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N-FUZZY COMPLEMENT OPERATIONS ON COMPLEX NUMBER

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Abstract:

In this paper, we define N-fuzzy complex numbers and N-fuzzy complex sets and investigated someof itsproperties on them.

Key Words: Fuzzy set, complex fuzzy set, complex fuzzy number, addition, multiplication and division, N-fuzzy set.

1. Introduction:

Complex fuzzy set and logic, which are extensions of fuzzy sets and logic respectively, were first proposed by Ramot et al. [2003]. According to their definition, a complex fuzzy set is characterized by a complex grade of membership, which is a combination of a traditional fuzzy degree of membership, 108 referred to as the amplitude term, with the addition of an extra term: the phase term. The range of Ramot complex grade of membership is a unit disk in a complex plane. Ramot at al. discuss several complex fuzzy sets operations, such as complement, union, and intersection [2002]. The Ramot type of complex fuzzy set can effectively handle cyclic or periodic fuzzy data. Note, however, that the complex fuzzy set concept introduced by Ramot et al. is different from the one defined by Buckley and Zhang [2007]. Zhang et al. [2007] studied other operations and delta-equalities of complex fuzzy sets and complex fuzzy relations. Dick studied complex fuzzy sets and complex fuzzy logics and proposed several applications [2006]. Indeed, the applications of complex fuzzy sets span various fields, such as signal processing physical phenomena and economics.The elements of a crisp set are pairwise different but if we allow repeated occurrences of any element, then we get a mathematical structure. This mathematical structure is called multiset [1991]. The numerous applications of multisets have found in mathematics and computer science. In addition multisets are used in concurrency theory [1985].If we allow repeated occurrences of any element of a set integral number of times (includesa negative number of times), we get a structure that has been called hybrid set. Loeb [1992]introduced this mathematical structure and shown that one can use a hybrid set to describe theroots of a rational functions where elements that occur a positive number of times and describe the poles of a rational function where elements that occur a negative number of times. Yager [1994] developed the concept of fuzzy multisets from crisp multisets and it is a fuzzy subsets whose elements may occur more than once. In this paper, we define N-fuzzy complex numbers and N-fuzzy complex sets and investigated some of its properties.

2. Preliminaries:

From the literature, we recall the following definitions for the development of N-fuzzy complex numbers and N-fuzzy complex fuzzy sets.

Definition 2.1:

Let a set χ be fixed. An N-fuzzy set. A in χ is defined as an object of the following form $A=\{(x,\delta_A(x))/x\in X\}$Where the function $\delta_A : X \to [-1,0]$ in called the degree of membership of the element $x\in X$ to the set A in X and for every $x \in X$, $-1\le \delta_A(x)\le 0$.

Definition 2.2:

A N-fuzzy complex number Z is defined by its membership function $\delta(Z): C \to [-1,0]$, where C is the set of all complex numbers. An $\alpha - cut$ of Z is $Z^{\alpha}=\{Z/\delta(Z)<\alpha\}$, where $-1\le\alpha<0$ and separately $Z'=\{Z/\delta(Z)=-1\}$.

Definition 2.3:

Z is an N-fuzzy complex number if it satisfies the following three conditions:

- ✓ $\delta(Z): C\to[-1,0]$ is continuous
- ✓ $Z^{\alpha}, -1\le\alpha\le 0$ Is open, bounded, connected and simply connected.
- ✓ Z^{-1} Is non- empty, compact, arc wise connected and simply connected.

Definition 2.4:

A function $T:[-1,0]\times[-1,0]\to[-1,0]$ is said to be a T-norm if for all $a,b,c\in[-1,0]$.

- ✓ $T(a,-1)=a$
- ✓ $T(a,b)=T(b,a)$
- ✓ $T(a,T(b,c))=T(T(a,b),c)$
- ✓ $T(a,b)\le T(a,c)$ If $b\le c$.

Similarly T-co-norm (S-norm) is a commutative, associative and non-decreasing mapping $S:[-1,0]\times[-1,0]\to[-1,0]$ that satisfy the boundary condition $S(a,0)=a$ for all $a\in[-1,0]$.

