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ON PRODUCT OF COMPLEX DOUBLE FRAMED FUZZY SOFT STRUCTURES

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Abstract:
In this paper, we are initiating the study of complex double-framed fuzzy soft semi groups. We define characteristic function of complex double-framed fuzzy soft set direct product and complex double-framed fuzzy soft interior ideals and proved some results.

Key Words: Soft Set, Fuzzy Set, Complex Fuzzy Soft Set, Semi Group, Double Framed Fuzzy Soft Set, Interior Ideal & Cartesian Product

1. Introduction:

Soft set theory was introduced by Molodtsov [1999] for modelling vagueness and uncertainty and it has been received much attention since Maji et al [2002], Ali et al [2009] and Sezgin and Atagun [2011] introduced and studied operations of soft sets. Soft set theory has also potential applications especially in decision making as in [2010]. This theory has started to progress in the mean of algebraic structures, since Aktas¸ and Cagman [2007] defined and studied soft groups. Since then, soft substructures of rings, fields and modules [2011], union soft substructures of near-rings and near-ring modules [2011], normalistic soft groups [2011] are defined and studied in detailed.

Moreover, Atagun and Sezgin [2011] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji*et al.* [2002] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [2009] introduced several operations of soft sets and Sezgin and Atagun [2009] studied on soft set operations as well. In this paper, we are initiating the study of complex double-framed fuzzy soft semi groups. We define characteristic function of complex double-framed fuzzy soft set dissect product and complex double-framed fuzzy soft interior ideals and proved some results.

2. Preliminaries:

Here in this part, we gathered some basic helping materials.

2.1 Definition: [Moltdosov] A pair (F,A) is called a soft set over U, where F is a mapping given byF: A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively.

2.2 Definition: [Moltdosov] The relative complement of the soft set F_A over U is denoted by $F^r{}_A$, where $F^r{}_A$: A → P(U) is a mapping given as $F^r{}_A(a)$ =U \\$F_A(a)$, for all a ∈A.

2.3 Definition: [Moltdosov] Let F_A and G_B be two soft sets over U such that A∩B ≠ ∅,. The restricted intersection of F_A and G_B is denoted by $F_A \Cup G_B$, and is defined as $F_A \Cup G_B$=(H,C), where C = A∩B and for all c ∈C, H(c) = F(c)∩G(c).

2.4 Definition: [Moltdosov] Let F_A and G_B be two soft sets over U such that A∩B ≠ ∅,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B$ = (H,C),where C = A∩B and for all c ∈C, H(c) = F(c)∪G(c).

2.4 Definition: [Zadeh] A function 'f' is defined from a universe X to a closed interval [0, 1] is called a fuzzy set. (i,e) a mapping f: X→[0,1].

2.5 Definition: [Remote] A complex fuzzy soft set (CFSS) C over the universe X is defined on object having of the form.

$$C = \{(x, \sigma_c(x))/xEX\}$$

Where $\sigma_c(x) = r_c(x).e^{iw_c(x)}$ here the amplitude term rc(x) and phase term wc(x) are real valued function for every x ε X, the amplitudes term: x →[0,1] and phase term wc(x)lying in the interval [0,2∏]

2.6 Definition: A double-framed pair$\langle(\bar{\alpha},\bar{\lambda}) : G\rangle$ is called a double-framed soft set (briefly DFS-set) over U where $\bar{\alpha}$ and $\bar{\lambda}$ are mapping from A to P(U).

For a DFS-set $\langle(\bar{\alpha},\bar{\lambda}) : G\rangle$ over U and two subsets γ andδ of U, the γ-inclusive set and the δ-exclusive set of $\langle(\bar{\alpha},\bar{\lambda}) : G\rangle$, denoted by i_A ($\bar{\alpha}$; γ) and $e_A(\bar{\lambda}, \delta)$ respectively, are defined as follows.

i_A $(\bar{\alpha}; \gamma)$ = { x∈A / $\gamma \subseteq \bar{\alpha}(x)$ } and $e_A(\bar{\lambda}, \delta)$ = { x∈A / $\delta \subseteq \bar{\lambda}(x)$ } respectively.

