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TEMPORAL INTERVAL VALUED BI FUZZY SUBGROUP OVER CERTAINNORMS

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Abstract:

The notion of temporal bi fuzzy subgroups (TBFS) is introduced, and related properties are investigated. Characterizations of a temporalbi fuzzy subgroup (TBFG) are established, and how the images or inverse images of temproal bi fuzzy subgroups become temporal bi fuzzy subgroups is studied.

Key Words: Fuzzy Set, Interval Number, Interval Valued Fuzzy Set, Temporal Bi Fuzzy Set, Temporal Bi Fuzzy Subgroup, Level Set & Pre-Image Homomorphism.

1. Introduction:

Atagün and Sezgin [2016] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Also union soft sub structures of near-rings and near-ring modules are studied in [2012]. Maji et al. [2002] presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets [2007]. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. The theory of fuzzy sets proposed by L.A. Zadeh [1965] in 1965, has achieved a great success in various fields. In 1999, Molodtsov [1999] introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory is free from the difficulties affecting existing methods. Presently, works on soft set theory are progressing rapidly. Maji et al. [2002] de.ned several operations on soft sets and made a theoretical study on the theory of soft sets. Roy and Maji [2003] gave some applications of fs-sets. Aktas. And Cagman [2007] compared soft sets with the related concepts of fuzzy sets and rough sets. In this paper, the notion of temporal bi fuzzy subgroups (TBFS) is introduced, and related properties are investigated. Characterizations of a temporal l bi fuzzy subgroup (TBFG) are established, and how the images or inverse images of temporal bi fuzzy subgroups become temporal bi fuzzy subgroups is studied.

2. Preliminaries:

Let I be a closed Interval, i.e., I = [0,1]. By an Interval number we mean a closed sub interval $\overline{a} = [\overline{a}, a^+]$ of I, where $0 \le \overline{a} \le a^+ \le 1$, denoted by D[0,1] the set of interval l numbers. Let us define what is known as refined minimum of two elements in D[0,1]. We also define the symbols " ≤ " , " ≥ " , " = " in case of two elements in D[0,1].

Consider two Interval numbers $\overline{a}_1 = [\overline{a}_1, a_1^+]$ and $\overline{a}_2 = [\overline{a}_2, a_2^+]$, then ´Ymin{ $\overline{a}_1, \overline{a}_2$ } = { min { $\overline{a}_1, \overline{a}_2$ } , min { a_1^+, a_2^+ }}, $\overline{a}_1 \ge \overline{a}_2$ if and only if $\overline{a}_1 \ge \overline{a}_2$ and $a_1^+ \ge a_2^+$ and Similarly we may have $\overline{a}_1 \le \overline{a}_2$ and $\overline{a}_1 = \overline{a}_2$

Let $\overline{a}_i \varepsilon$ D[0,1] when i ε Λ . We define ´Y{ $\inf_{i \varepsilon \Lambda}$ } $\overline{a}_I$ = [{ $\inf_{i \varepsilon \Lambda} \overline{a}_i$} , { $\inf_{i \varepsilon \Lambda} a_i^+$}] and ´Y{ $\sup_{i \varepsilon \Lambda}$ } $\overline{a}_I$ = [{ $\sup_{i \varepsilon \Lambda} \overline{a}_i$} , { $\sup_{i \varepsilon \Lambda} a_i^+$}] .

An interval valued fuzzy set (IVFS) δ^{-}_A defined on a non empty set X is given by δ^{-}_A = { (x , δ^{-}_A (x) , δ^{+}_A) / x ε X }, which is briefly denoted by $\delta^{-}_A = [\delta^{-}_A , \delta^{+}_A$ } where δ^{-}_A and $\delta^{+}_A$are two fuzzy sets in X such that $\delta^{-}_A (x) \le \delta^{+}_A (x)$ for all x ε X. δ^{-}_A (x) are called the degree of membership of an element x to δ^{-}_A , in which δ^{-}_A (x) and δ^{+}_A (x) are refered tp as the lower and upperdegrees respectively of membership of x to δ^{-}_A .

Definition 2.1:

Let E be the Universe and T be a non-empty set of time moments. Then a temporal fuzzy set (TFS) is an object having the form A(T) = { (x, δ_A(x,t) / (x,t) ε E X T } here

- ✓ A C E is a fixed set.
- ✓ δ_A(x,t) denote the degree of membership of the element (x, t) such that $0 \le \delta_A(x,t) \le 1$ for all (x,t) ε E X T.

