International Journal of Applied and Advanced Scientific Research (IJAASR)
Impact Factor: 5.655, ISSN (Online): 2456 - 3080
(www.dvpublication.com) Volume 3, Issue 1, 2018

COMPLEX TEMPORAL FUZZY SOFT INTERIOR IDEALS OVER SEMIGROUPS

Dr. R. Ramadoss* & N. Ramashankari**

* Professor, Department of Mathematics, PRIST University, Tanjore, Tamilnadu

** Research Scholar, Department of Mathematics, PRIST University, Tanjore, Tamilnadu

Cite This Article: Dr. R. Ramadoss & N. Ramashankari, "Complex Temporal Fuzzy Soft Interior Ideals Over Semigroups", International Journal of Applied and Advanced Scientific Research, Volume 3, Issue 1, Page Number 283-287, 2018.

Abstract:

In this paper, we introduce the notion of Complex temporal fuzzy soft semi groups (CTFSG) and itsCartesian product with the help of suitable example, also we define direct product, ideals and prove some results.

Key Words: Soft Set, Fuzzy Soft Set, Temporal Fuzzy Set, Left Ideal, Interior Ideal, Union and Intersection

1. Introduction:

The theory of fuzzy sets proposed by L.A. Zadeh [1965] in 1965, has achieved a great success in various fields. In 1999, Molodtsov [1999] introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory is free from the difficulties affecting existing methods. Presently, works on soft set theory are progressing rapidly. Maji et al. [2002] de.ned several operations on soft sets and made a theoretical study on the theory of soft sets. Atagün and Sezgin [2016] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Also union soft sub structures of near-rings and near-ring modules are studied in [2012]. Maji et al. [2002] presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets [2007]. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. Roy and Maji [2003] gave some applications of fs-sets. Aktas. And Cagman [2007] compared soft sets with the related concepts of fuzzy sets and rough sets. In this paper we introduce the notion of Complex temporal fuzzy soft semi groups (CTFSG) and its cartesian product with the help of suitable example, Also we define direct product, ideals and prove some results.

2. Preliminaries:

In this section, we recall basic definitions of soft set theory that are useful for subsequent sections. For more detail see the papers [1999].

Through out the paper, U refers to an initial universe, E is a set of parameters and P(U) is the power set of U. $\subset$ and $\supset$ stand for proper subset and super set, respectively.

Definition 2.1: For any subset A of E, a soft set λ_A over U is a set, defined by a functionλ_A , representing the mapping λ_A: E $\rightarrow$ P(U). A soft set over U can also be represented by the set of ordered pairs $\lambda_A = \{(x, \lambda_A(x)); x \in E, \lambda_A(x) \in P(U)\}$. Note that the set of all soft sets over U will be denoted by S(U).

Definition 2.2: Let $\lambda, \mu \in S(U)$. Then

- ✓ If $\lambda(e) = \emptyset$ for all $e \in E$, λ is said to be a null soft set, denoted by $\emptyset$.
- ✓ If $\lambda(e) = U$ for all $e \in E$, λ is said to be an absolute soft set, denoted by U.
- ✓ $\lambda$is a soft subset of μ , denoted $\lambda \subseteq \mu$, if $\lambda(e) \subseteq \mu(e)$ for all $e \in E$.
- ✓ Soft union of λ and μ , denoted by $\lambda \cup \mu$, is a soft set over U and defined by $\lambda \cup \mu$: E $\rightarrow$ P(U) such that $(\lambda \cup \mu)(e) = \lambda(e) \cup \mu(e)$ for all $e \in E$.
- ✓ $\lambda = \mu$, if $\lambda \subseteq \mu$ and $\lambda \supseteq \mu$.
- ✓ Soft intersection of λ and μ, denoted by $\lambda \cap \mu$, is a soft set over U and defined by $\lambda \cap \mu$: E $\rightarrow$ P(U) such that $(\lambda \cap \mu)(e) = \lambda(e) \cap \mu(e)$ for all $e \in E$.
- ✓ Soft complement of λ is denoted by λ^C and defined by λ^C: E $\rightarrow$ P(U) such that $\lambda^C(e) = \cup/\lambda(e)$ for all $e \in E$.

