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EXACTNESS OF COMMUTATIVE (IMPLICATIVE) SOFT G-MODULES

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Abstract:

In this paper, we obtain some characterizations for soft sub modules of modules with respect to image, pre image and upper α-inclusion.

Key Words: Soft Set, Soft Homomorphism, Epimorphism, Soft Modules, UP-Module, Partially Ordered Set, Pre Image & Bounded

Introduction:

The notions of ideals in BCK-algebras and positive implicative ideals in BCK-algebras (i.e Isekis implicative ideals) were introduced by Iseki [2]. The notions of commutative (sub-commutative) ideals in BCK-algebras, positive implicative and implicative (Sub-implicative), ideals in BCK-algebras were introduced by [4, 5]. In 1991, Xi [15] applied this concept to BCK-algebras, and he introduced the notion of fuzzy sub - algebras (ideals) of the BCK-algebras. Prabpayak and Leerawat [13, 14] introduced a new algebraic structure which is called KU-algebra. They gave the concept of homomorphisms of KU-algebras and investigated some related properties. Mostafa et al. [9] introduced the notion of sub implicative (sub-commutative) ideals of KU-algebras and investigated of their properties. BCK-algebras form an important class of logical algebras introduced by Iseki [2] and was extensively investigated by several researchers. It is an important way to research the algebras by its ideals. Zadeh [16] introduced the notion of fuzzy sets. At present this concept has been applied to many mathematical branches, such as group, functional analysis, probability theory, topology, and so on.Mostafa et al. [9] introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Senapati et al. [7,8] introduced the notion of fuzzy KU-subalgebras (fuzzy KU-ideals) of KU-algebras with respect to a given t-norm, intuitionistic fuzzy bi-normed KU-ideals of a KU-algebra and obtained some of their properties.In this paper, we obtain some characterizations for soft sub modules of a modules with respect to image, pre image and upper α-inclusion.

Basic Definitions and Preliminary Concepts:

In this section, we provide basic definitions and results regarding UP-algebra's (modules), soft UP-sub modules and UP-module homomorphisms.

Definition 2.1: [UP-algebra]: [A. Lampan]: An algebra A = (A, •, 0) of type (2, 0) is called a UP-algebra, where A is a nonempty set, · is a binary operation on A, and 0 is a fixed element of A (i.e., a nullary operation) if it satisfies the following axioms: for any $x, y, z \in A$,

(UP-1): $(y \bullet z) \bullet ((x \bullet y) \bullet (x \bullet z)) = 0$,
(UP-2): $0 \bullet x = x$,
(UP-3): $x \bullet 0 = 0$, and
(UP-4): $x \bullet y = y \bullet x = 0$ implies $x = y$. From [6], we know that the notion of UP-algebras is a generalization of KUalgebras.

Example 2.2 [A. Lampan]: Let A = {0, 1, 2, 3} be a set with a binary operation '•'defined by the following Cayley table: ·

•	0	1	2	3
0	0	1	2	3
1	0	0	0	0
2	0	1	0	3
3	0	1	2	0

Then (A, ·, 0) is a UP-algebra. Let (X, *, 0) be an UP-algebra. Then X is a partially ordered set with the following partial ordering $\leq$ defined on X by $x \leq y$ if and only if $x * y = 0$. X is said to be bounded if there is an element $1 \,\varepsilon\, X$ such that $x \leq 1$ for all $x \,\varepsilon\, X$. X is said to be commutative (implicative) if $x \wedge y = y \wedge x$ $(x * (y * x) = x)$ for all $x,y \,\varepsilon\, X$, where $x \wedge y = y * (y * x)$.

Example 2.3: Let X = { 0,1,2,3,4} and * be a binary operation on X defined as $x_1 * x_2 = x_1 = \min(x_1, x_2)$ for all $x_1, x_2 \,\varepsilon\, X$. Then (X, *, 0) forms a commutative UP-algebra.