Example 2.2:

Let X = [x_1. x_2. x_3} be the set and T be the non – empty set of time moments.

t → x ↓	t_1	t_2
x_1	0.4	0.4

x_2	1.0	0.2
x_3	0.0	0.1

For brevity we write A instead of A(T), the hesitation degree of a TFs is defined as $\pi_A(x,t) = 1 - \delta_A(x,t)$. Obviously, every ordinary Fuzzy set can be regarded as TFs for which T is a singleton set. All operations and operators can be defined for TFs in Atanassov. K.T. [1991].

Definitions 2.3:

Let A (T ') = {(x, $\delta_A(x,t)$ / E x T ' }

A (T '') = {(x, $\delta_A(x,t)$ / E x T''} where T ' and T'' have finite number of distinct time – elements or they are time interval. Then A (T ') $\bigcap$ B (T'') = { (x, min { $\delta^-_A(x,t)$, $\delta^-_B(x,t)$ }

A (T ') $\bigcup$ B (T'') = { (x, max { $\delta^-_A(x,t)$, $\delta^-_B(x,t)$ }

Also from definition of subset in Fuzzy fit theory subsets of TFs can be defined as the following:

A(T') $\subseteq$ B (T '') $\Leftrightarrow$ $\delta^-_A(x,t)$, $\delta^-_B(x,t)$ for every (x,t) ε (T ' $\bigcup$ T''),

Where $\delta^-_A(x,t)$ = { $\delta_A(x,t)$ if t ε T ''
= 0 if t ε T '' – T ' and

$\delta^-_B(x,t)$ = $\delta_B(x,t)$ if t ε T ''
= 0 if t ε T ' – T ''

It is obviously seen that if T ' = T ''. $\delta^-_A(x,t)$ = { $\delta_A(x,t)$, $\delta^-_B(x,t)$ = $\delta_B(x,t)$.

Let J be an arbitrary index set, then we define that T = $\bigcup i\varepsilon J$ Ti where Ti is a time set for each j ε J. Thus we extend the definition of Union and interjection of TFs family F = { Ai(Ti))= { (x, $\delta_{Ai}(x,t)$ / x ε E x Ti , i ε J}

As follows ($\bigcup i\varepsilon J$)A (Ti) = {(x, $(\max)_{j\in J}(\delta^-_A(x,t)$ / (x,t) € E x T }

($\bigcap j \in J$) A(Ti) = {(x, $(\min)_{j\in J}(\delta^-_A(x,t)$ / (x,t) € E x T }

Where ($\delta^-_A(x,t)$ = $\delta_A(x,t)$ if t € Tj
= 0 if t € T – Tj

In what follows let X denote the group under otherwise specified.

Definition 2.4:

Let ' X ' be a non empty set. A temporal bi fuzzy set (TBFS) A in a set X is a structure A = { (x, $\delta^-_A(x,t)$, $\lambda(x,t)$ / (x,t) € X x T } where is briefly denoted by A = < δ^-_A , λ > where δ^-_A = [δ^-_A , δ^+_A] is an temporal interval Fuzzy set (TIVFS) in X and λ is a fuzzy set (FS) in X. Denote C (X , T) the family of temporal bi fuzzy sets in X.

Definition 2.5:

A TBFS A = < δ^-_A, λ > in X is called a temporal bi fuzzy sub group (TBFSG) of X if it satisfies

- ✓ $\delta^-_A(xy,t) \geq$ Y min { $\delta^-_A(x,t)$, $\delta^-_A(y,t)$ } and $\delta^-_A(x^{-1},t) \geq \delta^-_A(x,t)$ }
- ✓ $\lambda(xy,t) \leq$ max { $\lambda(x,t)$, $\lambda(y,t)$} and $\lambda(x^{-1},t) \leq \lambda(x,t)$, for all x,y € X and t€T.

