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ROUGH INTUITIONISTIC FUZZY IDEALS IN Γ - SEMIGROUPS

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Abstract:

In this paper we apply the concept of rough intuitionistic fuzzy set in Γ-semigoup and discuss some of its properties. We introduce the notion of rough intuitionistic fuzzy left (right, two-sided,bi-,(1,2)-) ideals in Γ-semigoup and give some properties of such ideals.

Index Terms: Rough Intuitionistic Fuzzy Set, Rough Intuitionistic Fuzzy Left (Right, Two-Sided, Bi-, (1,2) Ideals, Rough Intuitionistic Fuzzy Bi-Ideal, Rough Intuitionistic Fuzzy (1,2) ideal.

1. Introduction:

Uncertainty is present in many real life situations due to imprecise or incomplete knowledge, usually the concept of information system is used to represent knowledge and this representation has many applications in artificial intelligence and determining. The uncertainty in studying information systems is characterized by a boundary or uncertain region, which for all of its elements if it is non-empty, we cannot certainly decide whether it belongs or does not belong to a concept. This kind of uncertainity is not avoidable in many applications, so the research community proposed several tools to handle them such as theory of fuzzy sets by Zadeh [13, 14] theory of rough sets by Pawlak [7-9]. W.J.Liu [6] studied fuzzy invarient Subgroups and fuzzy ideals In 1986 Atanassov[1] presented the intuitionistic fuzzy sets as a generalization of fuzzy sets[9]. K.H. Kim et. al [4] introduced fuzzy ideals of semigroups. The algebraic approach of rough set was studeied in [5, 10]. Thillaigovindan et. al [11,12] discussed rough ideals in Γ-semigroup. D.Dubois et.al.,[2]., introduced rough fuzzy sets. The rough intuitionistic fuzzy ideals was introduced by Jayanta Ghosh et.al.,[3] in semigroups. In this paper we substitute Γ-semigroup instead of the universe in Pawlak approximation space and define lower and upper approximation of an intuitionistic fuzzy set in Γ-semigroup and also discuss some important properties. We define rough intuitionistic fuzzy left (right, two-sided, bi-, (1,2)-) ideals in Γ-semigroup and verify its some basic properties.

2. Preliminaries:

This section contains some basic definitions which will be needed in the sequal. In this paper, unless otherwise stated explicitly M always denote a -semigroup with idendity.

Definition 2.1: [7] Let (U,R) be an approximation space where U is the non-empty universe , R is an equivalence relation and let X be any nonempty subset of U. Then the sets

$\mathrm{R}_*(X) = \{x \in U/[x]_R \subseteq X\}$ and $\mathrm{R}^*(X) = \{x \in U/[x]_R \cap X \neq \phi\}$ are respectively called the *lower* and *upper approximation* of the set X with respect to R where $[x]_R$ denotes the equivalence class containing the element $x \in X$ with respect to R. X is called R-definable if $\mathrm{R}_*(X) = \mathrm{R}^*(X)$, in the opposite case , i.e., if $\mathrm{R}_*(X) \neq \mathrm{R}^*(X)$ then X is called rough set with respect to R.

Definition 2.2: [1] An *intuitionistic fuzzy set* (briefly IFS), A in a nonempty set X is an object having the form, $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle |\ x \in X\}$ where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership and dgree of non membership of the element $x \in X$ to A, respectively and satisfy $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$. The family of all IFS in X is denoted by $\mathcal{IFS}(x)$.

Definition 2.3: [1] If $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ are any two $\mathcal{IFS}$ in X then

1. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x) \forall x \in X$.
2. $A = B \Leftrightarrow \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x) \forall x \in X$.
3. $A^c = (\mu_A, \nu_A)$
4. $A \cap B = (\mu_A \cap \mu_B, \nu_A \cup \nu_B)$ for all $x \in X$
   $(\mu_A \cap \mu_B)(x) = \mu_A(x) \wedge \mu_B(x)$ and $(\nu_A \cap \nu_B)(x) = \nu_A(x) \wedge \nu_B(x)$
5. $A \cup B = (\mu_A \cup \mu_B, \nu_A \cap \nu_B)$ for all $x \in X$
   $(\mu_A \cup \mu_B)(x) = \mu_A(x) \vee \mu_B(x)$ and
   $(\nu_A \cap \nu_B)(x) = \nu_A(x) \wedge \nu_B(x)$

Definition 2.4: [1] Let $L = (\mu_L, \nu_L)$ and $R = (\mu_R, \nu_R)$ are any two $\mathcal{IFS}$ of M. Then the *composition* $L \circ R$ is defined by $L \circ R = (\mu_L \circ \mu_L,\ \nu_R \circ \nu_R)$ where for all $x \in M$

$$(\mu_L \circ \mu_R)(x) = \vee_{x=y\gamma z}[\mu_L(y) \wedge \mu_R(z)]$$
$$(\nu_L \circ \nu_R)(x) = \wedge_{x=y\gamma z}[\nu_L(y) \vee \nu_R(z)]$$

Definition 2.5: [4] An $\mathcal{IFS}$ $L = (\mu_L, \nu_L)$of M is called an intuitionistic fuzzy Γ-semigroup of M if for all

$x, y \in M$ and $\in$ г , $\mu_L(x\gamma y) \geq \mu_L(x) \wedge \mu_L(y)$ and $\nu_L(x\gamma y) \geq \nu_L(x) \wedge \nu_L(y)$.