The set $DF_A(\bar{\alpha}, \bar{\lambda})_{(\gamma,\delta)}$ = { x∈A / $\gamma \subseteq \bar{\alpha}(x)$, $\delta \subseteq \bar{\lambda}(x)$ } is called a double framed including set of <$(\bar{\alpha}, \bar{\lambda})$: G>. It is clear that $DF_A(\bar{\alpha}, \bar{\lambda})_{(\gamma,\delta)} = i_A(\bar{\alpha}; \gamma) \cap e_A(\bar{\lambda}, \delta)$.

From now on, we will take G, as set of parameters, which is a group unless otherwise specified.

2.7 Note:

Let λ_S = ($\overline{\alpha_S}$, $\overline{\beta_S}$,E) be a double framed soft set over U. We will say that $\lambda_S(e)$= ($\overline{\alpha_S}(e)$, $\overline{\beta_S}(e)$) is image of parameter e∈ E.

2.8 Definition: Let λ_A and λ_B ∈ $DFS_E(U)$ then,

- ✓ If $\alpha_A(e) = \emptyset$ and $\beta_A(e)=U$ for all e∈ E , λ_A is said to be a null double-framed soft set, denoted by $\emptyset_b$ = (∅,U,E).
- ✓ If $\alpha_A(e) = U$ and $\beta_A(e)=\emptyset$ for all e∈ E , λ_A is said to be an absolute double-framed soft set, denoted by $\emptyset_b$ = (U,∅,E).
- ✓ λ_A is double-framed soft subset of λ_B, denoted by $\lambda_A \subseteq \lambda_B$, if $\alpha_A(e) \subseteq \alpha_B(e)$ and $\beta_A(e) \supseteq \beta_B(e)$ for all e∈ E.
- ✓ Double – framed soft union and intersection of λ_A and λ_B, denoted by

$$(\alpha_A \cup \alpha_B): A\cup B \rightarrow P(U) \text{ such that } (\alpha_A \cup \alpha_B)(e) = \alpha_A(e) \cup \alpha_B(e) \text{ and}$$
$$(\beta_A \cap \beta_B)(e) = \beta_A(e) \cap \beta_B(e) \text{ for all } e\in E.$$
$$\text{Also}(\alpha_A \cap \alpha_B): A\cap B \rightarrow P(U) \text{ such that } (\alpha_A \cap \alpha_B)(e) = \alpha_A(e) \cap \alpha_B(e) \text{ and}$$
$$(\beta_A \cup \beta_B)(e) = \beta_A(e) \cup \beta_B(e) \text{ for all } e\in E.$$

- ✓ Double – framed soft complement of λ_A is denoted by $\lambda_A{}^C$ and defined by

$$\lambda_A{}^C : E \rightarrow P(U) \times P(U) \text{ such that } \lambda_A{}^C(e) = \{(e,\alpha_A(e),\beta_A(e)): e\in E\}.$$

2.9 Definition: Let X be a universe of discourse and Xfx, A complex double framed fuzzy softset (CDFSS), C in X is

$$C = \left\{(x, C_T(x) = p_c(x)\, e^{i\sigma_c(x)},\ C_F(x) = r_c(x) e^{iw_c(x)})/xEX\right\}$$

Where $C_T(x) = p_C(x)\, e^{i\sigma_c(x)}$ and $C_F(x) = r_c(x) e^{iw_c(x)}$ ax complex truth membership function and complex falsity membership function. The valves C+(x), CF(X) may lie within the unit circle in the complex plane, where Pc(x), Rc(x) and $\sigma_c(x)$

Ac(x) amplitude terms and phase terms respectively and Pc(x),Rc(x) E [0.1] such that O≤ Pc(x)+Rc(x)≤1 and $\sigma_c(x)$,Ac(x)E[0,2∏]