Definition 2.3: Let E be a parameter set, $S \subset E$ and λ: S $\rightarrow$ E be an injection function. Then $S \cup \lambda(s)$ is called extended parameter set of S and denoted by ξ_S. If S=E, then extended parameter set of S will be denoted byξ.

Definition 2.4: A temporal fuzzy set is an object of the form $A(T) = \{h(x, t), \mu_A(x, t) : (x, t) \in E \times T\}$ where (a) $A \subset E$ is a fixed set (b) $0 \leq \mu_A(x, t) \leq 1$ for every $(x, t) \in E \times T$. (c) $\mu A(x, t)$ are the degrees of membershipof the element $x \in E$ at the time-moment $t \in T$.

Example 2.5: Suppose that is a temporal fuzzy set (Table 1) defined on = $\{x_1, x_2 ,x_3\}$ with respect to the time moment set = $\{t_1,t_2\}$:

Table 1: Temporal fuzzy set A

t/x	t_1	t_2
x_1	0.2	0.5
x_2	1.0	0.7

x_3	0.5	0.4

Definition 2.6: A Complex temporal l fuzzy soft set C = { < Pc . **e** $^{i\delta_c}$> } on a semi group S is known as a Complex temporalfuzzy soft left ideal of S, if C(xy,t) ≥ C(y,t) . i.e.,

$$Pc\,(xy,t)\,.\,e^{i\delta_{c(xy,t)}} \geq Pc\,(y,t)\,e^{i\delta_{c(x,t)}} \text{ for all } x,y\,\varepsilon\, S \text{ and } t\,\varepsilon\, T$$

Example 2.7: Let S = {1, 2, 3, 4} be a semigroup with the following multiplication table.

●	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	2	1
4	1	1	2	2

Consider a Complex temporal fuzzy soft set C on S are: C = {< 1, 0.3 e $^{0.7\pi i}$>, < 1, 0.3 e $^{0.8\pi i}$>, < 1, 0.3 e $^{0.6\pi i}$>, < 1, 0.3 e $^{0.3\pi i}$>} then C is a complex temporalfuzzy left ideal of S.

Definition 2.8: A Complex temporal fuzzy soft set C={<Pc . **e** $^{i\delta_c}$>} on a semi group S is known as a complex temporal fuzzy soft right ideal of S, if C(xy,t) ≥ C(x,t) , ie., Pc (xy,t) . e $^{i\delta_{c(xy,t)}}$ ≥ Pc (x,t) e $^{i\delta_{c(x,t)}}$ for all x,y ε S and t ε T.

Definition 2.9: A Complex temporal fuzzy soft set C = {< Pc. **e** $^{i\delta_c}$>} on a semi group S is known as a complex temporalfuzzy soft right ideal of S, if it both a complex temporal fuzzy soft left ideal of S, if it both a complex temporal fuzzy soft left ideal and a complex temporal fuzzy soft right ideal of S.

Example 2.10: Let S = {1, 2, 3, 4} be a semigroup with the following Cayle table.

●	1	2	3
1	1	1	1
2	1	1	1
3	1	1	3

Consider a Complex temporal fuzzy soft set C on S as: C = { < 1, 0.7 e $^{0.7\pi i}$>, < 2, 0.3 e $^{0.2\pi i}$>,< 3, 0.4 e $^{0.7\pi i}$> } then obviously C is a complex temporal fuzzy soft ideal of S.

Remark 2.11: Every Complex temporal fuzzy soft left (resp., right) ideal is a complex temporal fuzzy soft semi group. But the Converse may not be true as seen in the following example.

Example 2.12: Let S = {a, b, c, d } be a semigroup with the following Cayle table.