Definition 2.6: [4] An $\mathcal{IFS}$ $I = (\mu_I, \nu_I)$of M is called an intuitionistic fuzzy left ideal of M if for all $x, y \in M$ and $\in$ г , $\mu_I(x\gamma y) \geq \mu_I(y)$ and $\nu_I(x\gamma y) \leq \nu_I(y)$.

Definition 2.7: [4] An $\mathcal{IFS}$ $I = (\mu_I, \nu_I)$ of M is called an intuitionistic fuzzy right ideal of M if for all $x, y \in M$ and $\in$ г , $\mu_I(x\gamma y) \geq \mu_I(x)$ and $\nu_I(x\gamma y) \leq \nu_I(x)$.

Definition 2.8: [4] An $\mathcal{IFS}$ $L = (\mu_L, \nu_L)$of M is called an intuitionistic fuzzy two sided ideal of M if for all $x, y \in M$ and $\in$ г , $\mu_I(x\gamma y) \geq \mu_I(x) \vee \mu_I(y)$ and $\nu_I(x\gamma y) \leq \nu_I(x) \wedge \nu_I(y)$.

Definition 2.9: [4] An intuitionistic fuzzy subsemigoup $I = (\mu_I, \nu_I)$of M is called an intuitionistic fuzzy bi-ideal of M if for all $x, y, z, w \in M$ and $\alpha, \beta, \gamma \in$ г , $\mu_I(x\alpha w\beta y) \geq \mu_I(x) \vee \mu_I(y)$ and $\nu_I(x\alpha w\beta y) \leq \nu_I(x) \wedge \nu_I(y)$.

Definition 2.10: [4] An intuitionistic fuzzy subsemigoup $I = (\mu_I, \nu_I)$of M is called an intuitionistic fuzzy (1,2)-ideal of M if for all $x, y, w \in M$ and $\alpha, \beta \in$ г, $\mu_I(x\alpha w\beta(y\gamma z)) \geq \mu_I(x) \wedge \mu_I(y) \wedge \mu_I(y)$ and $\nu_I(x\alpha w\beta(y\gamma z)) \leq \nu_I(x) \vee \nu_I(y) \vee \nu_I(y)$.

3. Rough Intuitionistic Fuzzy Subsets (Rifs) in a Γ-Semigroup:

Let θ be a congruence relation on M, ie., θ is an equivalence relation on M such that

$$(a, b) \in \theta \Rightarrow (a\gamma x, b\gamma x) \in \theta \text{ and } (x\gamma a, x\gamma b) \in \theta \ \forall x \in M.$$

We denote the θ-congruence class containing the element $a \in M$ by $[a]_\theta$. For a congruence relation θ on M,

We have $[a]_\theta \Gamma [b]_\theta \subseteq [a\Gamma b]_\theta \forall a, b \in M$. A congruence relation θ on Mis called complete if $[a]_\theta \Gamma [b]_\theta = [a\Gamma b]_\theta \ for \ all \ a, b \in M$.

Let $L = (\mu_L, \nu_L)$ be an $\mathcal{IFS}$ of M.

Then the $\mathcal{IFS}$, $\theta_*(L) = (\theta_*(\mu_L), \theta_*(\nu_L))$ and $\theta^*(L) = (\theta^*(\mu_L), \theta^*(\nu_L))$ are respectively called θ-lower and θ-upper approximation of the $\mathcal{IFS}$ $L = (\mu_L, \nu_L)$ where $\forall x \in M$,

$$\theta_*(\mu_L)(x) = \bigwedge_{a \in [x]_\theta} \mu_L(a), \quad \theta_*(\nu_L)(x) = \bigvee_{a \in [x]_\theta} \nu_L(a) \text{ and}$$

$$\theta^*(\mu_L)(x) = \bigvee_{a \in [x]_\theta} \mu_L(a) \ , \theta^*(\nu_L)(x) = \bigwedge_{a \in [x]_\theta} \nu_L(a) \text{ for an}$$

$\mathcal{IFS}$ $L = (\mu_L, \nu_L)$ of M. $\theta(L) = (\theta_*(L), \theta^*(L))$ is called rough intuitionistic fuzzy set with respect to θ if $\theta_*(L) \neq \theta^*(L)$

Note: $\theta_*(\mu_L)(x) + \theta_*(\nu_L)(x) = \bigwedge_{a \in [x]_\theta} \mu_L(a) + \bigvee_{a \in [x]_\theta} \nu_L(a) \leq \bigwedge_{a \in [x]_\theta} \mu_L(a) + \bigvee_{a \in [x]_\theta} \left(1 - \mu_L(a)\right)$

$$= \bigwedge_{a \in [x]_\theta} \mu_L(a) + 1 - \bigwedge_{a \in [x]_\theta} \mu_L(a) = 1$$

Similarly $\theta^*(\mu_L)(x) + \theta^*(\nu_L)(x) \leq 1$ for all $x \in M$. This justify that $\theta_*(L) = (\theta_*(\mu_L), \theta_*(\nu_L))$ and $\theta^*(L) = \left(\theta^*(\mu_L), \theta^*(\nu_L)\right)$ are $\mathcal{IFS}$ of M.

