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CONSTRUCTION OF COMPLEX Q-FUZZY SOFT INTERSECTION GROUP BY UPPER AND LOWER T-SUBGROUPS

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Abstract:

In this article, we construct a soft intersection group under certain conditions and a soft union group is investigated by this upper t-subgroups and lower t-subgroups with suitable examples.

Key Words: Soft Set, Soft Group, Soft Intersection Group, Soft Union Group, Upper T-Subgroup & Lower T-Subgroup.

Introduction:

Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [15] described the application of soft set theory to a decision making problem. Maji et al. [15] also studied several operations on the theory of soft sets. The notion of soft set was introduced in 1999 by Molodtsov [14]. Since its inception, Maji et al. [15] and Ali et al. [2] introduced several operations of soft sets and Sezgin et al. [21] studied on soft set operations in more detail. And the theory of soft set has gone through remarkably rapid strides with a wide-ranging applications especially in decision making [6, 7, 16, 17, 23, 24, 19, 36, 25]. There have been also attempt to softificate various mathematical structures like groups [3], semirings [8], BCK/BCI-algebras [9, 10], p-ideals [11], BCH-algebras [12], rings [1], near-rings [20], mappings [18], soft lattices [13], substructures of rings, fields and modules [4]. Cagman et al. [5] introduced the concept of soft int-group which is based on the inclusion relation and the intersection of sets, and therefore more functional for developing the theory of soft groups. With the motivation of soft int-group, Sezgin et al. [22] moreover defined soft uni-group which is based on the inclusion relation and the union of sets and its basic properties of soft uni groups as regards normal soft uni-groups as well. In this article, we construct a soft intersection group under certain conditions and a soft union group is investigated by this upper t-subgroups and lower t-subgroups with suitable examples.

Preliminaries and Basic Concepts:

Definition 2.1: A pair (F, A) is called a soft set over U, where F is a mapping given by F: $A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively.

Definition 2.2: The relative complement of the soft set F_A over U is denoted by $F^r{}_A$, where $F^r{}_A : A \rightarrow P(U)$ is a mapping given as $F^r{}_A(a) = U \setminus F_A(a)$, for all $a \in A$.

Definition 2.3: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \Phi$. The restricted intersection of F_A and G_B is denoted by $F_A \Cup G_B$ and is defined as $F_A \Cup G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

Definition 2.4: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \Phi$. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$ and is defined as $F_A \cup_R G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

Definition 2.5: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi(F_A)$ over U, where $\psi(F_A) : B \rightarrow P(U)$ is a set valued function defined by $\psi(F_A)(b) = \bigcup\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$,

If $\psi^{-1}(b) \neq \Phi$, = 0 otherwise for all $b \in B$. Here, $\psi(F_A)$ is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B) : A \rightarrow P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all $a \in A$. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

Definition 2.6: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^{\star}(F_A)$ over U, where $\psi^{\star}(F_A) : B \rightarrow P(U)$ is a set-valued function defined by $\psi^{\star}(F_A)(b) = \bigcap\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \Phi$, =0 otherwise for all $b \in B$. Here, $\psi^{\star}(F_A)$ is called the soft anti image of F_A under ψ.

Definition 2.7: Let f_A be a soft set over U and α be a subset of U. Then, upper α-inclusion of a soft set f_A, denoted by $f^{\alpha}A$, is defined as $f^{\alpha}A = \{x \in A : f_A(x) \supseteq \alpha\}$

Definition 2.8: Let X be any non-empty set. A mapping $\delta : X \rightarrow [0,1]$ is called fuzzy set on X.

Definition 2.9: A complex fuzzy soft subset A, defined on a universe of discourse X, is characterized by a membership function τ_A(x) that assigns any element x∈X and a complex valued grade of membership in A. The values of τ_A(x) all lie within the unit circle in the complex plane and are thus all of the form P_A(x) $e^{j\mu_A(x)}$ where P_A(x) and $e^{j\mu_A(x)}$ are both real valued and P_A(x) ∈[0,1]. Here P_A(x) is termed as amplitude term and $e^{j\mu_A(x)}$ is termed as phase term.

