde{F}(e_1)=\mathcal{A}_1=<A_{e_1}(p),\lambda_{e_1}(p)>$	$\tilde{F}(e_2)=\mathcal{A}_2=<A_{e_2}(p),\lambda_{e_2}(p)>$
P_1	[0.4,0.3], 0.3	[0.3,0.6], 0.7
P_2	[0.3,0.6], 0.8	[0.7,1], 0.8
P_3	[0.5,0.8], 0.7	[0.3,0.6], 0.6
P_4	[0.6,0.9], 0.5	[0.2,0.4], 0.9

$(\tilde{G},I)=\tilde{G}(e_i)=\mathfrak{B}=\{<x, B_{e_i}(p), \mu_{e_i}(p)>: p\in X\} e_i\in J, i=1,2,3$

P	$\tilde{G}(e_1)=\mathfrak{B}_1=$ $<B_{e_1}(p),\mu_{e_1}(p)>$	$\tilde{G}(e_2)=\mathfrak{B}_2$ $<B_{e_2}(p),\mu_{e_2}(p)>$	$\tilde{G}(e_3)=\mathfrak{B}_3=$ $<B_{e_3}(p),\mu_{e_3}(p)>$
P_1	[0.3,0.6], 0.2	[0.2,0.5], 0.7	[0.6,0.9], 0.7
P_2	[0.2,0.5], 0.7	[0.6,0.8], 0.8	[0.3,0.6], 0.5
P_3	[0.4,0.7], 0.5	[0.2,0.5], 0.9	[0.1,0.4], 0.9
P_4	[0.5,0.8], 0.3	[0.1,0.3], 0.6	[0.4,0.7], 0.6

$(i)\,(\tilde{F},I)\cup_P((\tilde{F},I)\cap_P(\tilde{G},J))=(\tilde{F},I) \quad if \ (\tilde{F},I)\supset(\tilde{G},J)$

First we take, $(\tilde{F},I)\cap_P(\tilde{G},J)$

P	$\tilde{F}(e_1)\wedge_P \tilde{G}(e_1)=\{<x, r\min\{A_{e_1}(p), B_{e_1}(p)\}, (\lambda_{e_1}(p)\wedge \mu_{e_1}(p))>: p\in X\}$
P_1	[0.3,0.6], 0.2
P_2	[0.2,0.5], 0.7
P_3	[0.4,0.7], 0.5
P_4	[0.5,0.8], 0.3

P	$\tilde{F}(e_2)\wedge_P \tilde{G}(e_2)=\{<x, r\min\{A_{e_2}(p), B_{e_{2i}}(p)\}, (\lambda_{e_2}(p)\wedge_{e_2}(p))>: p\in X\}$
P_1	[0.2,0.5], 0.5
P_2	[0.6,0.8], 0.7
P_3	[0.2,0.5], 0.4
P_4	[0.1,0.3], 0.6

P	$\tilde{G}(e_3)=\mathfrak{B}_3=<B_{e_3}(p), \mu_{e_3}(p)>$
P_1	[0.6,0.9], 0.7
P_2	[0.3,0.6], 0.5
P_3	[0.1,0.4], 0.9
P_4	[0.4,0.7], 0.6

Now we take,

$$\left(\tilde{F}, I\right)\cup_P\left(\left(\tilde{F}, I\right)\cap_P\left(\tilde{G}, J\right)\right)$$

P	$\tilde{F}(e_1)\vee_P\left(\tilde{F}(e_1)\wedge_P\tilde{G}(e_1)\right)$
P_1	[0.4,0.7], 0.3
P_2	[0.3,0.6], 0.8
P_3	[0.5,0.8], 0.7
P_4	[0.6,0.9], 0.5

P	$\tilde{F}(e_2)\vee_P\left(\tilde{F}(e_2)\wedge_P\tilde{G}(e_2)\right)$
P_1	[0.3,0.6], 0.7
P_2	[0.7,1], 0.8
P_3	[0.3,0.6], 0.6
P_4	[0.2,0.4], 0.9

P	$\tilde{G}(e_3)=\mathfrak{B}_3=<B_{e_3}(p), \mu_{e_3}(p)>$
P_1	[0.6,0.9], 0.7
P_2	[0.3,0.6], 0.5
P_3	[0.1,0.4], 0.9
P_4	[0.4,0.7], 0.6

Hence $\left(\tilde{F}, I\right)\cup_P\left(\left(\tilde{F}, I\right)\cap_P\left(\tilde{G}, J\right)\right)=\left(\tilde{F}, I\right)$.