Theorem 3.2:

Let θ and φ be two congruence relation of M. If $L = (\mu_L, \nu_L) R = (\mu_R, \nu_R)$ are any two $\mathcal{IFS}$ of M, then the following hold:

- $\theta_*(L) \subseteq L \subseteq \theta^*(L)$
- $\theta_*(\theta_*(L) = \theta_*(L))$
- $\theta^*\left(\theta^*(L)\right) = \theta^*(L)$
- $\theta^*\left(\theta_*(L)\right) = \theta_*(L)$
- $\theta_*\left(\theta^*(L)\right) = \theta^*(L)$
- $(\theta^*(L^c))^c = \theta_*(L)$
- $(\theta_*(L^c))^c = \theta^*(L)$
- $\theta_*(L \cap R) = \theta_*(L) \cap \theta_*(R)$
- $\theta^*(L \cap R) = \theta^*(L) \cap \theta^*(R)$
- $\theta^*(L \cup R) = \theta^*(L) \cup \theta^*(R)$
- $\theta_*(L \cup R) \supseteq \theta_*(L) \cup \theta_*(R)$
- $L \subseteq R \Rightarrow \theta_*(L) \subseteq \theta_*(R)$
- $L \subseteq R \Rightarrow \theta^*(L) \subseteq \theta^*(R)$
- $\theta \subseteq \varphi \Rightarrow \theta_*(L) \supseteq \varphi_*(L)$
- $\theta \subseteq \varphi \Rightarrow \theta^*(L) \subseteq \varphi^*(L)$

Proof:

This is easily obtained from Definition 2.3 and definition of lower and upper approximation of an intuitionistic fuzzy set.

Theorem 3.3:

Let θ be a congruence relation of M. If A and B are any two $\mathcal{IFS}$ of M, then $\theta^*(L) \circ \theta^*(R) \subseteq \theta^*(L \circ R)$.

Proof:

Let $L = (\mu_L, \nu_L), R = (\mu_R, \nu_R)$ be any two $\mathcal{JFS}$ of M. Then $\theta^*(L) \circ \theta^*(R) = (\theta^*(\mu_L) \circ \theta^*(\mu_R), \theta^*(\nu_L) \circ \theta^*(\nu_R))$ and $\theta^*(L \circ R) = (\theta^*(\mu_L \circ \mu_R), \theta^*(\nu_L \circ \nu_R))$.
To show $\theta^*(L) \circ \theta^*(R) \subseteq \theta^*(L \circ R)$, we have to prove that for all $\in M$.
$(\theta^*(\mu_L) \circ \theta^*(\mu_R))(x) \leq \theta^*(\mu_L \circ \mu_R)(x)$ and $(\theta^*(\nu_L) \circ \theta^*(\nu_R)(x) \leq \theta^*(\nu_L \circ \nu_R)(x)$.
Now for all $x \in M$

$$\begin{aligned}
(\theta^*(\mu_L) \circ \theta^*(\mu_R))(x) &= \vee_{x=y\gamma z}[\theta^*(\mu_L)(y) \wedge \theta^*(\mu_R)(z)] \\
&= \vee_{x=y\gamma z}\left[\left(\vee_{a\in[y]_\theta} \mu_L(a)\right) \wedge \left(\vee_{b\in[z]_\theta} \mu_R(b)\right)\right] \\
&= \vee_{x=y\gamma z}\left[\vee_{a\in[y]_\theta, b\in[z]_\theta}\left(\mu_L(a) \wedge \mu_R(b)\right)\right] \\
&\leq \vee_{x=y\gamma z}\left[\vee_{a\gamma b\in[y\gamma z]_\theta}\left(\mu_L(a) \wedge \mu_R(b)\right)\right], \text{ since } a\gamma b \in [y]_\theta \Gamma [z]_\theta \subseteq [y\Gamma z]_\theta \\
&= \vee_{a\gamma b\in[x]_\theta}\left(\mu_L(a) \wedge \mu_R(b)\right), \text{ since } x = y\gamma z \\
&= \vee_{\alpha\in[x]_\theta, \alpha=a\gamma b}\left(\mu_L(a) \wedge \mu_R(b)\right) \\
&= \vee_{\alpha\in[x]_\theta} \vee_{\alpha=a\gamma b}\left(\mu_L(a) \wedge \mu_R(b)\right) \\
&= \vee_{\alpha\in[x]_\theta}(\mu_L \circ \mu_R)(\alpha) \\
&= \theta^*(\mu_L \circ \mu_R)(x)
\end{aligned}$$

Again,

$$\begin{aligned}
(\theta^*(\nu_L) \circ \theta^*(\nu_R))(x) &= \wedge_{x=y\gamma z}[\theta^*(\nu_L)(y) \vee \theta^*(\nu_R)(z)] \\
&= \wedge_{x=y\gamma z}\left[\left(\wedge_{a\in[y]_\theta} \nu_L(a)\right) \vee \left(\wedge_{b\in[z]_\theta} \nu_R(b)\right)\right] \\
&= \wedge_{x=y\gamma z}\left[\wedge_{a\in[y]_\theta, b\in[z]_\theta}[\nu_L(a) \vee \nu_R(b)]\right] \\
&\geq \wedge_{x=y\gamma z}\left[\wedge_{a\gamma b\in[y\gamma z]_\theta}\left(\nu_L(a) \vee \nu_R(b)\right)\right], \text{ since } a\gamma b \in [y]_\theta \Gamma [z]_\theta \subseteq [y\Gamma z]_\theta \\
&= \wedge_{a\gamma b\in[x]_\theta}\left(\nu_L(a) \vee \nu_R(b)\right), \text{ since } x = y\gamma z \\
&= \wedge_{\alpha\in[x]_\theta, \alpha=a\gamma b}\left(\nu_L(a) \vee \nu_R(b)\right) \\
&= \wedge_{\alpha\in[x]_\theta} \wedge_{\alpha=a\gamma b}\left(\nu_L(a) \vee \nu_R(b)\right) \\
&= \wedge_{\alpha\in[x]_\theta}(\nu_L \circ \nu_R)(\alpha) = \theta^*(\nu_L \circ \nu_R)
\end{aligned}$$

Thus we have $\theta^*(L) \circ \theta^*(R) \subseteq \theta^*(L \circ R)$.