Example 2.6:

Let X be the Klein's four group. We have X = { e, a, b, ab } where $a^2 = e = b^2$ and ab = ba . We define δ^-_A = [δ^-_A , δ^+_A] and λ by

$$\delta^-_A = \begin{pmatrix} e & a & b & ab \\ [0.3,0.4] & [0.3,0.8] & [0.7,0.4] & [0.9,0.3] \end{pmatrix}$$

$$\text{and } \lambda = \begin{pmatrix} e & a & b & ab \\ 0.1 & 0.7 & 0.9 & 0.3 \end{pmatrix}$$

Then A = (δ^-_A , λ) is a TBFSG of X.

Note 2.7:

For any x, y € X and $\delta^-_A(y,t) > \delta^-_A(x,t)$ and $\lambda(y,t) < \lambda(x,t)$, then the equalities $\delta^-_A(xy,t) = \delta^-_A(x,t) = \delta^-_A(yx,t)$ and $\lambda(xy,t) = \lambda(x,t) = \lambda(yx,t)$ are not true.

Example 2.8:

In the Klein's four group X = { e,a,b,ab), we define δ^-_A = [δ^-_A , δ^+_A] and λ by

$$\delta^-_A = \begin{pmatrix} e & a & b & ab \\ [0.3,0.9] & [0.1,0.7] & [0.1,0.9] & [0.3,0.7] \end{pmatrix}$$

$$\text{and } \lambda = \begin{pmatrix} e & a & b & ab \\ 0.2 & 0.6 & 0.4 & 0.6 \end{pmatrix}$$

Then A = (δ^-_A , λ) is a TBFSG of X.

Note that δ^{-}_{A} (b,t) = [0.,0.9] > [0.1,0.7]= δ^{-}_{A} (a,t) and
λ (b,t)= 0.4 < 0.6 = λ (a,t). But δ^{-}_{A} (ab) = [0.3,0.7] $\neq$ [0.1, 0.7]= δ^{-}_{A} (a,t).

3. Properties of Temporal Bi Fuzzy Soft Subgroups:

In this section, we have proved the following propositions based on the above definitions.

Proposition 3.1:

Let A =< δ^{-}_{A} , λ > be a TBFSG of X. Then that δ^{-}_{A} (x^{-1},t) = δ^{-}_{A} (x,t) and λ (x^{-1},t) =λ (x,t) for all x € X and t € T.

Proof: For any x € X and t € T, We have δ^{-}_{A} (x ,t) = $\delta^{-}_{A}((x^{-1})^{-1},t) \geq \delta^{-}_{A}(x^{-1},t) \geq \delta^{-}_{A}(x,t)$ and

$$\lambda(x,t) = \lambda((x^{-1})^{-1},t) \leq \lambda(x^{-1},t) \leq \lambda(x,t)$$

Hence δ^{-}_{A} (x^{-1},t) = δ^{-}_{A} (x,t) and λ (x^{-1},t) =λ (x,t) .

Proposition 3.2:

Let A ==< δ^{-}_{A} , λ > be a TBFSG of X. For any x,y € X and t € T if =< δ^{-}_{A} (xy^{-1},t) = = δ^{-}_{A} (e,t) and $\lambda(xy^{-1},t)$ = λ (e), then = δ^{-}_{A} (x,t) = δ^{-}_{A} (y,t) and λ(x,t) = λ(y,t).

Proof:

Let x,y € X and t € T be such that δ^{-}_{A} (xy^{-1},t) = δ^{-}_{A} (e,t) and $\lambda(xy^{-1},t)$ = λ(e,t). Using Proposition 3.1, we get δ^{-}_{A} (x ,t) = δ^{-}_{A} $((xy^{-1})\mathbf{y},t)$ $\geq$ r min { δ^{-}_{A} (e ,t), δ^{-}_{A} (y,t) and λ(x ,t) = $\lambda(xy^{-1},t)$ $\leq$ max { λ(e ,t) = λ(y,t)} = λ(y,t) for all x,y € X and t€T. Similarly, δ^{-}_{A} (y, t) $\geq$ δ^{-}_{A} (x,t) and λ(y,t) $\leq$ λ (x,t). Hence the proof.

Theorem 3.3:

A TBFS A =< δ^{-}_{A}, λ) in X is a TBFSG of X if and only if it satisfies

- ✓ δ^{-}_{A} (xy^{-1},t) $\geq$ r min {δ^{-}_{A} (x,t), δ^{-}_{A} (y,t)
- ✓ $\lambda(xy^{-1},t)$ $\leq$ max{λ(x,t), λ(y,t) } for all x,y € X and t € T.