The complex fuzzy soft set may be represented in the set form as A = {(x, τ_A(x)) / x∈X }. It is denoted by CFSS. The phase term of complex membership function belongs to (0,2π). Now we take those forms which presented to define the game of winner, neutral and loser.

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & \text{if } p_A > p_B \\ \mu_B(x) & \text{if } p_A < p_B \end{cases}.$$

This is a novel concept and it is the generalization of the concept "winner take all" for the union of phase terms.

Definition 2.10: Let X = {x_1, x_2, x_3} be a universe of discourse. Let A and B be complex fuzzy soft sets in X as shown below.

$$A = \{0.6\, e^{i(0.8)},\ 0.3\, e^{i\frac{3\pi}{4}},\ 0.5\, e^{i(0.3)}\}$$
$$B = \{0.8\, e^{i(0.9)},\ 0.1\, e^{i\frac{\pi}{4}},\ 0.4\, e^{i(0.5)}\}$$
$$A\cup B = \{0.8\, e^{i(0.9)},\ 0.3\, e^{i\frac{3\pi}{4}},\ 0.5\, e^{i(0.3)}\}$$

We can easily calculate the phase terms $e^{i\mu_{A\cap B}(x)}$ on the same line by winner, neutral and loser game.

Definition 2.11: Here a complex fuzzy soft subset A of a group G is said to be a complex fuzzy soft subgroup of G if for all x ,y ∈ G

(CFSSG1) A(xy) ≥ min { A(x), A(y) },

(CFSSG2) A(x^{-1}) ≥ A(x) where the product x and y is denoted by xy and the inverse of x by x^{-1}. It is well known and easy to see that a complex fuzzy soft subgroup G satisfies A(x)≤ A(e) and A(x^{-1}) = A(x) for all x ∈ G, where 'e' is the identity of G.

Some Applications of Complex Soft Intersection Structures:

Theorem 3.1: Let δ_G be a complex Q-fuzzy soft set over U, $\delta_G^{\supseteq t}$ be upper t-subgroups of δ_G for each t $\subseteq U$ and the set Im(δ_G) be ordered by combination. Then, δ_G is a complex fuzzy soft intersection group over U.

Proof: Let x,y € G and q € Q and δ_G(x,q)=t_1 and δ_G(y,q)=t_2.

Suppose that $t_1 \subseteq t_2$. It is obvious that x € $\delta_G^{\supseteq t_1}$ and y € $\delta_G^{\supseteq t_2}$.

Since $t_1 \subseteq t_2$, It is follows that x,y € $\delta_G^{\supseteq t_1}$ and since $\delta_G^{\supseteq t}$ is a subgroup of G for all t $\subseteq U$, it follows that x y^{-1} € $\delta_G^{\supseteq t_1}$. Hence δ_G(x y^{-1} ,q) ≥ t_1 = $t_1 \cap t_2$ = min { δ_G(x,q), δ_G(y,q)} . Thus δ_G is a complex Q-fuzzy soft intersection group over U.

Theorem 3.2: Let δ_G be a Complex Q-fuzzy soft set over U. $\delta_G^{\subseteq t}$ be lower t-subgroups of δ_G for each t $\subseteq U$ and the set Im(δ_G) by ordered by combination. Then δ_G is a complex Q – fuzzy soft Union group over U.

Proof: Let x,y € G, q € Q and δ_G (x,q) = t_1 and δ_G (y,q) = t_2

Suppose that $t_1 \subseteq t_2$. It is obvious that x € $\delta_G^{\subseteq t_2}$ and since $\delta_G^{\subseteq t}$ is a subgroup of G for all t ⊆ U, it follows that xy^{-1} € $\delta_G^{\subseteq t_2}$

Hence δ_G (xy^{-1} ,q) ≤ t_2 ≤ t_1 U t_2 = max { δ_G (x ,q), δ_G (y ,q) }

Therefore, δ_G is a CQFSU – Group over U. From now on, it will be more connected with a CQFSI – group and its upper t – subgroups.

Note 3.4: If δ_G is a CQFSI – group over U, then the set Im (δ_G) does not need to be ordered by combination as in the case of the following example.