Theorem 3.4:

Let θ be a congruence relation of M. If A and B are any two $\mathcal{IFS}$ of M, then $\theta_*(L) \circ \theta_*(R) \subseteq \theta_*(L \circ R)$.

Proof:

Since θ is complete congruence relation on M, $[a]_\theta \Gamma [b]_\theta = [a\Gamma b]_\theta$ *for every* $a, b \in M$.
Let $L = (\mu_L, \nu_L), R = (\mu_R, \nu_R)$ be any two $\mathcal{JFS}$ of M. Then $\theta_*(L) \circ \theta_*(R) = (\theta_*(\mu_L) \circ \theta_*(\mu_R), \theta_*(\nu_L) \circ \theta_*(\nu_R))$ and $\theta_*(L \circ R) = (\theta_*(\mu_L \circ \mu_R), \theta_*(\nu_L \circ \nu_R))$.
To show $\theta_*(L) \circ \theta_*(R) \subseteq \theta_*(L \circ R)$.
We have to prove that for all $x \in M$

$$\begin{aligned}
(\theta^*(\mu_L) \circ \theta^*(\mu_R))(x) &= \vee_{x=y\gamma z}[\theta^*(\mu_L)(y) \wedge \theta^*(\mu_R)(z)] \\
&= \vee_{x=y\gamma z}\left[\left(\vee_{a\in[y]_\theta} \mu_L(a)\right) \wedge \left(\vee_{b\in[z]_\theta} \mu_R(b)\right)\right] \\
&= \vee_{x=y\gamma z}\left[\vee_{a\in[y]_\theta, b\in[z]_\theta}\left(\mu_L(a) \wedge \mu_R(b)\right)\right] \\
&\leq \vee_{x=y\gamma z}\left[\vee_{a\gamma b\in[y\gamma z]_\theta}\left(\mu_L(a) \wedge \mu_R(b)\right)\right], \text{ since } a\gamma b \in [y]_\theta \Gamma [z]_\theta \subseteq [y\Gamma z]_\theta \\
&= \vee_{a\gamma b\in[x]_\theta}\left(\mu_L(a) \wedge \mu_R(b)\right), \text{ since } x = y\gamma z \\
&= \vee_{\alpha\in[x]_\theta, \alpha=a\gamma b}\left(\mu_L(a) \wedge \mu_R(b)\right) \\
&= \vee_{\alpha\in[x]_\theta} \vee_{\alpha=a\gamma b}\left(\mu_L(a) \wedge \mu_R(b)\right) \\
&= \vee_{\alpha\in[x]_\theta}(\mu_L \circ \mu_R)(\alpha) \\
&= \theta_*(\mu_L \circ \mu_R)(x)
\end{aligned}$$

Again,

$$\begin{aligned}
(\theta_*(\nu_L) \circ \theta_*(\nu_R))(x) &= \wedge_{x=y\gamma z}[\theta_*(\nu_L)(y) \vee \theta_*(\nu_R)(z)] \\
&= \wedge_{x=y\gamma z}\left[\left(\wedge_{a\in[y]_\theta} \nu_L(a)\right) \vee \left(\wedge_{b\in[z]_\theta} \nu_R(b)\right)\right] \\
&= \wedge_{x=y\gamma z}\left[\wedge_{a\in[y]_\theta, b\in[z]_\theta}[\nu_L(a) \vee \nu_R(b)]\right] \\
&\geq \wedge_{x=y\gamma z}\left[\wedge_{a\gamma b\in[y\gamma z]_\theta}\left(\nu_L(a) \vee \nu_R(b)\right)\right], \text{ since } a\gamma b \in [y]_\theta \Gamma [z]_\theta \subseteq [y\Gamma z]_\theta \\
&= \wedge_{a\gamma b\in[x]_\theta}\left(\nu_L(a) \vee \nu_R(b)\right), \text{ since } x = y\gamma z \\
&= \wedge_{\alpha\in[x]_\theta, \alpha=a\gamma b}\left(\nu_L(a) \vee \nu_R(b)\right) \\
&= \wedge_{\alpha\in[x]_\theta} \wedge_{\alpha=a\gamma b}\left(\nu_L(a) \vee \nu_R(b)\right) \\
&= \wedge_{\alpha\in[x]_\theta}(\nu_L \circ \nu_R)(\alpha) = \theta_*(\nu_L \circ \nu_R)
\end{aligned}$$

Thus we have $\theta_*(L) \circ \theta_*(R) \subseteq \theta_*(L \circ R)$.

Theorem 3.5:

Let θ and φ be two congruence relation of M. If L is an $\mathcal{IFS}$ of M then $(\theta \cap \varphi)^*(L) \subseteq \theta^*(L) \cap \varphi^*(L)$.