●	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	b
d	a	a	b	c

Consider a Complex temporal fuzzy soft set C on S as: C = {< a, 0.8 e $^{0.6\pi i}$>, < b, 0.6 e $^{0.6\pi i}$>, < c, 0.8 e $^{0.5\pi i}$>, < d, 0.4 e $^{0.4\pi i}$>} then obviously C is a complex temporal fuzzy soft semi group, however it is not a right ideal of S, because μ(cd) =μ_c(b) = 0.6 e $^{\mathbf{0.6\pi i}}$(not ≥) 0.8e $^{\mathbf{0.5\pi i}}$ = μ_c(c)

Definition 2.13 : A Complex temporal fuzzy soft set on a semi group S is known as a complex temporal l fuzzy soft interior ideal of S, if C (xky,y) ≥ C (k,t)

$$\text{i.e.) } Pc\,(xky,t)\,.\,e^{i\delta_{c(xky,t)}} \geq Pc\,(k,t)\,e^{i\delta_{c(kx,t)}} \text{ for all } x,k,\, y\ \varepsilon\ S \text{ and } t\ \varepsilon\, T.$$

Example 2.14: Let S = {a, b, c, d} be a semigroup then

●	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	

C = { < a, 0.7 e $^{0.6\pi i}$>, < b, 0.3 e $^{0.7\pi i}$>,< c, 0.3 e $^{0.7\pi i}$>, < d, 0.0 e $^{0.0\pi i}$>} then obviously C is a complex temporal fuzzy soft interior ideal of S.

Remark 2.15: Every Complex temproal fuzzy soft interior ideal is a complex temporal fuzzy soft interior ideal. But the converse may not be true as seen in the above example. For

Left Ideal: μ_c(dc,t) = μ $_c$(b,t) = 0 (not ≥) 0.3 e $^{0.7\pi i}$ = μ $_c$(c,t)

Right ideal: μ $_c$(dc,t)= μ $_c$(b,t) = 0 ≥ μ $_c$(d) So it is a Complex temporal fuzzy soft right ideal but not a left ideal. Hence C is not a Complex temporal fuzzy soft ideal. ⊗

3. Results:

The following results based on the above definitions

Proposition 3.1: A complex temporal fuzzy soft set C on a semi group S, the following are equivalent;

✓ C is a Complex temporal fuzzy soft ideal of S.

- ✓ $S \otimes C \subseteq C$.

Proof:

(i) $\Rightarrow$ (ii)

Assume that C is a complex temporal fuzzy soft ideal of S.

Let x € S, and t € T, such that $(S \otimes C)(x,t) = 0$; then it is clear $S \otimes C \subseteq C$, whenever there exist any two element y,k € S such that x = yk.

Then $(Is \bullet Pc.\, e^{i\delta_c})(x,t) = \max_{(x=yk)} [\inf \{ Is(y,t) \bullet Pc(k,t) \bullet e^{i\delta_c}(k,t) \}$

$\leq \max_{(x=yk)} [\inf \{ I \bullet ei2\pi \bullet Pc(yk,t) \bullet e^{i\delta_c}(yx,t) \}$

$= Pc(x,t) \bullet e^{i\delta_c}(x,t)$

(ii) $\Rightarrow$ (i)

Suppose $S \otimes \mathbf{C} \subseteq C$. For any element y,k € S, let x = yk, then

$Pc(yk,t) \bullet e^{i\delta_{c(yk,t)}} = Pc(k,t) \bullet e^{i\delta_{c(x,t)}}$

$\geq \{ Is \bullet Pc \bullet e^{i\delta_c})(x,t)$

$= \max_{(x=yk)} [\inf \{ Is(y,t) \bullet Pc(k,t) \bullet e^{i\delta_c}(k,t) \}$

$= Pc(x,t) \bullet e^{i\delta_c}(x,t)$

Hence C is a Complex temporal l fuzzy soft left ideal of S.

Proposition 3.2: A Complex temporalfuzzy soft set C on a semi group S, the following are equivalent.

- ✓ C is a complex temporal right ideal of S.
- ✓ (ii) $C \otimes S \subseteq C$.

