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TRIANGULAR MEMBERSHIP FUNCTIONS OF N-FUZZY SOFT REGULAR BI-IDEALS

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Abstract:

In this paper, we investigated the new contribution to soft set theory called N- fuzzy neutrosophic soft theory over soft regular groupoids. Also complemented, N-level set and various ideal structures have been studied with suitable examples.

Index Terms: Fuzzy Set, Soft Set, Union and Intersection of N-Fuzzy Soft Set, Left (Right) Ideals & Arbitrary Intersections.

1. Introduction:

The theory of Neutrosophic set which is the generalization of the classical sets, conventional fuzzy set [9], intuitionistic fuzzy set [1] and interval valued fuzzy set [9] was introduced by F. Smarandache [7]. This concept has recently motivated new approach in several directions such as databases, medical diagonses problems, decision making problem, topology, control theory and so on. The concept of neutrosophic set handles indeterminate data where as fuzzy theory and intuitionistic fuzzy set theory failed when the relations are indeterminate. The concept of Neutrosophic set was introduced by F. Smarandache [8] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In this paper, we investigated the new contribution to soft set theory called N- fuzzy neutrosophic soft theory over soft regular groupoids. Also complemented, N-level set and various ideal structures has been studied with suitable examples.

2. Preliminaries:

Definition 2.1: Let U be a non-empty set. Then by a fuzzy set on U is meant a function A : U $\rightarrow$ [0, 1]. A is called the membership function, A(x) is called the membership grade of x in A. We also write A = {(x, A(x)) : x $\in$ U }.

Example 2.2: Consider U = {a, b, c, d} and A : U $\rightarrow$ [0, 1] defined by A(a) = 0, A(b) = 0.1, A(c) = 0.9, A(d) = 1.

Definition 2.3: Let U be the initial universe set and E be the set of parameters. Let P(U) denote the power set of U . Consider a non empty set A, A$\subset$ E. A pair (F, A) is called a soft set over U, where F : A $\rightarrow$P (U).

Example 2.4: Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . . , h_5} .Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . . , e_8} , where e_1, e_2, . . . , e_8 stand for the attributes "expensive", "beautiful", "wooden", "cheap", "mod- ern" and "in bad", "repair" respectively.

Definition 2.5: A negative fuzzy (N-Fuzzy)neutrosophic soft set $\boldsymbol{\mathcal{A}}$ on the universe of discourse X is defined as $\boldsymbol{\mathcal{A}} = \langle x, \bar{\delta}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x)\rangle$, $x \in \mathcal{X}$, where δ, I, Δ : $\boldsymbol{\mathcal{X}} \rightarrow [-1, 0]$ and $-1 \leq \bar{\delta}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x) \leq 0$.

Definition 2.6: A N-Fuzzy neutrosophic soft set $\boldsymbol{\mathcal{A}}$ over the universe $\boldsymbol{\mathcal{X}}$ is said to be absolute (universe) N-Fuzzy neutrosophic soft set if $\delta_{\mathcal{A}}(x) = 0, I_{\mathcal{A}}(x) = 0, \Delta_{\mathcal{A}}(x) = -1$ for all $x \in \mathcal{X}$. It is denoted by 0_N.

Definition 2.7: A N-Fuzzy neutrosophic soft set $\boldsymbol{\mathcal{A}}$ over the universe $\boldsymbol{\mathcal{X}}$ is said to be null or empty N-Fuzzy neutrosophic soft set if $\delta_{\mathcal{A}}(x) = -1, I_{\mathcal{A}}(x) = -1, \Delta_{\mathcal{A}}(x) = 0$ for all $\in \mathcal{X}$. It is denoted by -1_N.