Proof:

It is easily to observe that $\theta \cap \varphi$ is also a congruence relation of M and it is also clear that $\theta \cap \varphi \subseteq \theta$ and $\theta \cap \varphi \subseteq \varphi$. Let $A = (\mu_L, \nu_L)$ be an $\mathcal{IFS}$ of M. Then by using Theorem 3.2(15), we obtain $(\theta \cap \varphi)^*(L) \subseteq \theta^*(L)$ and $(\theta \cap \varphi)^*(L) \subseteq \varphi^*(L)$. Therefore, using definition 2.3(1),(4) ,we have

$$(\theta \cap \varphi)^*(L) \subseteq \theta^*(L) \cap \varphi^*(L).$$

Theorem 3.6:

Let θ and φ be two congruence relation of M. If L is an $\mathcal{IFS}$ of M then$(\theta \cap \varphi)_*(L) \subseteq \theta_*(L) \cap \varphi_*(L)$.

Proof:

It is easily to observe that $\theta \cap \varphi$ is also a congruence relation of M and it is also clear that $\theta \cap \varphi \subseteq \theta$ and $\theta \cap \varphi \subseteq \varphi$. Let $L = (\mu_L, \nu_L)$ be an $\mathcal{IFS}$ of M.

Then by using Theorem 3.2(14),we obtain

$(\theta \cap \varphi)_*(L)\theta_*(L)$ and $(\theta \cap \varphi)_*(L)\varphi_*(L)$. Therefore, using definition 2.3(1),(5) ,we have

$$(\theta \cap \varphi)_*(L)\theta_*(L) \cup \varphi_*(L)$$

4. Rough Intuitionistic Fuzzy Ideals (Rifi) in a Γ-Semigroup:

Definition 4.1: Let θ be a congruence relation on M. An $\mathcal{IFS}$ L of M is called an upper (lower) rough intuitionistic fuzzy sub Γ-semigroup of M if $\theta_*(L)(\theta_*(L))$ is an intuitionistic fuzzy sub Γ-semigroup of M. L is called a rough intuitionistic fuzzy sub Γ-semigroup of M if both $\theta^*(L)$ and $\theta_*(L)$ are both intuitionistic fuzzy sub Γ-semigroup of M.

Definition 4.2: Let θ be a congruence relation on M. An $\mathcal{IFS}$ I of M is called an upper (lower) rough intuitionistic fuzzy left (right, two-sided)-ideal of M if $\theta_*(I)(\theta_*(I))$ is an intuitionistic fuzzy left(right, two-sided)-ideal of M.

Definition 4.3: Let θ be a congruence relation on M. An $\mathcal{IFS}$ B of M is called an upper (lower) rough intuitionistic fuzzy bi-ideal of M if $\theta_*(B)(\theta_*(B))$ is an intuitionistic fuzzy bi-ideal of M.

Definition 4.4: Let θ be a congruence relation on M. An $\mathcal{IFS}$ I of M is called an upper (lower) rough intuitionistic fuzzy (1,2)-ideal of M if $\theta_*(B)(\theta_*(B))$ is an intuitionistic fuzzy (1,2)-ideal of M.

Definition 4.5: Let θ be a congruence relation on M. An $\mathcal{IFS}$ I of M is called an rough intuitionistic fuzzy left (right, two-sided, bi-(1,2)-)ideal of M if $\theta_*(I) and\ \theta_*(I))$ are both intuitionistic fuzzy left (right,two-sided,bi-(1,2)-) ideal of M.

Theorem 4.6:

Let θ be a congruence relation on M. If I is an intuitionistic fuzzy sub Γ-semigroup of M, then

- I is an upper rough intuitionistic fuzzy sub Γ-semigroup of M.
- If θ is complete,then I is lower rough intuitionistic fuzzy sub Γ-semigroup of M.

Proof:

Since θ is a congruence relation on M. $[a]_\theta \Gamma [b]_\theta \subseteq [a\Gamma b]_\theta \forall a, b \in M.$

(i) Let $I = (\mu_I, \nu_I)$ be an rough intuitionistic fuzzy sub Γ-semigroup of M and $x, y \in M$and $\gamma \in \Gamma$. Now $\theta^*(I) = (\theta^*(\mu_I), \theta^*(\nu_I))$. Then

$$\begin{aligned}
\theta^*(\mu_I)(x\gamma y) &= \vee_{z\in[x\gamma y]_\theta} \mu_I(z) \\
&\geq \vee_{z\in[x]_\theta\gamma[y]_\theta} \mu_I(z) \\
&= \vee_{a\gamma b\in[x]_\theta\gamma[y]_\theta} \mu_I(a\gamma b) \\
&\geq \vee_{a\in[x]_\theta, b\in[y]_\theta} [\mu_I(a) \wedge \mu_I(b)] \\
&= \vee_{a\in[x]_\theta} \mu_I(a) \wedge \vee_{b\in[y]_\theta} \mu_I(b) \\
&= \theta^*(\mu_I)(x) \wedge \theta^*(\mu_I)(y).
\end{aligned}$$

$$\begin{aligned}
\theta^*(\nu_I)(x\gamma y) &= \wedge_{z\in[x\gamma y]_\theta} \nu_I(z) \\
&\leq \wedge_{z\in[x]_\theta\gamma[y]_\theta} \nu_I(z) \\
&= \wedge_{a\gamma b\in[x]_\theta\gamma[y]_\theta} \nu_I(a\gamma b) \\
&\leq \wedge_{a\in[x]_\theta, b\in[y]_\theta} [\nu_I(a) \vee \nu_I(b)] \\
&= \wedge_{a\in[x]_\theta} \nu_I(a) \vee \wedge_{b\in[y]_\theta} \nu_I(b) \\
&= \theta^*(\nu_I)(x) \vee \theta^*(\nu_I)(y)
\end{aligned}$$