Definition 2.5:

A function $C:[-1,0]\to[-1,0]$ is called a N-fuzzy complement operation, if it satisfies the following conditions;

- ✓ $C(-1)=0$ and $C(0)=-1$
- ✓ $C(a)\le C(b)$ if $a>b$ for all $a,b\in[-1,0]$

Definition 2.6: A T-norm T and a T-co-norm S are dual with respect to a N-fuzzy complement operation C if and only if $C(T(a,b))=S(c(a),c(b))$ and $C(S(a,b))=T(c(a),c(b))$ for all $a,b\in[-1,0]$.

Let Us define $Z=x+iy$, where $x,y\in X$ .of X_r and Y_r are real fuzzy numbers with membership function $\delta_r(x)$ and $\delta_r(y)$ respectively, then $Z_r=X_r+iY_r$ is a N-fuzzy complex number with membership function. $\delta_r(z)=\inf\{\delta_r(x),\delta_r(y)\}$, Similarly, we can define the N-fuzzy complex numbers $Z_g=X_g+iY_g$ with membership function $\delta_g(z)=\inf\{\delta_g(x),\delta_g(y)\}$ and $Z_b=X_b+iY_b$ with membership function $\delta_b(z)=\inf\{\delta_b(x),\delta_b(y)\}$ respectively.

Now, we consider a threedimensional fuzzy membership complex function $(\delta_r(z),\delta_g(z),\delta_b(z))$ between two pixels $x,y\in X$ and it can be represented as a N-fuzzy complex set

$$A=\{(Z,\delta_r(z),\delta_g(z),\delta_b(z))/z=x+iy and x,y\in x\}.$$

In this part of the paper, we try to define the arithmetic operations on a N-fuzzy complex set A. Let $f:C\times C\to C$ defined by $f(z_1z_2)=W$.

Now, we extend f to $\overline{A}\times\overline{A}$ into $\overline{A}$ by $f(\overline{z_1}\,\overline{z_2})=\overline{W}$ if

$$\sigma_k(z)=\max\{\pi_k(z_1,z_2)/f(z_1,z_2)=w\}$$

Where $\pi_k(z_1,z_2)=\inf\{\delta_k(z_1),\delta_k(z_2)\}$ for $k\in p$.

If we Use $f(z_1,z_2)=z_1+z_2$ or $f(z_1z_2)=z_1z_2$, then we obtain $\overline{w}=\overline{z_1}+\overline{z_2}$ or $\overline{w}=\overline{z_1}\,\overline{z_2}$, where $\overline{w}=\{(w,\delta_1(w)/w\in c,k\in p\}$, $\overline{z_1}=\{(z_1,\delta_1(z_1))/k\in N\},\overline{z_2}=\{(z_2,\delta_2(z_2))/k\in p\}$

Now, define-z for negation as

$-z=\{(z,\delta_1(z)/z\in c\}=\{c-z,\delta_1(-z)/z\in c\}$

Thus, the difference $\overline{z_1}-\overline{z_2}$, $\overline{z_1},\overline{z_2}\in\overline{A}$ is defined by $\overline{z_1}-\overline{z_2}=\overline{z_1}+\overline{(-z_2)}$

The reciprocal of $\overline{z}\in\overline{A}$ is denoted by z^{-1} and is defined by

$$z^{-1}=\{(z,\delta_1(z)/z\in c\}=\{(z^{-1},\delta_1(z^{-1})/z\in c\}$$

Thus the division of two N-fuzzy complex number $\overline{z_1},\overline{z_2}\in\overline{A}$ is denoted by $\dfrac{\overline{z_1}}{\overline{z_2}}$ and is defined by $\dfrac{\overline{z_1}}{\overline{z_2}}=\overline{z_1}\,\overline{z_2^{-1}}$.