Definition 2.4: [M. Aslamet. al]: Let M be an UP-algebra. Then by left M-module (abbreviated M-module) we mean an abelian group G with an operation $M \times G \rightarrow G$ with $(m,g) \rightarrow mg$ satisfies the following axioms for all $m, n \,\varepsilon\, M$ and $g_1, g_2 \,\varepsilon\, G$;

- ✓ $(m \wedge n)\, g_1 = m\,(n g_1)$

- ✓ $m(g_1 + g_2) = mg_1 + mg_2$
- ✓ $0g = 0$.

Moreover, if M is bounded and G satisfies 1.g = g , for all g ε G , then G is said to be unitary.

Example 2.5: Any bounded implicative UP-algebra M of a M-module, where '+ ' is defined as m + n = (m * n) ν (n *m) and mn = m ∧ n. A subgroup N of a M-module G is called sub module of G if N is also a M-module.

Theorem 2.6 [M. Bakhshi]: A subset N of a UP-module G is a UP- sub module of G if and only if $h_1 – h_2$ ε H and mh ε H for all h_1, h_2, h ε H and m ε M.

Definition 2.7 [M. Aslam et. al]: Let G and H be modules over an UP-algebra M. A mapping F: G → H is called M-homomorphism if

- ✓ $F(g_1 + g_2) = f(g_1) + f(g_2)$
- ✓ F (mg) = m f(g) for all g_1, g_2, g ε G and m ε M.

An UP-module homomorphism is said to be monomorphism (epimorphism) if it is one –one (on to). If it is both one-one and onto, then we say that it is an isomorphism.

Definition 2.8 [Soft set]: A pair (F, A) is called a soft set (S-set) over U, where F is a mapping given by F : A → P(U). The pair (F, A) is sometimes called as F_A.

Definition 2.9: A soft set Φ of an UP-module M is said to be a soft UP-submodule if it satisfies

(SUPM-1) Φ (a +b) ≥ T {Φ (a) , Φ (b)}
(SUPM-2) Φ (-a) = Φ (a) and
(SUPM-3) Φ (xa) ≥ Φ(a).
For the sake of simplicity, we shall use the symbols SS(M) and S(M) for the set of all soft sets of M and the set of all soft UP-modules of M , respectively.

Definition 2.10: Let F_A, F_B, $F_C$bein soft UP- modules of $S(M_1)$, $S(M_2)$ and $S(M_3)$ respectively. Then a sequence $F_A{}^{(\Phi 1,\,\Phi 1)}\!\!\rightarrow F_B{}^{(\Phi 2,\Phi 2)}\!\!\rightarrow$ FC of S(M)-homomorphism is said to be exact soft UP-module (S(M)-Exact) at F_B if the following conditions are satisfied.

- ✓ $M_1{}^{(\Phi 1)}\!\!\rightarrow M_2{}^{(\Phi 2)} \rightarrow M_3$ is exact.
- ✓ $A_1{}^{(\theta 1)}\!\!\rightarrow A_2{}^{(\theta 2)}\!\!\rightarrow A_3$ is exact.

Definition 2.11: Let M=0 and A = 0 , then $F_A = 0$. we call FA is a zero UP-modules.

Theorem 2.12: A soft set Φ ε S(M) if and only if

- ✓ Φ (xa) ≥ Φ(a)
- ✓ Φ(a-b) ≥ T { Φ(a) , Φ(b)} for all a,b ε M and x ε X.

Definition 2.13: Let f : M → N be an UP-module homomorphism and let Φ ε SS(M) be a soft set on M. Then the image of f(Φ) of Φ under f is defined as follows

$$f(\Phi)(x) = \begin{cases} \max \Phi(m) & \text{if } f^{-1}(x) \neq 0 \\ 0 & \text{if } f^{-1}(x) = 0 \end{cases}$$

We shall call f(Φ) the homomorphic image of Φ under f .