Example 3.5: Assume that U = S_3 is the universal set and G = Z_6 is the set of parameters.

If a CQFSS is constructed by δ_G (0 ,q) = S_3 ;

δ_G (1,q) = δ_G (5,q) = {(12),(13),(132)}

δ_G (2 ,q) = δ_G (4 ,q) = {(12),(13), (23), (123), (132) }

δ_G (3 ,q) = { (1), (12), (13), (132) }

then δ_G is a CQFSI – group over U, whose image set is not ordered by combination.

Note that $\delta_G^{\supseteq S3}$ = { 0 }

$$\delta_G^{\supseteq\{(1),(12),(13),(132)\}} = \{ 0,3 \}$$
$$\delta_G^{\supseteq\{(12),(13),(23),(123)(132)\}} = \{ 0,2,4\}$$
$$\delta_G^{\supseteq\{(1),(12),(13),(132)\}} = Z_6$$

Thus upper t – subgroup {0} attains an infimum on 0, {0,3} on 3, {0,2,4} on 2 and 4, since δ_G (2 ,q) = δ_G (4 ,q) and Z_6 on 1 and 5.

Theorem 3.6: Any subgroup H of a group G can be realized as an upper t – subgroup of some CQFSI – group over U.

Proof: Let δ_G be a CQFS – set over U defined by

$\delta_G(x,q) = t$, if $x \in H$

$= \Phi$, if $x \notin H$ then δ_G is a CQFSI – group over U.

Let a , b € G.

Case (i) : Suppose a € H and b € H , then ab € H.

It follows that $\delta_G(ab,q) = t$ and $\delta_G(a,q) = \delta_G(b,q) = t$.

Thus, $\delta_G(ab,q) \geq \min\{\delta_G(a,q), \delta_G(b,q)\}$.

And also if a € H , then so is $\mathbf{a}^{-1}$, thus $\delta_G(a,q) = \delta_G(a^{-1},q) = t$.

Case (ii) : Now, suppose a € H and b $\notin$ H then ab $\notin$ H. It follows that

$\delta_G(a,q) = t$ and $\delta_G(b,q) = \delta_G(ab,q) = \Phi$.

Therefore, $\delta_G(ab,q) \geq \min\{\delta_G(a,q), \delta_G(b,q)\}$,

Furthermore $\delta_G(a,q) = \delta_G(a^{-1},q)$ if a € H or a $\notin$ H.

Case (iii) : Now, suppose that a $\notin$ H and b $\notin$ H . Then either a,b € H or a,b $\notin$ H. It is easy to show that in any cases, $\delta_G(ab,q) \geq \min\{\delta_G(a,q), \delta_G(b,q)\}$,

And $\delta_G(a,q) = \delta_G(a^{-1},q)$.

Hence δ_G is a CQFSI – group over U. Therefore, for this CQFSI group, $\delta_G^{\supseteq t} = H$.

Note 3.7: It is known that if δ_G is a CQFSI group over U, then $\delta_G(e,q) \geq \delta_G(x,q)$ for all x € G and q €nQ.

Let $\delta_G(e,q) \geq t_e$, then it turns out to be an interesting case to investigate the upper $t_{\mathbf{e}}$ subgroup of $\delta_G^{\supseteq t}e$ of G. Because it x € $\delta_G^{\supseteq t}e$, then $\delta_G(x,q) \geq t = \delta_G(e,q)$ and it appears that only e € $\delta_G^{\supseteq t}e$. But that is not always the case as seen in the following example.

Example 3.8: Consider the CQFSI – group in theorem 3.6. Assume that H $\neq$ { e } and H $\neq$ G. It is known that δ_G is a CQFSI – group over U and Im (δ_G) = { Φ , t }. Thus, two upper t – subgroups are $\delta_G^{\supseteq\phi} = G$ and $\delta_G^{\supseteq t}$ = { x € G / $\delta_G(x,q) \geq t$ } = H. Since e € H, $\delta_G(e,q)= t$; but $\delta_G^{\supseteq t} = H$, which is not equal to "e".