$(ii)\left(\tilde{F}, I\right)\cap_P\left(\left(\tilde{F}, I\right)\cup_P\left(\tilde{G}, J\right)\right)=\left(\tilde{F}, I\right)$ $\quad if\ (\tilde{F}, I)\supset(\tilde{G}, J)$

First we take, $\left(\tilde{F}, I\right)\cup_P\left(\tilde{G}, J\right)$

P	$\tilde{F}(e_1)\vee_P\tilde{G}(e_1)=\{<x, r\max\{A_{e_1}(p), B_{e_1}(p)\}, (\lambda_{e_1}(p)\vee\mu_{e_1}(p))>: p\in X\}$
P_1	[0.4,0.7], 0.3
P_2	[0.3,0.6], 0.8
P_3	[0.5,0.8], 0.7
P_4	[0.6,0.9], 0.5

P	$\tilde{F}(e_2)\vee_P\tilde{G}(e_2)=\{<x, r\max\{A_{e_2}(p), B_{e_{2i}}(p)\}, (\lambda_{e_2}(p)\vee_{e_2}(p))>: p\in X\}$
P_1	[0.3,0.6], 0.7
P_2	[0.7,1], 0.8
P_3	[0.3,0.6], 0.6
P_4	[0.2,0.4], 0.9

P	$\tilde{G}(e_3)=\mathfrak{B}_3=<B_{e_3}(p), \mu_{e_3}(p)>$
P_1	[0.6,0.9], 0.7
P_2	[0.3,0.6], 0.5
P_3	[0.1,0.4], 0.9
P_4	[0.4,0.7], 0.6

Now we take, $\left(\tilde{F}, I\right)\cap_P\left(\left(\tilde{F}, I\right)\cup_P\left(\tilde{G}, J\right)\right)$

P	$\tilde{F}(e_1)\wedge_P\left(\tilde{F}(e_1)\vee_P\tilde{G}(e_1)\right)$
P_1	[0.4,0.7], 0.3

P$_2$	[0.3,0.6], 0.8
P$_3$	[0.5,0.8], 0.7
P$_4$	[0.6,0.9], 0.5

P	$\tilde{F}(e_2)\wedge_P\left(\tilde{F}(e_2)\vee_P\tilde{G}(e_2)\right)$
P$_1$	[0.3,0.6], 0.7
P$_2$	[0.7,1], 0.8
P$_3$	[0.3,0.6], 0.6
P$_4$	[0.2,0.4], 0.9

P	$\tilde{G}(e_3)=\mathfrak{B}_3=<B_{e_3}(p),\mu_{e_3}(p)>$
P$_1$	[0.6,0.9], 0.7
P$_2$	[0.3,0.6], 0.5
P$_3$	[0.1,0.4], 0.9
P$_4$	[0.4,0.7], 0.6

Hence $(\tilde{F},I)\cap_P\left((\tilde{F},I)\cup_P(\tilde{G},J)\right)=(\tilde{F},I)$.

Theorem: 3.5

Let $(\tilde{F},I)=\tilde{F}(e_i)=\mathcal{A}=\{<x,A_{e_i}(x),\lambda_{e_i}(x)>:x\in X\}e_i\in I$ and $(\tilde{G},I)=\tilde{G}(e_i)=\mathfrak{B}=\{<x,B_{e_i}(x),\mu_{e_i}(x)>:x\in X\}e_i\in J$ be an external cubic soft sets(ECSS). Then,

$(i)(\tilde{F},I)\cup_P(\tilde{G},J)$ is an ECSS.

$(ii)(\tilde{F},I)\cap_P(\tilde{G},J)$ is an ECSS.