Proof:

Proof is similar to the proposition 3.1

Theorem 3.3: If C is a Complex temporal fuzzy soft set of a semi group S. Then $S \otimes C$ (resp. $C \otimes S$) is a complex temporal fuzzy soft left (resp. Right) ideal of S.

Proof:

Since $S \otimes (S \otimes C) = (S \otimes S) \otimes C) \subseteq S \otimes C$

It follows from proposition 3.1, that $S \otimes C$ is a complex temporal fuzzy soft left ideal of S. Similarly $C \otimes S$ is a complex temporal fuzzy soft right ideal of S.

Theorem 3.4: Let S be a left zero semigroup of S. If C is a complex temporalfuzzy soft left ideal of S, then C(x,t) = C(y,t) for all x,y € S and t € T.

Proof:

Let x,y € S and t € T then xy=x and yx = y then

$Pc(x,t) \bullet e^{i\delta_c^{(x,t)}} = Pc(xy,t) \bullet e^{i\delta_c^{(xy,t)}}$

$\geq Pc(y,t) \bullet e^{i\delta_c^{(y,t)}}$

$= Pc(yx,t) \bullet e^{i\delta_c^{(yx,t)}}$

$= Pc(x,t) \bullet e^{i\delta_c^{(x,t)}}$

Therefore C(x,t) = C(y,t) for all x,y€S and t€ T.

Theorem 3.5: Let S be a right zero semigroup of S. If C is a complex temporal fuzzy soft right ideal of S, then C(x,t) = C(y,t) for all x,y € S and t € T.

Proof:

Proof is similar to the theorem 3.4.

Theorem 3.6: Let C be a complextemporal fuzzy soft left ideal of a semigroup S. If the set of idempotent elements of S form a left zero semi group of S, then C(x,t) = C(y,t) for all idempotent elements x and y of S and t € T.

Proof:

Let Idm (S) be the set of all idempotent elements of S and assume that Idm(S) is a left zero semigroup of S.

For any x,y € Idm (S) and t€ T,

We have xy = x and yx = y then

$Pc(x,t) \bullet e^{i\delta_c^{(x,t)}} = Pc(xy,t) \bullet e^{i\delta_c^{(xy,t)}}$

$\geq Pc(y,t) \bullet e^{i\delta_c^{(y,t)}}$

$= Pc(yx,t) \bullet e^{i\delta_c^{(yx,t)}}$

$= Pc(x,t) \bullet e^{i\delta_c^{(x,t)}}$

$$\geq Pc(x,t) \bullet e^{i\delta_c(x,t)}$$
$$= Pc(y,t) \bullet e^{i\delta_c(y,t)}$$

Therefore C(x,t) = C(y,t) for all x,y € Idm (S) and t € T.

Proposition 3.7: If S be a semi group**,** then the intersection of two complex temporal fuzzy soft soft left (respectively right) ideal of S is a Complex temporal fuzzy soft left (respectively right) ideal of S.

Proof:

Let C_1 and C_2 be any two complex temporal fuzzy soft left ideal of semigroup S, and x, y € S and t € T. Then

$$(Pc1\bullet e^{i\delta_{c1}} \bigcap Pc2\bullet e^{i\delta_{c2}})(xy,t) = \inf\{ Pc_1(xy,t)\bullet e^{i\delta_{c_1}(xy,t)}, Pc_2(xy,t)\bullet e^{i\delta_{c_2}(xy,t)}\}$$
$$\geq \inf\{ Pc_1(y,t)\bullet e^{i\delta_{c}(xy,t)}, Pc_2(xy,t)\bullet e^{i\delta_{c}(y,t)}\}$$
$$= \{ Pc_1\bullet e^{i\delta_{c_1}}, Pc_2\bullet e^{i\delta_{c_2}}\}(y,t)$$

Thus $C_1 \bigcap C_2$ is a complex temporal fuzzy soft left ideal of S.

Proposition 3.8: If S be a semi group, then the union of two Complex temporalfuzzy soft left (resplyright) ideals of S is a complex temporal fuzzy soft left (respectively right) ideal of S.