Example: Let X={a, b, c} be the universel set and C be a complex double framed fuzzy soft set which is given by

$$C = \begin{Bmatrix} <a_1, 0.3e^{0.7\Pi i}, 0.5e^{0.2\Pi i}> \\ <a_2, 0.7e^{0.2\Pi i}, 0.1e^{0.6\Pi i}> \\ <a_3, 0.7e^{0.4\Pi i}, 0.2e^{0.8\Pi i}> \end{Bmatrix}$$

2.10 Note: at should be noted that throughout in this part we use 'a' capital letter C to denote a complex double framed fuzzy soft set;

$$C = \left\{<T_c = P_c e^{i\sigma_c}, F_c = r_c e^{i\triangle_c}>\right\}$$

2.11 Definition: A complex double-framed fuzzy soft set C on a semi group S is known as complex double-framed fuzzy soft semigroup (CDFFSG). If it satisfy the following condition;

$$C(xy) \succcurlyeq \inf\{c(x), c(y)\}$$

(ie)

✓ $$p_c(xy).e^{i\sigma_c(xy)} \succcurlyeq \inf\left\{p_c(x)e^{i\sigma_c(x)}, p_c(y)e^{i\sigma_c(y)}\right\}$$

✓ $$r_c(xy), e^{i\triangle_c(xy)} \preccurlyeq \sup\left\{r_c(x)e^{i\triangle_c(x)}, r_c(y)e^{i\triangle_c(y)}\right\} \text{ for all } x, y \in s.$$

2.12 Example: Let S= {a, b, c} be a semigroup with the following multiplication table

.	a	b	c
a	a	b	c
b	b	a	c
c	c	c	c

From the example is definition (2.5), C is clearly complex double-framed fuzzy soft semigroup of S.

2.13 Definition: Cartecian product of complex double-framed fuzzy soft semigroups. Let

$$C_1 = \left\{<C_1T = P_{c_1}e^{i\sigma_{c_1}(x)}, C_1F = r_{c_1}e^{i\triangle_{c_1}}>\right\}$$
$$C_2 = \left\{<C_2T = P_{c_2}e^{i\sigma_{c_2}(x)}, C_2F = r_{c_2}e^{i\triangle_{c_2}}>\right\}$$

Be any two complex double-framed fazzy soft semi groups S_1 and S_2 respectively. Then

$$C_1 \times C_2 = \left\{ \prec (x,y), (c_1 \times c_2)_T(x,y), (c_1 \times c_2)_F(x,y) \succ / \nabla\ xES_1, yES_2 \right\}$$

Where

$$(c_1 \times c_2)_T(x,y) = \inf\{c_{1T}(x), c_{2T}(y)\}$$
$$(c_1 \times c_2)_F(x,y) = \sup\{c_{1F}(x), c_{2F}(y)\}$$

For all x in S_1 and y in S_2.

2.14 Example: Let S_1= {a, b, c} and S_2 = {ℓ, m, n} ax any two semigroups with the following multiplication tables

.	a	b	c
a	c	b	a
b	b	b	c
c	c	c	c

.	ℓ	m	n
ℓ	n	m	ℓ
m	m	m	ℓ
n	n	n	n

Confider

$$C_1 = \begin{Bmatrix} \prec 1,\ 0.3e^{0.6\Pi i}, 0.3e^{0.2\Pi i} \succ \\ \prec m, 0.4e^{0.8\Pi i}, 0.3e^{0.7\Pi i} \succ \\ \prec n.\ 0.1e^{0.7\Pi i}, 0.3e^{0.2\Pi i} \succ \end{Bmatrix}$$

$$C_2 = \begin{Bmatrix} \prec a,\ 0.9e^{0.3\Pi i}, 0.4e^{0.4\Pi i} \succ \\ \prec b, 0.2e^{0.7\Pi i}, 0.9e^{0.8\Pi i} \succ \\ \prec c.\ 0.1e^{0.9\Pi i}, 0.9e^{0.1\Pi i} \succ \end{Bmatrix}$$

Be only two complex double-framed fuzzy soft semigroups S_1 and S_2respectically.then

$$C_1 \times C_2 = \left\{ \prec \inf\left\{0.3e^{0.6\Pi i}, 0.9e^{0.3\Pi i}\right\}, \inf\left\{0.3e^{0.2\Pi i}, 0.4e^{0.4\Pi i}\right\} .... \right\}$$
$$= \left\{ \prec 0.3e^{0.6\Pi i}, 0.3e^{0.2\Pi i} \succ, ... \right\}$$

3. Complex Double-Framed Fuzzy Soft Ideals:

In the section, we define some ideals namely complex double-framed (left, right, interior) ideal in semigroup, with help of examples and study some of its related results.