Definition 2.8: The complement of N-Fuzzy neutrosophic soft set $\boldsymbol{\mathcal{A}}$ is denoted by $\mathcal{A}^C$ and is defined as $\mathcal{A}^C = \langle x, \delta_{\mathcal{A}^c}(x), I_{\mathcal{A}^c}(x), \Delta_{\mathcal{A}^c}(x)\rangle$ where $\delta_{\mathcal{A}^c}(x) = \Delta_{\mathcal{A}}(x), I_{\mathcal{A}^c}(x) = 1\text{-}I_{\mathcal{A}}(x)$, $\Delta_{\mathcal{A}^c}(x) = \delta_{\mathcal{A}}(x)$. The complement of a N-Fuzzy neutrosophic soft set A can also be defined as $0_N - \boldsymbol{\mathcal{A}}$.

Definition 2.9: Let (G, .) be a groupoid and let $-1_N \neq \boldsymbol{\mathcal{A}} \in$NFNS(G), then $\boldsymbol{\mathcal{A}}$ is called is called N-Fuzzy neutrosophic soft subgroupoid (in short NFNSS in G) if

$$\delta_{\mathcal{A}}(xy) \leq \max\{\delta_{\mathcal{A}}(x), \delta_{\mathcal{A}}(y)\}$$
$$I_{\mathcal{A}}(xy) \leq \max\{I_{\mathcal{A}}(x), I_{\mathcal{A}}(y)\}$$
$$\Delta_{\mathcal{A}}(xy) \geq \min\{\Delta_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(y)\} \text{ for all x, y} \in \text{G}.$$

A N-Fuzzy neutrosophic soft set $\mathcal{A} = \{\langle x, \bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}}\rangle, x \in \mathcal{X}\}$in $\mathcal{X}$ can be identified to an ordered pair $(\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ in F ($\mathcal{X}$, [−1, 0]) × F ($\mathcal{X}$, [−1, 0]) × F ($\mathcal{X}$, [−1, 0])

Where F ($\mathcal{X}$, [−1, 0]) denotes the set of all functions from $\mathcal{X}$ to [−1, 0]. For the sake of simplicity, we shall use the notation $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ instead of $\mathcal{A} = \{(x: \bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})/x \in \mathcal{X}\}$.

Definition 2.10: Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be an NFNSS in $\mathcal{X}$. Then the set N$\{(\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}}): (p, q, r)\}$ = $\{x \in \mathcal{X} / \bar{\delta}_{\mathcal{A}} \leq p, \bar{I}_{\mathcal{A}} \leq q, \bar{\Delta}_{\mathcal{A}} \geq r\}$ where $p, q, r \in$ [-1,0] with $p + q + r \geq 1$ is called an N-level set of $\mathcal{A}$.

Definition 2.11: Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ and Let $\mathcal{B} = (\bar{\delta}_{\mathcal{B}}, \bar{I}_{\mathcal{B}}, \bar{\Delta}_{\mathcal{B}})$ be two NFNSS in $\mathcal{X}$. If for all $x \in \mathcal{X}$, $\bar{\delta}_{\mathcal{A}}(x) \geq \bar{\delta}_{\mathcal{B}}(x)$, $\bar{I}_{\mathcal{A}}(x) \geq \bar{I}_{\mathcal{B}}(x)$ and $\bar{\Delta}_{\mathcal{A}}(x) \leq \bar{\Delta}_{\mathcal{B}}(x)$, then $\mathcal{A}$ is called an NFNSS subset of $\mathcal{B}$ and is written as $\mathcal{A} \sqsubseteq \mathcal{B}$. We say that $\mathcal{A} = \mathcal{B}$ if and only if $\mathcal{A} \sqsubseteq \mathcal{B}$ and $\sqsupseteq \mathcal{B}$.

Definition 2.12: Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ and Let $\mathcal{B} = (\bar{\delta}_{\mathcal{B}}, \bar{I}_{\mathcal{B}}, \bar{\Delta}_{\mathcal{B}})$ be two NFNSS in $\mathcal{X}$.

$$(\mathcal{A} \bar{\cup} \mathcal{B}) = \left\{x, min\left\{\bar{\delta}_{\mathcal{A}}(x), \bar{\delta}_{\mathcal{B}}(x)\right\}, min\left\{\bar{I}_{\mathcal{A}}(x), \bar{I}_{\mathcal{B}}(x)\right\}, max\left\{\bar{\Delta}_{\mathcal{A}}(x), \bar{\Delta}_{\mathcal{B}}(x)\right\}\right\}$$