The conjugate of $\overline{z}\in\overline{A}$ is denoted by $\overline{z^*}$ and is defined by

$$\overline{z^*}=\{(z,\delta_1(\overline{z})/z\in c\},\text{where } \overline{z}=x-iy \text{ if } z=x+iy.$$

The modulus of $|\overline{z}|$ of $\overline{z}$ is defined by $|\overline{z}|=\max\{\delta_k(r)/k\in p\}$, r is the modulus of $z\in c$ and $\delta_k(r)=\max\{\delta_k(z)/z\in c,|z|=r\}$ for $k\in p$.

Here we observe that the addition and multiplication of N-fuzzy complex numbers by using $\alpha-cuts$

Let $M^\alpha=\{z_1+z_2/(z_1,z_2)\in\overline{z_1^\alpha}\times\overline{z_2^\alpha}\}$ and $N^\alpha=\{z_1z_2/(z_1,z_2)\in\overline{z_1^\alpha}\times\overline{z_2^\alpha}\}$

These two $\alpha - cuts$ are needed to investigate the properties of N-fuzzy complex numbers.

3. Some Properties of N-Complex Fuzzy Number:

In this section, we have proved some important results

Theorem 3.1:

If $\overline{w} = \overline{z_1} + \overline{z_2}$, then $\overline{w^\alpha} = M^\alpha$ and if $\overline{w} = \overline{z_1}\,\overline{z_2}$, then $\overline{w^\alpha} = p^\alpha$.

Proof:

In this theorem we have two cases arise.

Case (i): $-1 \le \alpha < 0$

Let $w \in \overline{w^\alpha}$ .Then we can find $z_1 \in \overline{z_1}$ and $z_2 \in \overline{z_2}$

Because $\overline{w} = \overline{z_1} + \overline{z_2}$ such that $z_1 + z_2 = w$ and $\pi_k(z_1, z_2) < \alpha$

Since $\delta_k(w) < \alpha$ for $k \in p.$ This implies that $\delta_k(z_i) < \alpha$ for i =1, 2 and $k \in p$

$\Rightarrow z_1 \in \overline{z_1^\alpha} and z_2 \in \overline{z_2^\alpha}. \Rightarrow (z_1, z_2) \in \overline{z_1^\alpha} \times \overline{z_2^\alpha}, where \overline{z_1^\alpha} \times \overline{z_2^\alpha}$ is the Cartesian product between $\overline{z_1^\alpha} and \overline{z_2^\alpha}.$

Thus from the given definition of $\mu^\alpha, \omega \in \mu^\alpha$ .Conversely, let $\omega \in \mu^\alpha$.

Then $\omega = z_1 + z_2$ with $\delta_k(z_i) < \alpha$ for $i = 1,2 and k \in p$

$$\Rightarrow \pi_k(z_1, z_2) < \alpha \Rightarrow \max\{\pi_k(z_1, z_2) / z_1 + z_2 = w\} < \alpha \Rightarrow \pi_k(\omega) < \alpha \text{ for all } k \in p$$

$$\Rightarrow \omega \in \overline{\omega^\alpha} \text{ .Hence } \overline{\omega^\alpha} = p^\alpha$$

Case (ii): $\alpha = -1$

Let $\omega \in \mu^{-1}$. Then there exist $z_1 and z_2$ such that $z_1 + z_2 = \omega and \pi_k(z_1, z_2) = -1 for all k \in p$.

Thus, $\delta_k(\omega) = -1 for all k \in \rho \Rightarrow \omega \in \overline{\omega^{-1}}.$

Conversely,

Let $\omega \in \overline{\omega^{-1}}$ then we can find z in, supp $(\overline{z_1}) and (\overline{z_{2n}}) in \sup p(\overline{z_2}) for n = 1,2,3.....$

Such that $z_{1n} + z_{2n} = \omega and \pi_k(z_{1n}, z_{2n}) < -1 - \frac{1}{n} for all p \in P.$

Since the support are compact we may choose a subsequence $z_{1nk} \to z_1 and z_{2nk} \to z_2$ with $z_1 + z_2 = \omega and \pi_k(z_1, z_2) \le -1$)for all $k \in \rho$,because π_k is continuous.