Proof:

Assume that A = <δ^{-}_{A} , λ> is a TBFSG of X and let x,y € X and t€T. Then

δ^{-}_{A} (xy^{-1},t) $\geq$ r min {δ^{-}_{A} (x,t), δ^{-}_{A} (y,t)} = r min {δ^{-}_{A} (x ,t) ,δ^{-}_{A} (y,t)} and
$\lambda(xy^{-1},t)$ $\leq$ max {λ(x,t), $\lambda(y^{-1},t)$} = max {λ(x ,t) ,λ(y ,t)} by proposition 1

Conversely, Suppose that (i) and (ii) are valid. If we take y = x in (i) and (ii), then

$$\delta^{-}_{A}(e,t) = \delta^{-}_{A}(xx^{-1},t) \geq r\min\{\delta^{-}_{A}(x,t), \delta^{-}_{A}(x,t)\} = \delta^{-}_{A}(x,t)$$
$$\lambda(e,t) = \lambda(xx^{-1},t) \leq \max\{\lambda(x,t), \lambda(y,t)\} = \lambda(x,t)$$

It follows from (i) and (ii) that

$$\delta^{-}_{A}(y^{-1},t) = \delta^{-}_{A}(ey^{-1},t) \geq r\min\{\delta^{-}_{A}(e,t), \delta^{-}_{A}(y,t)\} = \delta^{-}_{A}(y,t)$$
$$\lambda(y^{-1},t) = \lambda(ey^{-1},t) \leq \max\{\lambda(e,t), \lambda(y,t)\} = \lambda(y,t)$$

So that, $\delta^{-}_{A}(xy,t) = \delta^{-}_{A}(x(y^{-1})^{-1},t) \geq r\min\{\delta^{-}_{A}(x,t), \delta^{-}_{A}(y^{-1},t)\}$

$$\geq r\min\{\delta^{-}_{A}(x,t), \delta^{-}_{A}(y,t)\}$$
$$\lambda(xy,t) = \lambda(x(y^{-1})^{-1},t) \leq \max\{\lambda(x,t), \lambda(y^{-1},t)\} \leq \max\{\lambda(x,t), \lambda(y,t)\}$$

Therefore A =< δ^{-}_{A} , λ) in X is a TBFSG of X.

Theorem 3.4:

If A = < δ^{-}_{A} , λ) in X is a TBFSG of X then the set S = {x € X, t € T / δ^{-}_{A}(x,t) = δ^{-}_{A}(e,t) =λ(x,t) = λ(y,t) } is a subgroup of X.

Proof:

Let x, y € S and t € T. Then δ^{-}_{A}(x,t) = δ^{-}_{A}(e,t) = δ^{-}_{A}(y,t) and λ(x,t) = λ(e,t) = λ(y,t). It follows from theorem 1 that

$$\delta^{-}_{A}(xy^{-1},t) \geq r\min\{\delta^{-}_{A}(x,t), \delta^{-}_{A}(y,t)\} = \delta^{-}_{A}(e,t)$$
$$\lambda(xy^{-1},t) \leq \max\{\lambda(x,t), \lambda(y,t)\} = \lambda(e,t)$$

So from Proposition 3.2 that $\delta^{-}_{A}(xy^{-1},t) = \delta^{-}_{A}(e,t)$ and $\lambda(xy^{-1},t) = \lambda(e,t)$.

Hence $x y^{-1}$ € S, and so S is a subgroup of X.

Definition 3.5:

Let A = < δ^{-}_{A}, λ> in X is a TBFS in a set X, r € [0,1] and (α, β) € [0,1]. The set U (A/[α,β],r) = { x € X and t € T / δ^{-}_{A} (x,t) $\geq$ [α,β] , λ(x,t) $\leq$ r}. Is called the temporal bi fuzzy level set of A.

Theorem 3.6:

For a A = < δ^{-}_{A}, λ) in X, the following or equivalent.

- ✓ A = < δ^{-}_{A}, λ) is a TBFSG of X.

The non empty temporal bi fuzzy level set A If A = < δ^{-}_{A} , λ > is a subgroup of X.