Definition 3.9: Let δ_G be a CQFSU – group over U. Then e – set of δ_G, denoted by $G_{\delta G}$, is defined as $G_{\delta G}$ = { x € G / $\delta_G(x,q)= \delta_G(e,q)$ }

Theorem 3.10: Let δ_G be a CQFSI – group over U. If $\delta_G(e,q) = t_{\mathbf{e}}$, then $\delta_G^{\supseteq t}{}_e = G_{\delta G}$.

Proof: $\delta_G^{\supseteq t}{}_e$ = { x € G / $\delta_G(x,q) \geq t_e$ }

= { x € G / $\delta_G(x,q) = t_e$ }

Since te $\geq$ $\delta_G(x,q)$ for all x € G and q € Q. $\delta_G^{\supseteq t}{}_e$ = { x € G / $\delta_G(x,q)= \delta_G(e,q)$ } = $G_{\delta G}$.

Note 3.11: Let δ_G be a CQFSI – group over U and (to, t1, t2, t3,.....t_n} € Im (δ_G) which satisfying that $t_o \geq t_1 \geq t_2 \geq t_3 \geq \ldots\ldots \geq t_n$. Then the family of Upper – t – subgroups from a chain, which denoted by C (δ_G) = $\delta_G^{\supseteq t}{}_0 < \delta_G^{\supseteq t}{}_1 < \ldots\ldots\ \delta_G^{\supseteq t}{}_n$. Not to our surprise, only some of the upper t – subgroups of δ_G form a chain. Since all the subgroups of G, In general, does not form a chain that is, it makes no sense to hope all the upper t – subgroups form a chain. In the connection, see example 1 { 0, 3 } $\not\subset$ { 0,2,4} and { 0,2,4 } $\not\subset$ (0, 3}. Of course if the number of the Im (δ_G) forms a chain, so does the upper t – subgroups of δ_G . For further detail, refer to the following theorem.

Theorem 3.12: Let G be a finite group, δ_G be a CQFSI – group over U and I be an arbitrary finite index set and G ($\delta_G^{\supseteq t}$) = { $\delta_G^{\supseteq t}{}_i$ / i € I, ti € Im (δ_G) } then we have the followings:

(i) There exists a unique $i_{\mathbf{e}}$ € I such that $t_{\mathbf{ie}} \geq t_{\mathbf{i}}$, for all i.

(ii) $G\,\delta_G = \bigcap_{i \varepsilon I} \delta_G^{\supseteq ti} = \delta_G^{\supseteq t_{ie}}$

(iii) $G = \bigcup_{i \in I} \delta_G^{\supseteq ti}$

(iv) If the members of Im (δ_G) forms a chain, so do is G ($\delta_G^{\supseteq ti}$)

Proof:

(i) Since $\delta_G(e,q)$ € Im (δ_G) , there exists a unique . ie € I such that $\delta_G(e,q)= t_{\mathbf{ie}}$. We know that $\delta_G(e,q) \geq \delta_G(x,q)$ for all x € G and q € Q. It follows that $t_{\mathbf{ie}} \geq \delta_G(x,q)$, for all x € G and q € Q. Thus $t_{ie} \geq t_i$, for all i € I.

(ii) Since in theorem 3.11 , it is proved that

G δ_G = $\delta_G^{\supseteq t_{ie}}$, where $\delta_G(e,q) = t_{ie}$.

It is only shows that G $\delta_G^{\supseteq t_{ie}} = \bigcap_{i\varepsilon I} \delta_G^{\supseteq ti}$ .Since $t_{ie} \geq t_i$ for all i € I

G $\delta_G^{\supseteq t_{ie}} \leq$ G $\delta_G^{\supseteq t_{ie}}$, for all i. Thus G $\delta_G^{\supseteq t_{ie}} \subseteq \bigcap_{i\varepsilon I} \delta_G^{\supseteq ti}$. Now let x € $\bigcap_{i\varepsilon I} \delta_G^{\supseteq ti}$, then x € $\delta_G^{\supseteq t_{ie}}$, for all i € I. This completes the proof.

Conclusion:

We construct a soft intersection group under certain conditions and a soft union group is investigated by this upper t-subgroups and lower t-subgroups with suitable examples. One can obtain the similar idea in the field of rough set theory and vague set theory.