Their union and intersection is also a N-Fuzzy neutrosophic soft set in $\mathcal{X}$ defined as for all $x \in \mathcal{X}$. and

$$(\mathcal{A} \bar{\cap} \mathcal{B}) = \left\{x, max\left\{\bar{\delta}_{\mathcal{A}}(x), \bar{\delta}_{\mathcal{B}}(x)\right\}, max\left\{\bar{I}_{\mathcal{A}}(x), \bar{I}_{\mathcal{B}}(x)\right\}, min\left\{\bar{\Delta}_{\mathcal{A}}(x), \bar{\Delta}_{\mathcal{B}}(x)\right\}\right\}$$

Example 2.13:

Let $\mathcal{X}$ = *w,x,y,z* be a non-empty set. Define

$$\overline{\delta_{\mathcal{A}}} : \mathcal{X} \longrightarrow [-1, 0],$$
$$\overline{I_{\mathcal{A}}} : \mathcal{X} \longrightarrow [-1, 0],$$
$$\overline{\Delta_{\mathcal{A}}} : \mathcal{X} \longrightarrow [-1, 0]$$

as

$$\bar{\delta}_{\mathcal{A}}(w) = -0.9 \quad \bar{\delta}_{\mathcal{A}}(x) = -0.5 \quad \bar{\delta}_{\mathcal{A}}(y) = -0.7 \quad \bar{\delta}_{\mathcal{A}}(z) = -0.6$$
$$\bar{I}_{\mathcal{A}}(w) = -0.8 \quad \bar{I}_{\mathcal{A}}(x) = -0.4 \quad \bar{I}_{\mathcal{A}}(y) = -0.6 \quad \bar{I}_{\mathcal{A}}(z) = -0.3$$
$$\overline{\Delta}_{\mathcal{A}}(w) = -0.7 \quad \overline{\Delta}_{\mathcal{A}}(x) = -0.5 \quad \overline{\Delta}_{\mathcal{A}}(y) = -0.6 \quad \overline{\Delta}_{\mathcal{A}}(z) = -0.2$$

Then $\mathcal{A} = \{\langle w, -0.9, -0.8, -0.7\rangle, \langle x, -0.5, -0.4, -0.5\rangle, \langle y, -0.7, -0.6, -0.6\rangle, \langle z, -0.6, -0.3, -0.2\rangle\}$. It is easy to verify that $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ is N-Fuzzy neutrosophic soft set in X.

Definition 2.14: Let C be a non empty subset of $\mathcal{X}$. Then N-Fuzzy neutrosophic soft characteristic function of C is a function $\bar{\Psi}_C = (\bar{\delta}_{\bar{\Psi}_C}, \bar{I}_{\bar{\Psi}_C}, \bar{\Delta}_{\bar{\Psi}_C})$ defined as, for any $x \in \mathcal{X}$.

$$\bar{\delta}_{\bar{\Psi}_C}(x) = \begin{cases} -1 & \text{if } x \in C \\ 0 & \text{if } x \notin C \end{cases}$$
$$\bar{I}_{\bar{\Psi}_C}(x) = \begin{cases} 0 & \text{if } x \in C \\ 0.5 & \text{if } x \notin C \end{cases}$$
$$\bar{\Delta}_{\bar{\Psi}_C}(x) = \begin{cases} 0.5 & \text{if } x \in C \\ -1 & \text{if } x \notin C \end{cases}$$

We denote N-Fuzzy neutrosophic soft characteristic function of $\mathcal{X}$ by $\bar{\Psi}_x = (\bar{\delta}_x, \bar{I}_x, \bar{\Delta}_x)$.

Definition 2.15: A negative fuzzy set (briefly, N-Fuzzy set) in a non empty set $\mathcal{X}$ is a function $\bar{\mu} : \mathcal{X} \to [-1,0]$. Here we are using "–" for the negative fuzzy functions. Jun et.al ( ) used the term negative valued function and N-Function for negative fuzzy set and N-Fuzzy set.