Some Standard Results:

Theorem 3.1: Let f: M→ N be an UP-module epimorphism and let Φ be a soft sub module of M. Then the homomorphic image f (Φ) of Φ under f is a soft UP- sub module of N.

Definition 3.2: Let f: M → N be a homorphism of UP-modules and χ a soft set of N.Then the inverse image of χ, f-1(χ) is a soft set on M given by $f^{-1}(\chi) = \chi(f(m)0$ for all m ε M.

Theorem 3.3: Let f: M → N be a homorphism of UP-modules and χ ε S(M), and then the inverse image $f^{-1}(\chi)$ ε S(M).

Theorem 3.4: Let f: M → N be a epimorphism of UP-modules . If χ is a soft set on N such that the and the inverse image $f^{-1}(\chi)$ ε S(M), then χ ε S(M).

Theorem 3.5: For a soft set on M, the following are equivalent;

- ✓ Φ is soft UP-sub module on M.
- ✓ $m_{(\alpha,\beta)}(\Phi)$ is a soft UP-sub module of M for all α ,β ε [0,1].

Proof: In this theorem, we have

(i) implies (ii),. Now let α, β ε [0,1] be such that $m_{(\alpha,\beta)}(\Phi)$ is a soft sub module on M and let x ε X , m_1, m_2, m ε M. Then Φ(xm). α = m_α (Φ)(xm) ≥ m_α (Φ)(m) = Φ(m) . α

And sinceα ≠ 0 and β ≠ 0, then we have Φ (xm) ≥ Φ (m).

Now Φ (m_1-m_2) = m_α (Φ) (m_1-m_2)
≥ T { $m_\alpha(\Phi)(m_1)$, $m_\alpha(\Phi)(m_2)$}
= T { $\Phi(m_1)$.α , $\Phi(m_2)$.α} = T { $\Phi(m_1)$, $\Phi(m_2)$}.αwhich means that
Φ (m_1-m_2) ≥ T { $m\alpha(\Phi)(m_1)$, $m\alpha(\Phi)(m_2)$}.

Hence Φ is soft sub module on M.

Theorem 3.6: Let δ_N and Δ_k be a soft sets over M , where N and k are sub modules of M and χ be a module homomorphism fron N to k. If δ_N is soft sub module of M, then so is $\chi(\delta_N)$.

Proof: Let k_1, k_2 ε K.Since χ is surjective, there exists n_1, n_2 ε N such that $\chi(n_1) = k_1$ and $\chi(n_2) = k_2$.

Thus $\chi(\delta N)(k_1-k_2) \geq T\ \{\ \chi(\delta_N)(k_1), \chi(\delta_N)(k_2)\}$ is satisfied.

Now let r ε R and k ε K. Since χ is surjective, there exists n εN such that $\chi(n) = k$.

Then

$$\begin{aligned}
\chi(\delta_N)(rk) &= \cup\ \{\delta(x)\ /\ n\ \varepsilon\ N, \chi(n) = rk\} \\
&= \cup\ \{\delta(x)\ /\ n\ \varepsilon\ N, \chi n = \chi^{-1}(rk)\} \\
&= \cup\ \{\delta(x)\ /\ n\ \varepsilon\ N, i = \chi^{-1}(r\chi(n))\} \\
&= \cup\ \{\delta(x)\ /\ n\ \varepsilon\ N,\ i = \chi^{-1}(\chi(rn)) = rn\} \\
&= \cup\ \{\delta(x)\ /\ n\ \varepsilon\ N, \chi(n) = k\} \\
&\geq \cup\ \{\delta(x)\ /\ n\ \varepsilon\ N, \chi(n) = k\} \\
&= \chi(\delta_N)(k)
\end{aligned}$$

Hence $\chi(\delta_N)$ is a soft sub module of M.

Theorem 3.7: Letδ_N and Δ_k be a soft sets over M , where N and k are sub modules of M and χ be a module homomorphism from N to k. If Δk is soft sub module of M, then so is $\chi^{-1}(\Delta k)$.