Proof:

Assume that A = < δ^{-}_{A} , λ) is a TBFSG of X. Let x,y € U (A: [α,β],r) for all r€[0,1], [α,β] € [0,1] and t € T. Then If δ^{-}_{A} (x,t) $\geq$ [α,β], λ(x,t) $\leq$ r, δ^{-}_{A} (y,t) $\geq$ [α,β], λ(y,t) $\leq$ r,

It follows from proposition 3.1 that δ^{-}_{A} (xy^{-1},t) $\geq$ r min { δ^{-}_{A} (x,t), δ^{-}_{A} (y,t) } $\geq$ [α,β] and

$\lambda(xy^{-1},t)$ $\leq$ r max { λ(x,t), λ(y,t) } $\leq$ r so that xy^{-1} € U (A : [α,β],r).

Therefore A = <δ^{-}_{A}, λ > is a subgroup of X.

Conversely, let r € [0,1] and [α,β] € D[0,1] be such that U (A;[α,β],r) $\neq$ $\emptyset$ and U (A;[α,β],r) is a subgroup of X.

Suppose that theorem 3.1 (i) is not true and theorem 3.1 (ii) is valid. Then there exists $[\alpha_o, \beta_o]$ € D [0,1] and a,b € X and t € T such that

$\delta^-_A(ab^{-1},t) \le [\alpha_o,\beta_o] \le r\min\{\delta^-_A(a,t), \delta^-_A(b,t)\}$ and $\lambda(ab^{-1},t) \le \max\{\lambda(a,t), \lambda(b,t)\}$

It follows that a,b € U $(A;[\alpha_o,\beta_o],\max\{\lambda(a,t), \lambda(b,t)\}$ but $ab^{-1} \notin U(A;[\alpha_o,\beta_o],\max\{\lambda(a,t), \lambda(b,t)\}$

This is a contradiction. If theorem 3.1 (i) is true and theorem 3.1 (ii) is not valid then

$\delta^-_A(ab^{-1},t) \ge r\min\{\delta^-_A(a,t), \delta^-_A(b,t)\}$ and $\lambda(ab^{-1},t) \ge r_o \ge \max\{\lambda(a,t), \lambda(b,t)\}$ for some r_o € [0,1] and a,b€X, t€T.

Thus a,b € U $(A;[\alpha_o,\beta_o],r_o)$ but a,b∉ U ⊂ A; r min <{ $\delta^-_A(a,t)$, $\delta^-_A(b,t)$}, r_o) which is a contradiction.

Assume that there exist $[\alpha_o,\beta_o]$ € D[0,1], ro € [0,1] and a,b € X such that

$\delta^-_A(ab^{-1},t) < [\alpha_o,\beta_o] \le r\min\{\delta^-_A(a,t), \delta^-_A(b,t)\}$ and $\lambda(ab^{-1},t) > r_o \ge \max\{\lambda(a,t),\lambda(b,t)\}$. Then a,b € U $(A;[\alpha_o,\beta_o],r_o)$ but

$ab^{-1} \notin U(A;[\alpha_o,\beta_o],r_o)$. This is also a contradiction. Hence (i) and (ii) of theorem 3.1 are valid.

Therefore A = < δ^-_A , λ> is a TBFSG of X.

Definition 3.7:

Let X and Y be give crisp sets. A mapping of X x T → Y x T induces two mapping C : C(X,t) C(Y,t) , A = C(A) and C^{-1} : C(Y,t) → C(X,t). $B=C^{-1}$ (B), where

$$C(\delta^-_A(y,t)) = \begin{cases} r(\operatorname{Sup} \delta^-_A(x,t))_{y=f(x)} & \text{if } f^{-1}(y) \ne \emptyset \\ [0,0] \text{ otherwise} \end{cases}$$

$$C(\lambda(y,t)) = \begin{cases} \inf \lambda(x,t))_{y=f(x)} & \text{if } f^{-1}(y) \ne \emptyset \\ 1 \text{ otherwise} \end{cases}$$

For all (y,t) € Y x T and $C^{-1}(\delta^-_B(x,t)) = \delta^-_B(f(x),t)$ for all x € X and t € T.

A TBFS A = <δ^-_A , λ> in X has the property if for any subset P of X there exists xo € P such that $\delta^-_A(x_o,t) = r(\sup)_{x\in P}\, \delta^-_A(x,t)$ and $\lambda(x_0,t) = (\inf)_{x\in P}\lambda(x,t)$

Theorem 3.8:

For a homomorphism of f : X x T → Y x T of groups, let C:C(x,t)→C(y,t) and C^{-1} :C(y,t) →C(x,t) be the temporal bi fuzzy transformation and inverse temporal bi fuzzy transformation, respectively, induced by f.