$$\Rightarrow z_i \in \overline{z_i^{-1}} for i = 1,2$$

$$\Rightarrow \inf\{\delta_k(z_1), \delta_k(z_2)\} \le -1 \text{ for all } k \in \rho \Rightarrow \delta_k(z_i) \le -1 for i = 1,2 \text{ and for all } k \in \rho \Rightarrow (z_1, z_2) \in \overline{z_1^{-1}}, \overline{z_2^{-1}}$$

$$\Rightarrow \omega \in \rho^{-1}$$

.Hence, if $\overline{\omega} = \overline{z_1} + \overline{z_2} the \overline{\omega}^\alpha = \mu^\alpha.$

Before proceeding to the $m \times t$ theorem, we will discuss some important results which are needed for the upcoming results.

Lemma 3.2:

If $\overline{\omega} = \overline{z_1} + \overline{z_2} or \overline{\omega} = \overline{z_1}\,\overline{z_2}, then \overline{\omega^\alpha}$ is open for $-1 \le \alpha < 0$

Proof:

Let $\overline{\omega} = \overline{z_1} + \overline{z_2} and \omega \in \overline{\omega^\alpha} for -1 \le \alpha < 0$

Then by theorem $(3.1), \Im(z_1, z_2) \in \overline{z_1} \times \overline{z_2}$ such that $z_1 + z_2 = \omega.$ Since $\overline{z_2^\alpha}$ is open so, choose on disk $O(z_2, \in)$ with radius $\in > 0,$ cantered at z_2 such that $O(z_2, \in)$ contained in $\overline{z_2^\alpha}$ thus $z_1 + O(z_2, \in)$ is an open set containing $\omega = z_1 + z_2$ and contained in $\mu^\alpha = \overline{\omega^\alpha}.$ Therefore $\overline{\omega^\alpha}$ is open. If $\overline{\omega} = \overline{z_1}\,\overline{z_2},$ then by similar argument by choosing $\omega = z_1 z_2 and z_1.O(z_2, \in)$ in place of $\omega = z_1 + z_2$ and $z_1 + O(z_2, \in)$reply thus $\overline{\omega^\alpha}$ is open.

Lemma 3.3:

If $\overline{\omega} = \overline{z_1} + \overline{z_2} or\ \overline{\omega} = \overline{z_1 z_2}$ .Assuming that $\omega_n \in \overline{\omega^0}\ for n \in \rho$

Converges to $\omega and \delta_k(\omega_n)$Converges to $\lambda_k in[-1,0]\ for k \in \rho. then \delta_k(\omega) \le \lambda_k\ for all k \in \rho$

Proof:

Let $\overline{\omega} = \overline{z_1} + \overline{z_2}$. Then for every $\in > 0$, there is $z_{1n} \in \overline{z_1^0}$ and $z_{2n} \in \overline{z_2^0}$ such that $z_{1n} + z_{2n} = \omega_n$ and $\delta_k(\omega_n) - \in < \pi_k(z_{1n}, z_{2n}) \le \delta_k(\omega_n)$ for $k \in \rho$. Hence all z_{1n}, z_{2n} and $\omega_n\ for n \in \rho$ belongs to Compact sets. Thus, we can choose subsequence's such that $z_{1nk} \to z_1, z_{2nk} \to z_2, \omega_{nk} \to \omega and z_1 + z_2 = \omega$ with $\lambda_k - \in < \pi_k(z_1, z_2) \le \lambda_k$ for $k \in \rho$, because π_k is continuous. Since $\in$ is arbitrary, so $\pi_k(z_1, z_2) = \lambda_k\ for k \in \rho$ thus $\delta_k(\omega) \le \lambda_k\ for k \in \rho$. By similar argument, we can prove that if $\overline{\omega} = \overline{z_1 z_2}$ then $\delta_k(\omega) \le -k$ for $k \in \rho$.

Theorem 3.4:

If $\overline{z_1}$ and $\overline{z_2}$ are N-fuzzy complex numbers, then so $\overline{z_1} + \overline{z_2}, \overline{z_1 z_2}, \overline{z_1} - \overline{z_2} and \dfrac{\overline{z_1}}{\overline{z_2}}$.