Definition 2.16: Let G be a groupoid and let $\mathcal{A} \in$ NFNS (G). Then A is called

(i) N-Fuzzy neutrosophic soft left ideal (in short NFNSLI) of G if for any *x,y* ∈ G, $\mathcal{A}(xy) \leq \mathcal{A}(y)$, that is $\delta_{\mathcal{A}(xy)} \leq \delta_{\mathcal{A}(y)}$, $I_{\mathcal{A}(xy)} \leq I_{\mathcal{A}(y)}$ and $\Delta_{\mathcal{A}(xy)} \geq \Delta_{\mathcal{A}(y)}$
(ii) N-Fuzzy neutrosophic soft right ideal (in short NFNSRI) of G if for any x,y ∈ G, $\mathcal{A}(xy) \leq \mathcal{A}(x)$, that is $\delta_{\mathcal{A}(xy)} \leq \delta_{\mathcal{A}(x)}$, $I_{\mathcal{A}(xy)} \leq I_{\mathcal{A}(x)}$ and $\Delta_{\mathcal{A}(xy)} \geq \Delta_{\mathcal{A}(x)}$
(iii) N-Fuzzy neutrosophic soft ideal (in short NFNSI) of G if for any *x,y* ∈ G if it is both NFNSLI and NFNSRI. It is clear that, $\mathcal{A}$ is a NFNSI of G if and only if for any *x,y* ∈ G

$$\bar{\delta}_{\mathcal{A}(xy)} \leq min\{\bar{\delta}_{\mathcal{A}(x)}, \bar{\delta}_{\mathcal{A}(y)}\}$$
$$\bar{I}_{\mathcal{A}(xy)} \leq min\{\bar{I}_{\mathcal{A}(x)}, \bar{I}_{\mathcal{A}(y)}\}$$
$$\bar{\Delta}_{\mathcal{A}(xy)} \geq max\{\bar{\Delta}_{\mathcal{A}(x)}, \bar{\Delta}_{\mathcal{A}(y)}\}$$

However, a NFNSI (respectively NFNSLI, NFNSRI) is a NFNS (G) of G. Note that for any NFNS (G) A of G we have $\bar{\delta}_{\mathcal{A}(x^n)} \leq \bar{\delta}_{\mathcal{A}(y)}$, $\bar{I}_{\mathcal{A}(x^n)} \leq \bar{I}_{\mathcal{A}(y)}$ and $\bar{\Delta}_{\mathcal{A}(x^n)} \geq \bar{\Delta}_{\mathcal{A}(y)}$, where *x* is any composite of *x*'s.

Note: In a groupoid G

(i) A N-Fuzzy neutrosophic soft left ideal (right) of G is N-Fuzzy neutrosophic soft subgroupoid of G but the converse is not true.
(ii) A N-Fuzzy neutrosophic soft left ideal of G may not be a N-Fuzzy neutrosophic soft right ideal of G and vice versa.

Example 2.17:

Let G = {*e* ,*a*, *b*, *c*}. The G is a groupoid under the operation defined in the following table

	e	*a*	*b*	*c*
e	*e*	*a*	*b*	*c*
a	*a*	*e*	*c*	*b*

b	b	c	e	a
c	c	b	a	e

Define $\bar{\delta}_{\mathcal{A}}:\mathcal{X}\to[-1,0]$, $\bar{I}_{\mathcal{A}}:\mathcal{X}\to[-1,0]$, $\bar{\Delta}_{\mathcal{A}}:\mathcal{X}\to[-1,0]$ such that

$$\begin{aligned}\mathcal{A} &= \left(\bar{\delta}_{\mathcal{A}},\bar{I}_{\mathcal{A}},\bar{\Delta}_{\mathcal{A}}\right)\\ &= \{\langle e,-0.7,-0.5,-0.3\rangle,\langle a,-0.6,-0.2,-0.3\rangle,\langle b,-0.9,-0.5,-0.7\rangle,\langle c,-0.8,-0.3,-0.5\rangle\}\end{aligned}$$