Proof:

(i)Since $(\tilde{F},I)$and $(\tilde{G},J)$ external cubic soft sets,

So for, $(\tilde{F},I)$, we have, $\lambda_{e_i}(x)\notin(A^-_{e_i}(x),A^+_{e_i}(x)).\quad\forall e_i\in I$

Since, $\lambda_{e_i}(x)\notin(A^-_{e_i}(x),A^+_{e_i}(x))$. and $0\le A^-_{e_i}(x)\le A^+_{e_i}(x)\le 1$.

So we have $\lambda_{e_i}(x)\le A^-_{e_i}(x)$ *or* $A^+_{e_i}(x)\le\lambda_{e_i}(x)$.

AlSo for, $(\tilde{G},I)$, we have, $\mu_{e_i}(x)\notin(B^-_{e_i}(x),B^+_{e_i}(x)).\quad\forall e_i\in I$

Since, $\mu_{e_i}(x)\notin(B^-_{e_i}(x),B^+_{e_i}(x))$. and $0\le B^-_{e_i}(x)\le B^+_{e_i}(x)\le 1$.

So we have $\mu_{e_i}(x)\le B^-_{e_i}(x)$ *or* $B^+_{e_i}(x)\le\mu_{e_i}(x)$.

Then, we have,

$(\lambda_{e_i}\vee\mu_{e_i})\notin\{\max\{A^-_{e_i}(x),B^-_{e_i}(x)\},\max\{A^+_{e_i}(x),B^+_{e_i}(x)\}\}\quad\forall e_i\in I\cap J,\forall x\in X.$

Now by definition of P-union, $(\tilde{F},I)\cup_P(\tilde{G},J)=(\tilde{H},C)$ where $C=I\cup J$

$$\begin{aligned}\tilde{H}(e_i)&=\tilde{F}(e_i) && \text{if } e_i\in I-J\\ &=\tilde{G}(e_i) && \text{if } e_i\in J-I\\ &=\tilde{F}(e_i)\vee_P\tilde{G}(e_i) && \text{if } e_i\in I\cap J.\end{aligned}$$

If $e_i\in I\cap J$, $\tilde{F}(e_i)\vee_P\tilde{G}(e_i)$ is defined as,

$\tilde{F}(e_i)\vee_P\tilde{G}(e_i)=\{<x,r\max\{A_{e_i}(x),B_{e_i}(x)\},(\lambda_{e_i}(x)\vee\mu_{e_i}(x))>:x\in X\},e_i\in I\cap J.$

Thus, $(\tilde{F},I)\cup_P(\tilde{G},J)$ is an ECSS, if $e_i\in I\cap J$

$e_i\in I-J$ or $e_i\in J-I$. Then the result is trivial.

(ii)Since $(\tilde{F},I)$and $(\tilde{G},J)$ external cubic soft sets,

So for, $(\tilde{F},I)$, we have, $\lambda_{e_i}(x)\notin(A^-_{e_i}(x),A^+_{e_i}(x)).\quad\forall e_i\in I$

Since, $\lambda_{e_i}(x)\notin(A^-_{e_i}(x),A^+_{e_i}(x))$. and $0\le A^-_{e_i}(x)\le A^+_{e_i}(x)\le 1$.

So we have $\lambda_{e_i}(x)\le A^-_{e_i}(x)$ *or* $A^+_{e_i}(x)\le\lambda_{e_i}(x)$.

AlSo for, $(\tilde{G},I)$, we have, $\mu_{e_i}(x)\notin(B^-_{e_i}(x),B^+_{e_i}(x)).\quad\forall e_i\in I$

Since, $\mu_{e_i}(x)\notin(B^-_{e_i}(x),B^+_{e_i}(x))$. and $0\le B^-_{e_i}(x)\le B^+_{e_i}(x)\le 1$.

So we have $\mu_{e_i}(x)\le B^-_{e_i}(x)$ *or* $B^+_{e_i}(x)\le\mu_{e_i}(x)$.