(ii) Since θ is a complete congruence relation on M, $[a]_\theta \Gamma [b]_\theta = [a\Gamma b]_\theta \forall a, b \in M$. Let $I = (\mu_I, \nu_I)$ be an rough intuitionistic fuzzy sub Γ-semigroup of M and $x, y \in M$. Now $\theta_*(I) = (\theta_*(\mu_I), \theta_*(\nu_I))$. Then

$$\begin{aligned}
\theta_*(\mu_I)(x\gamma y) &= \wedge_{z\in[x\gamma y]_\theta} \mu_I(z) \\
&= \wedge_{z\in[x]_\theta\gamma[y]_\theta} \mu_I(z) \\
&= \wedge_{a\gamma b\in[x]_\theta\gamma[y]_\theta} \mu_I(a\gamma b)
\end{aligned}$$

$$\geq \bigwedge_{z\in[x]_\theta\gamma[y]_\theta}\mu_I(z)$$
$$= \bigwedge_{a\in[x]_\theta}\mu_I(a) \vee \bigwedge_{b\in[y]_\theta}\mu_I(b)$$
$$= \theta_*(\mu_I)(x)\wedge\theta_*(\mu_I)(y)$$
$$\theta_*(\nu_I)(x\gamma y) = \bigvee_{z\in[x\gamma y]_\theta}\nu_I(z)$$
$$= \bigvee_{z\in[x]_\theta\gamma[y]_\theta}\nu_I(z)$$
$$= \bigvee_{a\gamma b\in[x]_\theta\gamma[y]_\theta}\nu_I(a\gamma b)$$
$$\leq \bigvee_{a\in[x]_\theta,b\in[y]_\theta}[\nu_I(a)\vee\nu_I(b)]$$
$$= \bigvee_{a\in[x]_\theta}\nu_I(a)\vee\bigvee_{b\in[y]_\theta}\nu_I(b)$$
$$= \theta_*(\nu_I)(x)\vee\theta_*(\nu_I)(y).$$

This shows that θ_*(I) is an intuitionistic fuzzy sub Γ-semigroup of M.

Theorem 4.7:

Let θ be a congruence relation on M. If I is an intuitionistic fuzzy left (right,two-sided)ideal of M, Then

- I is an upper rough intuitionistic fuzzy left (right,two-sided) ideal of M.
- If θ is complete, then I is lower rough intuitionistic fuzzy left (right, two-sided) ideal of M.

Proof:

Let $I = (\mu_I, \nu_I)$ be an rough intuitionistic fuzzy left ideal of M $x, y \in M$and $\gamma \in \Gamma$.

(i) Now $\theta^*(I) = (\theta^*(\mu_I), \theta^*(\nu_I))$. Then

$$\theta^*(\mu_I)(x\gamma y) = \bigvee_{z\in[x\gamma y]_\theta}\mu_I(z)$$
$$\geq \bigvee_{z\in[x]_\theta\gamma[y]_\theta}\mu_I(z)$$
$$= \bigvee_{a\gamma b\in[x]_\theta\gamma[y]_\theta}\mu_I(a\gamma b)$$
$$\geq \bigvee_{a\in[x]_\theta,b\in[y]_\theta}\mu_I(b)$$
$$= \bigvee_{b\in[y]_\theta}\mu_I(b)$$
$$= \theta^*(\mu_I)(y).$$
$$\theta^*(\nu_I)(x\gamma y) = \bigwedge_{z\in[x\gamma y]_\theta}\nu_I(z)$$
$$\leq \bigwedge_{z\in[x]_\theta\gamma[y]_\theta}\nu_I(z)$$
$$= \bigwedge_{a\gamma b\in[x]_\theta\gamma[y]_\theta}\nu_I(a\gamma b)$$
$$\leq \bigwedge_{a\in[x]_\theta,b\in[y]_\theta}\nu_I(b)$$
$$= \bigwedge_{b\in[y]_\theta}\nu_I(b)$$
$$= \theta^*(\nu_I)(y)$$

This shows that $\theta^*(I)$ is an intuitionistic fuzzy left ideal of M. Therefore I is an upper rough intuitionistic fuzzy left ideal of M. Similarly we can prove the other cases.

(ii) Since θ is a complete congruence relation on M, $[a]_\theta\Gamma[b]_\theta = [a\Gamma b]_\theta \forall a, b \in M$. Now $\theta_*(I) = (\theta_*(\mu_I), \theta_*(\nu_I))$. Then

$$\theta_*(\mu_I)(x\gamma y) = \bigwedge_{z\in[x\gamma y]_\theta}\mu_I(z)$$
$$= \bigwedge_{z\in[x]_\theta\gamma[y]_\theta}\mu_I(z)$$
$$= \bigwedge_{a\gamma b\in[x]_\theta\gamma[y]_\theta}\mu_I(a\gamma b)$$
$$\geq \bigwedge_{z\in[x]_\theta\gamma[y]_\theta}\mu_I(b)$$
$$= \bigwedge_{b\in[y]_\theta}\mu_I(b)$$
$$= \theta_*(\mu_I)(y)$$
$$\theta_*(\nu_I)(x\gamma y) = \bigvee_{z\in[x\gamma y]_\theta}\nu_I(z)$$
$$= \bigvee_{z\in[x]_\theta\gamma[y]_\theta}\nu_I(z)$$
$$= \bigvee_{a\gamma b\in[x]_\theta\gamma[y]_\theta}\nu_I(a\gamma b)$$
$$\leq \bigvee_{a\in[x]_\theta,b\in[y]_\theta}\nu_I(b)$$
$$= \bigvee_{b\in[y]_\theta}\nu_I(b)$$
$$= \theta_*(\mu_I)(y).$$

This shows that θ_*(I) is an intuitionistic fuzzy left ideal of M. Therefore I is an upper rough intuitionistic fuzzy left ideal of M. Similarly we can prove the other cases.