3.1 Definition: A complex double-framed fuzzy soft set C on a semigroup S is known as complex double-framed fuzzy soft left ideal of S, if C(xy) >,C (y)

(ie)

- ✓ $p_c(x).e^{i\sigma_c(xy)} \succcurlyeq p_c(y)e^{i\sigma_c(y)}$
- ✓ $r_c(xy).e^{i\triangle_c(xy)} \succcurlyeq r_c(y)e^{i\triangle_c(y)}, \nabla\ x,yES.$

3.2 Example: Let S = {l. m. n, u} be a semigroup with the following multiplication table

.	l	m	n	u
l	l	l	l	l
m	l	l	l	l
n	l	l	m	l
u	l	l	m	m

Confider a complex double-framed soft set C on S as

$$C = \begin{Bmatrix} \prec l, 0.3e^{0.6\Pi i}, 0.4e^{0.7\Pi i} \succ \\ \prec m, 0.2e^{0.7\Pi i}, 0.3e^{0.8\Pi i} \succ \\ \prec n, 0.4e^{0.2\Pi i}, 0.7e^{0.1\Pi i} \succ \\ \prec u, 0.3e^{0.2\Pi i}, 0.9e^{0.8\Pi i} \succ \end{Bmatrix}$$

Then C is a complex double-framed fuzzy soft set left ideal of S. Fimilarly, we can define a complex double-framed fuzzy soft right ideal of S is

$$C(xy) \geq C(x)$$

(i,e)

- ✓ $p_c(xy).e^{i\sigma_c(xy)} \succcurlyeq p_c(x)e^{i\sigma_c(x)}$
- ✓ $r_c(xy).e^{i\triangle_c(xy)} \preccurlyeq r_c(x)e^{i\triangle_c(x)}, \nabla\ xyES.$

3.3 Definition: A complex double-framed fuzzy soft set C on a semigroup S is known as a complex double-framed fuzzy soft set ideal of S if it is both a complex double-framed left ideal and a complex double-framed right ideal of S. Example. Let S={l,m,n} be a semigroup with the following cayley table,

.	l	m	n
l	l	l	l
m	l	l	l
n	l	l	n

If we can define a complex double-framed fuzzy soft set C on S as;

$$C=\left\{\begin{matrix}<l\,,0.3e^{0.6\Pi i},0.9e^{0.8\Pi i}> \\ <m,0.7e^{0.7\Pi i},0.2e^{0.3\Pi i}> \\ <n,0.7e^{0.3\Pi i},0.2e^{0.2\Pi i}>\end{matrix}\right\}$$

Then clearly C is a complex double-framed fuzzy soft ideal of S.

3.4 Remark: Every complex double- framed fuzzy soft left (resp, right) ideal is a complex double- framed fuzzy soft semigroup. But the converse may not be true as seen in the following example.

3.5 Example: Let S = {1, 2, 3, 4} be a semigroup with the following cayley table,

.	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	2
4	1	1	2	3

Take a complex double- framed fuzzy soft set C on S as,

$$C=\left\{\begin{matrix}<1,0.8e^{0.6\Pi i},0.5e^{0.4\Pi i}> \\ <2,0.6e^{0.6\Pi i},0.6e^{0.4\Pi i}> \\ <3,0.8e^{0.5\Pi i},0.7e^{0.5\Pi i}> \\ <4,0.4e^{0.4\Pi i},0.7e^{0.5\Pi i}>\end{matrix}\right\}$$

Then clearly C is a complex double- framed fuzzy soft semigroup of S. however it is not a complex double-framed right ideal of S, because

$$T_C(nu)=T_c(m)=0.6e^{0.6\Pi i}\ngeq 0.8e^{0.5\Pi i}=T_c(c)$$

3.6 Proposition: A complex double- framed fuzzy soft set $C_{1,}$ C_2 and C_3 of a semigroup S, if $C_1\subseteq C_2$, Then $C_1 *C_2 \subseteq C_2 *C_3$ and $C_3* C_1\subseteq C_3 *C_2$.