Then, we have,

$(\lambda_{e_i} \wedge \mu_{e_i}) \notin \{\min\{A^-_{e_i}(x), B^-_{e_i}(x)\}, \min\{A^+_{e_i}(x), B^+_{e_i}(x)\}\} \quad \forall e_i \in I \cap J, \forall x \in X.$

Now by definition of P-intersection, $(\tilde{F}, I) \cap_P (\tilde{G}, J) = (\tilde{H}, C)$ where $C = I \cup J$

$$\tilde{H}(e_i) = \tilde{F}(e_i) \quad \text{if} \quad e_i \in I - J$$
$$= \tilde{G}(e_i) \quad \text{if} \quad e_i \in J - I$$
$$= \tilde{F}(e_i) \wedge_P \tilde{G}(e_i) \quad \text{if} \quad e_i \in I \cap J.$$

If $e_i \in I \cap J$, $\tilde{F}(e_i) \wedge_P \tilde{G}(e_i)$ is defined as,

$$\tilde{F}(e_i) \wedge_P \tilde{G}(e_i) = \{< x, r\min\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x) \wedge \mu_{e_i}(x)) >: x \in X\}, e_i \in I \cap J.$$

Thus, $(\tilde{F}, I) \cap_P (\tilde{G}, J)$ is an ECSS, if $e_i \in I \cap J$

$e_i \in I - J$ or $e_i \in J - I$. Then the result is trivial.

Theorem: 3.5

Let $(\tilde{F}, I)^c$ and $(\tilde{G}, I)^c$ be an internal cubic soft sets (ICSS). Then,

$(i)\ ((\tilde{F}, I) \cup_P (\tilde{G}, J))^c$ is an ICSS.

$(ii)\ ((\tilde{F}, I) \cap_P (\tilde{G}, J))^c$ is an ICSS.

Proof:

(i) Since $(\tilde{F}, I)^c$ and $(\tilde{G}, J)^c$ be an internal cubic soft sets, so for $(\tilde{F}, I)$ we have

$$A^-_{e_i}(x) \le \lambda_{e_i}(x) \le A^+_{e_i}(x). \quad \forall e_i \in I \text{ and } \forall x \in X.$$

This implies, $1 - A^-_{e_i}(x) \le 1 - \lambda_{e_i}(x) \le 1 - A^+_{e_i}(x). \quad \forall e_i \in I$ and $\forall x \in X$.

Also for $(\tilde{G}, J)$ we have

$$B^-_{e_i}(x) \le \mu_{e_i}(x) \le B^+_{e_i}(x). \quad \forall e_i \in J \text{ and } \forall x \in X.$$

This implies, $1 - B^-_{e_i}(x) \le 1 - \mu_{e_i}(x) \le 1 - B^+_{e_i}(x). \quad \forall e_i \in J$ and $\forall x \in X$.

Then we have

$$\max\{1 - A^-_{e_i}(x), 1 - B^-_{e_i}\} \le \{1 - \lambda_{e_i}(x) \vee 1 - \mu_{e_i}(x)\} \le \max\{1 - A^+_{e_i}(x), 1 - B^+_{e_i}(x)\}. \quad \forall e_i \in I \cup J$$

Now by definition,

$(\tilde{F}, I) \cup_P (\tilde{G}, J) = (\tilde{H}, C)$ where $C = I \cup J$

$$\tilde{H}(e_i) = \tilde{F}(e_i) \quad \text{if} \quad e_i \in I - J$$
$$= \tilde{G}(e_i) \quad \text{if} \quad e_i \in J - I$$
$$= \tilde{F}(e_i) \vee_P \tilde{G}(e_i) \quad \text{if} \quad e_i \in I \cap J.$$

Where, $\tilde{F}(e_i) \vee_P \tilde{G}(e_i) = \{< x, r\max\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i}(x) \vee \mu_{e_i}(x)) >: x \in X\}, e_i \in I \cap J.$

And also,

$((\tilde{F}, I) \cup_P (\tilde{G}, J))^c = (\tilde{H}, C)^c$. where $C = I \cup J$

$$\tilde{H}^c(e_i) = \tilde{F}^c(e_i) \quad \text{if} \quad e_i \in I - J$$
$$= \tilde{G}^c(e_i) \quad \text{if} \quad e_i \in J - I$$
$$= \tilde{F}^c(e_i) \vee_P \tilde{G}^c(e_i) \quad \text{if} \quad e_i \in I \cap J.$$

Where, $\tilde{F}^c(e_i) \vee_P \tilde{G}^c(e_i) = \{< x, r\max\{A^c_{e_i}(x), B^c_{e_i}(x)\}, (\lambda^c_{e_i}(x) \vee \mu^c_{e_i}(x)) >: x \in X\}, e_i \in I \cap J$

$$= \{< x, r\max\{1 - A_{e_i}(x), 1 - B_{e_i}(x)\}, (1 - \lambda_{e_i}(x) \vee 1 - \mu_{e_i}(x)) >: x \in X\}$$

Thus, $((\tilde{F}, I) \cup_P (\tilde{G}, J))^c$ is an ICSS, if $e_i \in I \cap J$

$e_i \in I - J$ or $e_i \in J - I$. Then the result is trivial.