Then A is N-Fuzzy neutrosophic soft groupoid of G. Now

$$\bar{\delta}_{\mathcal{A}}(ab)=\bar{\delta}_{\mathcal{A}}(c)=-0.8\nleq-0.9=\delta_{\mathcal{A}}(b).$$

Hence A is not N-Fuzzy neutrosophic soft left ideal of G. If we take

$$\begin{aligned}\mathcal{B} &= \left(\bar{\delta}_{\mathcal{B}},\bar{I}_{\mathcal{B}},\bar{\Delta}_{\mathcal{B}}\right)\\ &= \{\langle e,-0.8,-0.4,-0.6\rangle,\langle a,-0.9,-0.6,-0.7\rangle,\langle b,-0.7,-0.4,-0.5\rangle,\langle c,-0.8,-0.4,-0.7\rangle\}\end{aligned}$$

Then $\mathcal{B}$ is N-Fuzzy neutrosophic soft left and right ideal of G. Hence N-Fuzzy neutrosophic soft ideal of G. Obviously it is an N-Fuzzy neutrosophic soft subgroupoid of G.

3. Properties of N-Fuzzy Soft Subgroupoids:

Proposition 3.1: Let G be a groupoid. Then the intersection of any collection of N-Fuzzy neutrosophic soft subgroupoid of G is N-Fuzzy neutrosophic soft subgroupoid of G.

Proof:

Let $\{\mathcal{A}_i=(\bar{\delta}_{\mathcal{A}_i},\bar{I}_{\mathcal{A}_i},\bar{\Delta}_{\mathcal{A}_i}), i\in\Lambda\}$ be a collection of N-Fuzzy neutrosophic soft subgroupoids of G. Then $\bar{\delta}_{\mathcal{A}_i}(xy)\leq$ max $\{\bar{\delta}_{\mathcal{A}_i}(x),\bar{\delta}_{\mathcal{A}_i}(y)\}$, $\bar{I}_{\mathcal{A}_i}(xy)\leq$ max $\{\bar{I}_{\mathcal{A}_i}(x),\bar{I}_{\mathcal{A}_i}(y)\}$ and $\bar{\Delta}_{\mathcal{A}_i}(xy)\geq$ min $\{\bar{\Delta}_{\mathcal{A}_i}(x),\bar{\Delta}_{\mathcal{A}_i}(y)\}, i\in\Lambda$, $x,y\in G$

$$\begin{aligned}\left(\overline{\bigcap_{i\in\Lambda}}\bar{\delta}_{\mathcal{A}_i}\right)(xy) &= \overline{\bigcap_{i\in\Lambda}}\left(\bar{\delta}_{\mathcal{A}_i}(xy)\right)\\ &\leq \overline{\bigcap_{i\in\Lambda}}\left\{max\left(\bar{\delta}_{\mathcal{A}_i}(x),\bar{\delta}_{\mathcal{A}_i}(y)\right)\right\}\\ &= max\left\{\overline{\bigcap_{i\in\Lambda}}\bar{\delta}_{\mathcal{A}_i}(x),\overline{\bigcap_{i\in\Lambda}}\bar{\delta}_{\mathcal{A}_i}(y)\right\}\\ &= max\left\{\left(\overline{\bigcap_{i\in\Lambda}}\bar{\delta}_{\mathcal{A}_i}\right)(x),\left(\overline{\bigcap_{i\in\Lambda}}\bar{\delta}_{\mathcal{A}_i}\right)(y)\right\}\end{aligned}$$

$$\begin{aligned}\left(\overline{\bigcap_{i\in\Lambda}}\bar{I}_{\mathcal{A}_i}\right)(xy) &= \overline{\bigcap_{i\in\Lambda}}\left(\bar{I}_{\mathcal{A}_i}(xy)\right)\\ &\leq \overline{\bigcap_{i\in\Lambda}}\left\{max\left(\bar{I}_{\mathcal{A}_i}(x),\bar{I}_{\mathcal{A}_i}(y)\right)\right\}\\ &= max\left\{\overline{\bigcap_{i\in\Lambda}}\bar{I}_{\mathcal{A}_i}(x),\overline{\bigcap_{i\in\Lambda}}\bar{I}_{\mathcal{A}_i}(y)\right\}\\ &= max\left\{\left(\overline{\bigcap_{i\in\Lambda}}\bar{I}_{\mathcal{A}_i}\right)(x),\left(\overline{\bigcap_{i\in\Lambda}}\bar{I}_{\mathcal{A}_i}\right)(y)\right\}\end{aligned}$$