Proof:

We have to prove $C_1*C_3\subseteq C_2*C_3$ since C_1,C_2 and C_3 are complex double- framed fuzzy soft set of S. Let xES.

Case (1): Af x is not expressed as x=yp, then $(C_1*C_3)(x)=<0,1>$ and $(C_2*C_3)(x)=<0,1>$. Clearly $C_1*C_3\subseteq C_2*C_3$.

Case (2): Assume that these exists y,pES such that x=yp. Then

$$\begin{aligned}(p_c,op_{c3})(x).e^{i(\sigma_c,o\sigma_{c3})(x)} &= \max_{x=yp}\left\{\inf\{p_c(y)e^{i\sigma_c(y)}p_{c3}(p)e^{i\sigma_{c3}(p)}\}\right\} \\ &\preccurlyeq \max_{x=p}\left\{\min\{p_{c2}(y)e^{i\sigma_{c2}(y)},p_{c3}(p)e^{i\sigma_{c3}(p)}\}\right\} \\ &=(p_{c2}op_{c3})(x).e^{i(\sigma_{c2}o\sigma_{c3})(x)}\end{aligned}$$

Similarly,

$$\begin{aligned}(r_{c1,}or_{c3})(x) &= \min\left\{\sup\{r_{c1}(y)e^{i\triangle_c(y)},r_{c3}(p)e^{i\triangle_{c3}(p)}\}\right\} \\ &\succcurlyeq \min\left\{mex\{r_{c2}(y)e^{i\triangle_{c2}(y)},r_{c3}(p)e^{i\triangle_{c3}(y)}\}\right\} \\ &=(r_{c2},or_{c3})(x).e^{i(\triangle_{c2,}o\triangle_{c3})(x)}\end{aligned}$$

Therefore $C_1*C_3\subseteq C_2*C_3$. Similarly, we can prove $C_3* C_1\subseteq C_3 *C_2$.

3.7 Theorem: A complex double- framed fuzzy soft set C on a semigroup S is a complex double- framed fuzzy soft semigroup of S if and only if $C*C \subseteq C$.

Proof:

Let C be a complex double- framed fuzzy soft semigroup of S and xES.

Case (1): If x≠yp for any y,pES then obivously $C*C \subseteq C$.

Case (2): If x=yp, for any y,pES, then

$$\begin{aligned}(p_c,op_c)(x).e^{i(\sigma_c,o\sigma_c)(x)} &= \max_{x=yp}\left\{\inf\{p_c(y)e^{i\sigma_c(y)}p_c(p)e^{i\sigma_c(p)}\}\right\} \\ &\preccurlyeq \max_{x=p}\left\{p_c(yp)e^{i\sigma_c(yp)}\right\} \\ &= p_c(x).e^{i\sigma_c(x)}\end{aligned}$$

Similarly,

3.8 Remark: Every complex double-framed fuzzy softideal is a complex double-framed fuzzy soft interior ideal. But the covered may not true as seen in the example 3.5. For

$$\text{Left: } T_c(12)=T_c(2)=0\neq 0.7e^{0.5\pi i}=T_c(3)$$

$$\text{Right: } T_c(12) = T_c(2) = 0 \geq 0 = T_c(4)$$

So it is a complex double-framed fuzzy soft ideal but not a left. Hence c is not a complex double-formed fuzzy soft ideal.

Conclusion:

We are initiating the study of complex double-framed fuzzy soft semi groups. We define characteristic function of complex double-framed fuzzy soft set direct product and complex double-framed fuzzy soft interior ideals and proved some results.

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