Proof:

Let C_1 and C_2 be any two complex temporal fuzzy soft left ideals of S, and x,y € S, and t € T. Then

$$(Pc1\bullet e^{i\delta_{c1}} \bigcup Pc2\bullet e^{i\delta_{c2}})(xy,t) = \mathrm{Sup}\{ Pc_1(xy,t)\bullet e^{i\delta_{c_1}(xy,t)}, Pc_2(xy,t)\bullet e^{i\delta_{c_2}(xy,t)}\}$$
$$\geq \mathrm{Sup}\{ Pc_1(y,t)\bullet e^{i\delta_{c}(xy,t)}, Pc_2(xy,t)\bullet e^{i\delta_{c}(y,t)}\}$$
$$= \{Pc_1\bullet e^{i\delta_{c_1}} \bigcap Pc_2\bullet e^{i\delta_{c_2}}\}(y,t)$$

Thus $C_1 \bigcup C_2$ is a complex temporalfuzzy soft left ideal of S.

Theorem 3.9: If C_1 and C_2 be a complex temporal fuzzy soft left and right ideals of a semi group S, respectively, then $C_1 \otimes C_2 \subseteq C_1 \bigcap C_2$

Proof:

Let C_1 is complex temporalfuzzy soft right ideal and C_2 is any complex temporal fuzzy soft left ideal of S, then by proposition 3.1 and proposition 3.2, we have

$$C_1 \otimes C_2 \subseteq C_1 \otimes S \subseteq C_1 \text{ and } C_1 \otimes C_2 \subseteq S \otimes C_2$$

Conclusion:

In this paper, we define Complex temporal fuzzy soft left (respectively right) ideal in semi group and study some results prove on its.

References:

1. S. Abdullah, M. Aslam and K. Ullah, Bipolar fuzzy soft sets and its applications in decision making problem, J. Intell. Fuzzy Syst. 27:2, 729-742 (2014).
2. S. Abdullah, M. Aslam and K. Hila, A new generalization of fuzzy bi-idealsin semigroups and its Applications in fuzzy finite state machines, Multiple Valued Logic and Soft Computing, 27:2, 599-623 (2015).
3. H. Aktas. and N. Cagman, Soft sets and soft groups, Inform. Sci. 177, 2726-2735 (2007).
4. M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, on some new operationsin soft set theory, Comput. Math. Appl. 57, 1547-1553 (2009).
5. A.O. Atagun and A. Sezgin, Soft substructures of rings, fields and modules,Comput.Math. Appl. 61 (3), 592-601 (2011).
6. K. T. Atanassov, “Temporal intuitionistic fuzzy sets”, Comptes Rendus Acad. Bulgare Sci., 1991, 7, 5-7.
7. N. Cagman and S. Engino˘glu, Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207, 848-855 (2010).
8. N. Cagman, F. C¸ ıtak and H. Aktas¸, Soft int-groups and its applications togroup theory, Neural Comput. Appl. 21, 151-158 (2012).
9. F. Feng, X.Y. Liu, V. Leoreanu-Fotea, Y.B. Jun, Soft sets andsoft rough sets,Inform. Sci. 181 (6), 1125-1137 (2011).
10. P. K. Maji, R. Biswas and A.R. Roy, Soft set theory, Comput.Math. Appl. 45,555-562 (2003).
11. P. K. Maji, A.R. Roy and R. Biswas, An application of soft sets in a decisionmaking problem, Comput. Math. Appl. 44, 1077-1083 (2002).
12. D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37, 19-31(1999).
13. A. Rosenfeld, Fuzzy groups, J.M. Anal. And Appli. 35,512 - 517 (1971).
14. Shery Fernandez, Ph.D. thesis “A study of fuzzy g-modules” April 2004.

15. L.A. Zadeh, Fuzzy sets, Information Control 8, 338 - 353 (1965).
16. J. Zhan, N. Cagman, A. Sezgin Sezer, Applications of soft union sets to hemirings via SU-h-ideals, J. Intell. Fuzzy Syst.26, 1363-1370 (2014.