Theorem 4.8:

Let θ be a congruence relation on M. If I is an intuitionistic fuzzy bi-ideal of M, then I is an upper rough intuitionistic fuzzy bi-ideal of M.

Proof:

Since θ is a congruence relation on M. $[a]_\theta\Gamma[b]_\theta \subseteq [a\Gamma b]_\theta \forall a, b \in M$.

(i) Let $I = (\mu_I, \nu_I)$ be an rough intuitionistic fuzzy bi-ideal of M and $x, y, w \in M$ and $\alpha, \beta \in \Gamma$. Then

$$\theta^*(\mu_I)(x\alpha w\beta y) = \bigvee_{z\in[x\alpha w\beta y]_\theta}\mu_I(z)$$

$$\geq \vee_{z\in[x]_\theta\gamma[w]_\theta\gamma[y]_\theta} \mu_I(z)$$
$$= \vee_{a\alpha m\beta b\ \in[x]_\theta\gamma[w]_\theta\gamma[y]_\theta} \mu_I(a\alpha m\beta b)$$
$$\geq \vee_{a\in[x]_\theta, m\in[w]_\theta b\in[y]_\theta} [\mu_I(a)\wedge\mu_I(b)]$$
$$= \vee_{a\in[x]_\theta, b\in[y]_\theta} [\mu_I(a)\wedge\mu_I(b)]$$
$$= \vee_{a\in[x]_\theta} \mu_I(a)\wedge\vee_{b\in[y]_\theta}\mu_I(b)$$
$$= \theta^*(\mu_I)(x)\wedge\theta^*(\mu_I)(y).$$

Similarly we have proved $\theta^*(\nu_I)(x\alpha w\beta y)\leq \theta^*(\nu_I)(x)\vee\theta^*(\nu_I)(y)$. This shows that $\theta^*(I)$ is an intuitionistic fuzzy bi-ideal of M. Therefore I is an upper rough intuitionistic fuzzy bi-ideal of M.

Theorem 4.9:

Let θ be a congruence relation on M. If I is an intuitionistic fuzzy bi-ideal of M, then I is an lower rough intuitionistic fuzzy bi-ideal of M.

Proof:

Since θ is a congruence relation on M. $[a]_\theta\Gamma[b]_\theta\subseteq[a\Gamma b]_\theta\forall a,b\in M$.

(i) Let $I=(\mu_I,\nu_I)$ be an rough intuitionistic fuzzy bi-ideal of M and $x,y,w\in M$and $\alpha,\beta\in\Gamma$. Then

$$\theta_*(\mu_I)(x\alpha w\beta y) = \wedge_{z\in[x\alpha w\beta y\ ]_\theta}\mu_I(z)$$
$$= \wedge_{z\in[x]_\theta\gamma[w]_\theta\gamma[y]_\theta} \mu_I(z)$$
$$= \wedge_{a\alpha m\beta b\ \in[x]_\theta\gamma[w]_\theta\gamma[y]_\theta} \mu_I(a\alpha m\beta b)$$
$$\geq \wedge_{a\in[x]_\theta, m\in[w]_\theta b\in[y]_\theta} [\mu_I(a)\wedge\mu_I(b)]$$
$$= \wedge_{a\in[x]_\theta, b\in[y]_\theta} [\mu_I(a)\wedge\mu_I(b)]$$
$$= \wedge_{a\in[x]_\theta} \mu_I(a)\wedge\wedge_{b\in[y]_\theta}\mu_I(b)$$
$$= \theta_*(\mu_I)(x)\wedge\theta_*(\mu_I)(y)$$

Similarly we have proved $\theta_*(\nu_I)(x\alpha w\beta y)\leq\theta_*(\nu_I)(x)\vee\theta_*(\nu_I)(y)$. This shows that $\theta_*(\nu_I)$ is an intuitionistic fuzzy bi-ideal of M. Therefore I is an lower rough intuitionistic fuzzy bi-ideal of M.

Theorem 4.10:

Let θ be a congruence relation on M. If I is an intuitionistic fuzzy (1,2)-ideal of M, then it is an upper rough intuitionistic fuzzy (1,2)-ideal of M.

Proof:

Proof is similar to Theorem 4.8.

Theorem 4.11:

Let θ be a congruence relation on M. If I is an intuitionistic fuzzy (1,2)-ideal of M, then it is an lower rough intuitionistic fuzzy (1,2)-ideal of M.

Proof:

Proof is similar to Theorem 4.9.

Theorem 4.12:

Let θ be a congruence relation on M. If I and J are intuitionistic fuzzy left ideal and intuitionistic fuzzy right ideal of M, respectively, then $\theta^*(I\circ J)\subseteq\theta^*(I)\cap\theta^*(J)$.