Proof:

Let us assume that $\overline{\omega} = \overline{z_1} + \overline{z_2}$ we have to show $\overline{\omega}$ is a N-fuzzy complex number our first target is to show $\delta_k(\omega)$ is continuous for all $k \in p$ by arguing that $\omega_n \to \omega$ implies $\delta_k(\omega_n) \to \delta_k(\omega)\ for k \in \rho$.

If is sufficient to choose $\omega_n in \overline{\omega^0}$. since $\delta_k(\omega_n) \in [-1,0]$, so, there exists a subsequence

$$\delta_k(\omega_n) \to \lambda_k \in [-1,0]\ for k \in \rho.$$

We have, $\lim\min \delta_k(\omega_n) \le \lambda_k \le \lim\max\ for k \in \rho.$

Also by lemma-3.2, $\{\omega / \delta_k(\omega) \ge t\}$ is a closed set for all real t. Therefore, $\delta_k(\omega)\ for k \in \rho$ is lower semicontinous and we have, $\lim\min \delta_k(\omega_n) \ge \delta_k(\omega)$For $k \in N$.

Again by lemma-3.3, $\delta_k(\omega) \le \lambda_k\ for k \in p$. Thus $\lim\min \delta_k(\omega_n) = \lambda_k = \delta_k(\omega)\ for k \in p.$

Hence there is a subsequence $\delta_k(\omega_{nj}) \to \lim\max \delta_k(\omega_n)$.

Also by lemma-3.3 implies, $\delta_k(\omega) = \lim\max \delta_k(\omega_n)$

Therefore, $\lim\min \delta_k(\omega_n) = \delta_k(\omega) = \lim\max \delta_k(\omega_n)\ for all K \in p.$

$\Rightarrow \lim \delta_k(\omega_n) = \delta_k(\omega)\ for K \in p. \Rightarrow \delta_k(\omega)$is continuous for all $k \in p.$

Since $\overline{\omega^\alpha}\ for -1 \le \alpha < 1$ is the sum of two bounded sets by theorem-1,

So $\overline{\omega^\alpha}\ for -1 \le \alpha \le 0$ is bounded. Hence, $\overline{\omega}$ satisfies all the conditions of N-fuzzy complex number.

(ii) By replacing $\overline{\omega} = \overline{z_1} + \overline{z_2} by \overline{\omega} = \overline{z_1 z_2}$ in (i), we can prove that $\overline{\omega} = \overline{z_1 z_2}$ is a N-fuzzy complex number.

(iii) Since $\overline{z_2}$ is a N-fuzzy complex number, then so is $-\overline{z_2}$ and therefore $\overline{z_1} - \overline{z_2}$ is a N-fuzzy complex number.

(iv) Since the mapping $z \to z^{-1}, z \ne 0$ is continuous and $(z^{-1})^\alpha = \left(\overline{z^\alpha}\right)^{-1}$.

Thus z^{-1} is a N-fuzzy complex number. Hence $\dfrac{\overline{z_1}}{\overline{z_2}}$ is a N-fuzzy complex number.

By using theorem3.1, theorem3.3 and lemma3.1, lemma3.2, we can prove the following result;

Lemma 3.5:

$$\left|\overline{z}\right|^{\alpha} = \left|\overline{z^{\alpha}}\right| for -1 \le \alpha \le 0.$$

Theorem 3.6:

Let $\overline{z_1}, \overline{z_2}$ be N-fuzzy complex numbers. Then

- ✓ $\left|\overline{z_1} + \overline{z_2}\right| \le \left|\overline{z_1}\right| + \left|\overline{z_2}\right|$
- ✓ $\left|\overline{z_1}\,\overline{z_2}\right| = \left|\overline{z_1}\right|\left|\overline{z_2}\right|$
- ✓ $\left|\dfrac{\overline{z_1}}{\overline{z_2}}\right| = \dfrac{\left|\overline{z_1}\right|}{\left|\overline{z_2}\right|}.$