References:

1. Acar, U., Koyuncu, F. and Tanay, B., Soft sets and soft rings, Comput. Math. Appl. 59, (2010), 3458-3463.
2. Ali, M.I., Feng, F., Liu, X., Min, W.K, and Shabir, M., On some new operations in soft set theory, Comput. Math. Appl. 57, (2009), 1547-1553.
3. Aktas. , H. and C. a˘gman, N., Soft sets and soft groups, Inform. Sci. 177, (2007), 2726-2735.
4. Atag¨un, A.O. and Sezgin, A., Soft substructures of rings, fields and modules, Comput. Math. Appl. 61(3), (2011), 592-601.
5. Cagman, N., Cıtak, F., Akta¸s, H., Soft int-group and its applications to group theory, Neural Comput. Appl. 21, (Issue 1-Supplement), (2012), 151-158.
6. C.agman, N., and Engino˘glu S., Soft matrix theory and its decision making, Comput. Math. Appl. 59, (2010) 3308-3314.
7. C.agman, N. and Engino˘glu, S. Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207, (2010) 848-855.
8. Feng, F., Jun, Y.B. and X. Zhao, Soft semirings, Comput. Math. Appl. 56, (2008), 2621-2628.
9. Jun, Y.B., Soft BCK/BCI-algebras, Comput. Math. Appl. 56, (2008), 1408-1413.
10. Jun, Y.B and Park, C.H., Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform. Sci. 178, (2008), 2466-2475.
11. Jun, Y.B., Lee, K.J. and Zhan, J. Soft p-ideals of soft BCI-algebras, Comput. Math. Appl. 58, (2009), 2060-2068.
12. Kazancı, O., Yılmaz, S. and Yamak, S., Soft sets and soft BCH-algebras, Hacet. J. Math. Stat. 39, (2), (2010), 205-217.
13. Karaaslan, F., Cagman, N. and Enginoglu, S., Soft Lattices, Journal of New Results in Science 1 (2012) 5-17.
14. Molodtsov, D., Soft set theory-first results, Comput. Math. Appl. 37, (1999), 19-31.
15. Maji, P.K., Biswas R. and Roy, A.R., Soft set theory, Comput. Math. Appl. 45, (2003), 555-562.
16. Maji, P.K., Roy, A.R., Biswas, R., An application of soft sets in a decision making problem, Comput. Math. Appl. 44, (2002), 1077-1083.
17. Molodtsov, D.A., Yu., V, Leonov and Kovkov, D. V., Soft sets technique and its application, Nechetkie Sistemi i Myakie Vychisleniya 1-1 (2006), 8-39.
18. Majumdar, P. and Samanta, S.K., On soft mappings, Comput. Math. Appl. 60, (9), (2010), 2666-2672.
19. Ma, X., Liu, Q., Zhan, J., A survey of decision making methods based on certain hybrid soft set models, Artificial Intelligence Review 47, (2017), 507–530.
20. Sezgin, A., Atag¨un, A.O. and Ayg¨un, E., A note on soft near-rings and idealistic soft nearrings, Filomat 25, (2011), 53-68.
21. Sezgin A., and Atag¨un, A.O, On operations of soft sets, Comput. Math. Appl. 61(5), (2011), 1457-1467.
22. A. Sezgin,N. Cagman, Z. Kaya Turk , E. Mu¸stuoglu, Soft uni-groups and its applications to group theory, submitted.
23. Zou, Y. and Xiao, Z., Data analysis approaches of soft sets under incomplete information, Knowledge Based Systems 21, (2008), 941-945.
24. Zhan, J. Liu, Q., Herawan, T.,A novel soft rough set: soft rough hemirings and its multicriteria group decision making, Applied Soft Computing 54, (2017), 393–402.
25. Zhan, J., Ali, M.I, Mehmood, N., On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods, Applied Soft Computing 56, (2017), 446–457.
26. Zhan, J., Zhu, K., A novel soft rough fuzzy set: Z-soft rough fuzzy ideals of hemirings and corresponding decision making, Soft Computing 21, (2017), 1923–1936.