- ✓ If A = < δ^-_A, λ > € C(x,t) is a TBFSG of X which has the temporal bi fuzzy property, then C(A) is a TBFSG of Y.
- ✓ If B = < δ^-_B, k > € C(y,t) is a TBFSG of Y, then C^{-1} (B) is a TBFSG of X.

Proof:

(i) Given f(x,t) , f(y,t) € f(x,t), let x_o€ f^{-1}(f(x,t)) and y_o€ f^{-1}(f(y,t)) be such that

$\delta^-_A(x_o,t) = r(\operatorname{Sup})_{a\in f^{-1}(f(xo,t))}\, \delta^-_A(a,t)$, $\lambda(x_o,t) = (\inf)_{a\in f^{-1}(f(x,t))}\, \lambda(a,t)$

$\delta^-_A(y_o,t) = r(\operatorname{Sup})_{b\in f^{-1}(f(yo,t))}\, \delta^-_A(b,t)$, $\lambda(y_o,t) = (\inf)_{a\in f^{-1}(f(y,t))}\, \lambda(b,t)$

$$\begin{aligned}
C\{(\delta^-_A)(f(x,t)f(y,t)\} &= r\operatorname{Sup}_{z\in f^{-1}(f(x.t)f(y,t)}\, \delta^-_A(z,t) \ge \delta^-_A(x_oy_o,t) \\
&= r\min\{\delta^-_A(x_o,t), \delta^-_A(z_o,t)\} \\
&= r\min\{r(\operatorname{Sup})_{a\in f^{-1}(f(x,t)}, r(\operatorname{Sup})_{b\in f^{-1}(f(y,t)}\, \delta^-_A(b,t)\} \\
&= r\min\{C(\delta_A)f(x,t), C(\delta_A)(f(y,t)\}
\end{aligned}$$

$$\begin{aligned}
C\{(\delta_A)(f(x,t)^{-1}\} &= r\operatorname{Sup}_{z\in f^{-1}(f(x.t)-1}\, \delta^-_A(z,t) \\
&\ge \delta^-_A(x_o^{-1},t) \\
&\ge \delta^-_A(x_o,t) \\
&= C(C\,\delta_A)(f(x,t) \\
C\{(C\lambda)f(x,t)^{-1})\} &= \inf_{z\in f^{-1}(f(x.t)^{-1}}\lambda(z,t) \ge \lambda(x_o^{-1},t) \\
&= \lambda(x_o,t) \\
&= C(C\lambda)(f(x,t)
\end{aligned}$$

Therefore C(A) is a TBFSG of Y.

For any x,y € X and t € T, We have

$$\begin{aligned}
C^{-1}\{(C\,\delta_B)((xy,t)\} &= (\delta^-_B)(f(xy,t)\} \\
&= (\delta^-_B)(f(x,t)f(y,t)\} \\
&= r\min(C^{-1}\{(\delta^-_B)(x,t) \\
&= C^{-1}\{(\delta^-_B)(y,t)\} \\
C^{-1}\{(\delta_B)((x^{-1},t)\} &= (\delta^-_B)(f(x^{-1},t)\} \\
&= (\delta^-_B)(f(x,t))^{-1} \\
&= (\delta^-_B)f(x,t) \\
C^{-1}\{(k)((xy,t)\} &= k(f(xy,t) \\
&= k(f(x,t),f(y,t)\} \\
&\le \max\{K\, f(x,t), k\, f(y,t)\} \\
&= \max\{C^{-1}(k)(x,t), C^{-1}(k)(y,t)\}
\end{aligned}$$

and

$$\begin{aligned}
C^{-1}\{(k)((x^{-1},t)\} &= k(f(x^{-1},t) \\
&= k(f(x,t))^{-1} \\
&\le k(f(x,t))
\end{aligned}$$

$= C^{-1}(k)(x,t)$

Thus $C^{-1}(B)$ is a TBFSG of X.

Conclusion:

In the article, we apply the motion of temporal bi fuzzy subsets of a group, and introduced temporal bi fuzzy groups. Also we provide characterization of a TBFSG and study how the images or preimages of TBFSG become TBFSGS.

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