Proof:

Let $I=(\mu_I,\nu_I), J=(\mu_J,\nu_J)$ be intuitionistic fuzzy left ideal and intuitionistic fuzzy right ideal of M, respectively and $x\in M$. Then

$$\theta^*(I\circ J) = (\theta^*(\mu_I\circ\mu_J),\theta^*(\nu_I\circ\nu_J)) \text{ and } \theta^*(I)\cap\theta^*(J) = \left(\theta^*(\mu_I)\cap\theta^*(\mu_J),\ \theta^*(\nu_I)\cup\theta^*(\nu_J)\right).$$

To show: $\theta^*(I\circ J)\subseteq\theta^*(I)\cap\theta^*(J)$ wehave to prove for all $x\in M$and $\gamma\in\Gamma$,

$\theta^*(\mu_I\circ\mu_J)(x)\leq\left(\theta^*(\mu_I)\cap\theta^*(\mu_J)\right)(x)$ and $\theta^*(\nu_I\circ\nu_J)(x)\geq\left(\theta^*(\nu_I)\cap\theta^*(\nu_J)\right)(x)$. Now for all $x\in M$,

$$\theta^*(\mu_I\circ\mu_J)(x) = \vee_{a\in[x]_\theta}(\mu_I\circ\mu_J)(a)$$
$$= \vee_{a\in[x]_\theta}\vee_{a=y\gamma z}[\mu_I(y)\wedge\mu_J(z)]$$
$$\leq \vee_{a\in[x]_\theta}\vee_{a=y\gamma z}[\mu_I(y\gamma z)\wedge\mu_J(y\gamma z)]$$
$$= \vee_{a\in[x]_\theta}[\mu_I(a)\wedge\mu_J(a)]$$
$$\leq \vee_{a\in[x]_\theta, b\in[x]_\theta}[\mu_I(a)\wedge\mu_J(a)]$$
$$= \vee_{a\in[x]_\theta}\mu_I(a)\wedge\ \vee_{b\in[x]_\theta}\mu_J(b)$$
$$= \theta^*(\mu_I)(x)\wedge\theta^*(\mu_J)(x)$$
$$= \left(\theta^*(\mu_I)\cap\theta^*(\mu_J)\right)(x)$$

Similarly we have to prove $\theta^*(\nu_I\circ\nu_J)(x)\geq\left(\theta^*(\nu_I)\cap\theta^*(\nu_J)\right)(x)$.

Hence $\theta^*(I\circ J)\subseteq\theta^*(I)\cap\theta^*(J)$.

Theorem 4.13:

Let θ be a congruence relation on M. If I and J are intuitionistic fuzzy right ideal and intuitionistic fuzzy left ideal of M, respectively, then $\theta_*(I \circ J) \subseteq \theta_*(I) \cap \theta_*(J)$.

Proof:

Let $I = (\mu_I, \nu_I), J = (\mu_J, \nu_J)$ be intuitionistic fuzzy right ideal and intuitionistic fuzzy left ideal of M, respectively and $x \in M$.

Then $\theta_*(I \circ J) = (\theta_*(\mu_I \circ \mu_J), \theta_*(\nu_I \circ \nu_J))$ and $\theta_*(I) \cap \theta_*(J) = \left(\theta_*(\mu_I) \cap \theta_*(\mu_J), \theta_*(\nu_I) \cup \theta_*(\nu_J)\right)$.

To show: $\theta_*(I \circ J) \subseteq \theta_*(I) \cap \theta_*(J)$ wehave to prove for all $x \in M$and $\gamma \in \Gamma$,

$\theta_*(\mu_I \circ \mu_J)(x) \leq \left(\theta_*(\mu_I) \cap \theta_*(\mu_J)\right)(x)$ and $\theta_*(\nu_I \circ \nu_J)(x) \geq \left(\theta_*(\nu_I) \cap \theta_*(\nu_J)\right)(x)$. Now for all $x \in M$,

$$\begin{aligned}
\theta_*(\mu_I \circ \mu_J)(x) &= \wedge_{a \in [x]_\theta} (\mu_I \circ \mu_J)(a) \\
&= \wedge_{a \in [x]_\theta} \vee_{a = y\gamma z} [\mu_I(y) \wedge \mu_J(z)] \\
&\leq \wedge_{a \in [x]_\theta} \vee_{a = y\gamma z} [\mu_I(y\gamma z) \wedge \mu_J(y\gamma z)] \\
&= \wedge_{a \in [x]_\theta} [\mu_I(a) \wedge \mu_J(a)] \\
&= \wedge_{a \in [x]_\theta} \mu_I(a) \wedge \wedge_{b \in [x]_\theta} \mu_J(b) \\
&= \theta_*(\mu_I)(x) \wedge \theta_*(\mu_J)(x) \\
&= \left(\theta_*(\mu_I) \cap \theta_*(\mu_J)\right)(x)
\end{aligned}$$

Similarly we have to prove $\theta_*(\nu_I \circ \nu_J)(x) \geq \left(\theta_*(\nu_I) \cap \theta_*(\nu_J)\right)(x)$.

Hence $\theta_*(I \circ J) \subseteq \theta_*(I) \cap \theta^*(J)$.

5. Conclusions:

The main objective of this paper is to concentrate our study on algebraic properties of rough intuitionistic fuzzy ideals with respect to Γ-semigroups. The theory of Γ-semigroups, theory of rough sets and intuitionistic fuzzy sets have many applictions in various fields such as artificial intelegence , automata theory and knowledge discovery. In this paper we introduce the notion of rough intitionistic fuzzy ideals in Γ-semigroups.

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