Proof: Let n_1, n_2 ε N. Then

$\chi^{-1}(\Delta k)\ (n_1-n_2) \geq T\{\ \chi^{-1}(\Delta k)\ (n_1), \chi^{-1}(\Delta_k)\ (n_2)\}$ is satisfied.

Now let r ε R and n ε N. Then

$\chi^{-1}(\Delta_k)\ (rn) = \Delta\ (\chi(rn)) = \Delta\ (r\chi(n)) \geq \Delta\ (\chi(n)) = \chi^{-1}(\Delta_k)(n)$.

Hence $\chi^{-1}(\Delta_k)$ is a soft sub module of M.

Theorem 3.8: If δ_{N1} is a union soft sub module of M and δ_{N2} is union sub module of M, then so is δ_{N1} U δ_{N2}.

Theorem 3.9: If $\delta_N <$ M, then $\delta(0_M) \leq \delta(x)\}$ for all x ε N.

Theorem 3.10: If $\delta_N <$ M, then $N_\delta = \{x\ \varepsilon\ N\ /\ \delta(x) = \delta(0_M)\}$ is a sub module of N.

Theorem 3.11: Let δ_N be a soft set over M and α be a subset of M such that $\delta\ (0_M) \leq \alpha$. If δ_N is a union soft sub module of M, then $\delta_N\ (\alpha)$ is a sub module of M.

Definition 3.12: Let N be a sub module of M and δ_N be a soft set over M.Then δ_N is called a union soft sub module of M, denoted by $\delta_N <$ M if

(USM-1)$\delta(x\text{-}y)\ \leq S\ \{\ \delta(x)\ , \delta(y)\}$ and

(USM-2) $\delta(rx) \leq\ \delta(x)$, for all x,y ε N and r ε R.

Example 3.13: Consider the ring R = (Z_{12}, +, •), the left R-module M = (Z_{12}, +) with natural operation and the sub module N_1 = {0, 6} of M.

Let the soft δ_N over M where $\delta : N_1 \to P(M)$ is the set valued function defined by

$\delta(0) = \{0, 3, 4, 9\}$and $\delta(6) = \{\ 0,3,4,9,11\ \}$. Then it can be easily seen that $\delta_{N1} <$ M.

Now, let the submodule of M be N_2 = { 0,4,8} and the soft set Δ_{N2} over M,where $\Delta : N_2 \to \{0,4,8\} = P(M)$ is a set valued function defined by

$\Delta(0) = \{0,3,9\}$, $\Delta(4) = \{0,3,5,8,11\}$ and $\Delta(8) = \{0,3,5,8,9,11\}$. Then $\Delta(2 \bullet 4\ ) = \Delta\ (8) = \{0,3,5,8,9,11\}$ not less than or equal to $\Delta\ (4) = \{\ 0,3,5,8,11\}$. Therefore Δ_{N2} is not a union soft sub module of M.

Theorem 3.14: Let δ_N be a soft set over M. Then δ_N is a union soft sub module of M if and only if $\delta_N^{\,r}$ is a soft sub module of M.

Theorem 3.15: Let δ_N and Δ_k be a soft sets over M, where N and K are sub modules of M and χ be a module homomorphism from N to k. If Δ_k is a union soft sub module of M, then so is $\chi^{-1}(\Delta_k)$.

Theorem 3.16: Let δ_N and Δ_k be a soft sets over M, where N and K are sub modules of M and χ be a module isoomorphism from N to k. If δ_N is a union soft sub module of M, then so is $\chi^*(\delta_N)$.

Conclusion:

In this paper, we obtain some characterizations for soft sub modules of a modules with respect to image, pre image and upper α-inclusion. Also we provide basic definitions and results regarding UP-algebra's (modules), soft UP-sub modules and UP-module homomorphisms.

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