Hence, $((\tilde{F}, I) \cup_P (\tilde{G}, J))^c$ is an ICSS in all cases.

(ii) Since $(\tilde{F}, I)^c$ and $(\tilde{G}, J)^c$ be an internal cubic soft sets, so for $(\tilde{F}, I)$ we have

$$A^-_{e_i}(x) \le \lambda_{e_i}(x) \le A^+_{e_i}(x). \quad \forall e_i \in I \text{ and } \forall x \in X.$$

This implies, $1 - A^-_{e_i}(x) \le 1 - \lambda_{e_i}(x) \le 1 - A^+_{e_i}(x). \quad \forall e_i \in I$ and $\forall x \in X$.

Also for $(\tilde{G}, J)$ we have

$$B^-_{e_i}(x) \le \mu_{e_i}(x) \le B^+_{e_i}(x). \quad \forall e_i \in J \text{ and } \forall x \in X.$$

This implies, $1 - B^-_{e_i}(x) \le 1 - \mu_{e_i}(x) \le 1 - B^+_{e_i}(x). \quad \forall e_i \in J$ and $\forall x \in X$.

Then we have

$$\min\{1-A_{e_i}^-(x),1-B_{e_i}^-\}\leq\{1-\lambda_{e_i}(x)\vee 1-\mu_{e_i}(x)\}\leq\min\{1-A_{e_i}^+(x),1-B_{e_i}^+(x)\}\quad \forall e_i\in I\cup J$$

Now by definition,

$(\tilde{F},I)\cap_P(\tilde{G},J)=(\tilde{H},C)$ where $C=I\cup J$

$$\begin{aligned}\tilde{H}(e_i)&=\tilde{F}(e_i) && if\quad e_i\in I-J\\ &=\tilde{G}(e_i) && if\quad e_i\in J-I\\ &=\tilde{F}(e_i)\wedge_P\tilde{G}(e_i) && if\quad e_i\in I\cap J.\end{aligned}$$

Where, $\tilde{F}(e_i)\wedge_P\tilde{G}(e_i)=\{<x,r\min\{A_{e_i}(x),B_{e_i}(x)\},(\lambda_{e_i}(x)\vee\mu_{e_i}(x))>:x\in X\},e_i\in I\cap J.$

And also,

$((\tilde{F},I)\cap_P(\tilde{G},J))^c=(\tilde{H},C)^c$. where $C=I\cup J$

$$\begin{aligned}\tilde{H}^c(e_i)&=\tilde{F}^c(e_i) && if\quad e_i\in I-J\\ &=\tilde{G}^c(e_i) && if\quad e_i\in J-I\\ &=\tilde{F}^c(e_i)\wedge_P\tilde{G}^c(e_i) && if\quad e_i\in I\cap J.\end{aligned}$$

Where, $\tilde{F}^c(e_i)\wedge_P\tilde{G}^c(e_i)=\{<x,r\min\{A_{e_i}^c(x),B_{e_i}^c(x)\},(\lambda_{e_i}^c(x)\wedge\mu_{e_i}^c(x))>:x\in X\},e_i\in I\cap J$

$$=\{<x,r\min\{1-A_{e_i}(x),1-B_{e_i}(x)\},(1-\lambda_{e_i}(x)\wedge 1-\mu_{e_i}(x))>:x\in X\}$$

Thus, $((\tilde{F},I)\cap_P(\tilde{G},J))^c$ is an ICSS, if $e_i\in I\cap J$

$e_i\in I-J$ or $e_i\in J-I$. Then the result is trivial.

Hence, $((\tilde{F},I)\cap_P(\tilde{G},J))^c$ is an ICSS in all cases.

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