Proof: (i) By theorem 3.1 and lemma-3.3, we have

$$\left|\overline{z_1} + \overline{z_2}\right|^{\alpha} = \left|\left(\overline{z_1} + \overline{z_2}\right)^{\alpha}\right| = \left|\overline{z_1^{\alpha}} + \overline{z_2^{\alpha}}\right|$$

$$= \left\{\left|\overline{z_1} + \overline{z_2}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}}\right\} \quad (1)$$

Also, by lemma-3.3, we have

$$\left(\left|\overline{z_1}\right| + \left|\overline{z_2}\right|\right)^{\alpha} = \left|\overline{z_1}\right|^{\alpha} + \left|\overline{z_2}\right|^{\alpha} = \left|\overline{z_1^{\alpha}}\right| + \left|\overline{z_2^{\alpha}}\right|$$

$$= \left\{\left|\overline{z_1}\right| + \left|\overline{z_2}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z2}^{-\alpha}\right\} \quad (2)$$

Hence from (1) and (2),

$$\left|\overline{z} + \overline{z_2}\right| \le \left|\overline{z_1}\right| + \left|\overline{z_2}\right| forall z_1, z_2 \in C.$$

(ii) We have $\left|\overline{z_1}\,\overline{z_2}\right|^{\alpha} = \left|\left(\overline{z_1}\,\overline{z_2}\right)^{\alpha}\right| = \left|\overline{z_1^{\alpha}}\,\overline{z_2^{\alpha}}\right|$

$$= \left\{\left|\overline{z_1}\,\overline{z_2}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}}\right\} \quad (3)$$

Also

$$\left(\left|\overline{z_1}\right|\left|\overline{z_2}\right|\right)^{\alpha} = \left|\overline{z_1}\right|^{\alpha}\left|\overline{z_2}\right|^{\alpha} = \left|\overline{z_1^{\alpha}}\right|\left|\overline{z_2^{\alpha}}\right| = \left\{\left|\overline{z_1}\right|\left|\overline{z_2}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z2}^{-\alpha}\right\} \quad (4)$$

Hence from (3) & (4) $\left|\overline{z_1}\,\overline{z_2}\right| = \left|\overline{z_1}\right|\left|\overline{z_2}\right|.$

(iii) We see that

$$\left|\frac{\overline{z_1}}{\overline{z_2}}\right|^{\alpha} = \left|\overline{z_1}\,\overline{z_2^{-1}}\right|^{\alpha} = \left|\overline{z_1^{\alpha}}\left(\overline{z_2^{-1}}\right)^{\alpha}\right| = \left|\overline{z_1^{\alpha}}\left(\overline{z_2^{\alpha}}\right)^{-1}\right|$$

$$= \left\{\left|\frac{\overline{z_1}}{\overline{z_2}}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}}\right\} \quad (5)$$

Also

$$\left(\frac{\left|\overline{z_1}\right|}{\left|\overline{z_2}\right|}\right)^{\alpha} = \left(\left|\overline{z_1}\right|\left|\overline{z_2}\right|^{-1}\right)^{\alpha} = \left|\overline{z_1}\right|^{\alpha}\left(\left|\overline{z_2^{-1}}\right|\right)^{\alpha} = \left|\overline{z_1}\right|^{\alpha}\left(\left|\overline{z_2}\right|^{\alpha}\right)^{-1}$$

$$= \left\{\left|\frac{\left|\overline{z_1}\right|}{\left|\overline{z_2}\right|}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}}\right\} \quad (6)$$

From (5) and (6) $\alpha - cuts$ are Equal and hence $\left|\frac{\overline{z_1}}{\overline{z_2}}\right| = \frac{\left|\overline{z_1}\right|}{\left|\overline{z_2}\right|}$ is proved.

Conclusion:

The theory of N-fuzzy complex sets is an extension of Buckley fuzzy complex set theory. In this paper, first time we made an attempt to introduce the different of N-fuzzy complex numbers. Thus, the N-fuzzy complex numbers and sets are paving the way to large possibilities to future research.

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