$$\begin{aligned}\left(\overline{\bigcap_{i\in\Lambda}}\bar{\Delta}_{\mathcal{A}_i}\right)(xy) &= \overline{\bigcap_{i\in\Lambda}}\left(\bar{\Delta}_{\mathcal{A}_i}(xy)\right)\\ &\geq \overline{\bigcap_{i\in\Lambda}}\left\{min\left(\bar{\Delta}_{\mathcal{A}_i}(x),\bar{\Delta}_{\mathcal{A}_i}(y)\right)\right\}\\ &= min\left\{\overline{\bigcap_{i\in\Lambda}}\bar{\Delta}_{\mathcal{A}_i}(x),\overline{\bigcap_{i\in\Lambda}}\bar{\Delta}_{\mathcal{A}_i}(y)\right\}\\ &= min\left\{\left(\overline{\bigcap_{i\in\Lambda}}\bar{\Delta}_{\mathcal{A}_i}\right)(x),\left(\overline{\bigcap_{i\in\Lambda}}\bar{\Delta}_{\mathcal{A}_i}\right)(y)\right\}\end{aligned}$$

Hence $\overline{\bigcap_{i\in\Lambda}}\mathcal{A}_i$ is N-Fuzzy neutrosophic soft subgroupoid of G.

Proposition 3.2:

The intersection of any collection of N-Fuzzy neutrosophic soft left(right) ideal of G is N-Fuzzy neutrosophic soft left(right) of G. This proof is similar as propsotion-3.1.

Proposition 3.3: Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be N-Fuzzy neutrosophic soft set in G. Then

(i) $\mathcal{A}$ is N-Fuzzy neutrosophic soft subgroupoid of G if and only if $\bar{\delta}_{\mathcal{A}} o_{\mathcal{G}} \bar{\delta}_{\mathcal{A}} \geq \bar{\delta}_{\mathcal{A}}$, $\bar{I}_{\mathcal{A}} o_{\mathcal{G}} \bar{I}_{\mathcal{A}} \geq \bar{I}_{\mathcal{A}}$ and $\bar{\Delta}_{\mathcal{A}} o_{\mathcal{G}} \bar{\Delta}_{\mathcal{A}} \leq \bar{I}_{\mathcal{A}}$.

(ii) $\mathcal{A}$ is N-Fuzzy neutrosophic soft left ideal of G if and only if $\bar{\delta}_{\mathcal{T}} o_{\mathcal{G}} \bar{\delta}_{\mathcal{A}} \geq \bar{\delta}_{\mathcal{A}}$, $\bar{I}_{\mathcal{T}} o_{\mathcal{G}} \bar{I}_{\mathcal{A}} \geq \bar{I}_{\mathcal{A}}$ and $\bar{\Delta}_{\mathcal{T}} o_{\mathcal{G}} \bar{\Delta}_{\mathcal{A}} \leq \bar{I}_{\mathcal{A}}$.

(iii) A $\mathcal{A}$ is N-Fuzzy neutrosophic soft right ideal of G if and only if $\bar{\delta}_{\mathcal{A}} o_{\mathcal{G}} \bar{\delta}_{\mathcal{T}} \geq \bar{\delta}_{\mathcal{A}}$, $\bar{I}_{\mathcal{A}} o_{\mathcal{G}} \bar{I}_{\mathcal{T}} \geq \bar{I}_{\mathcal{A}}$ and $\bar{\Delta}_{\mathcal{A}} o_{\mathcal{G}} \bar{\Delta}_{\mathcal{T}} \leq \bar{I}_{\mathcal{A}}$.

Proof:

(i) Let A $\mathcal{A}$ is N-Fuzzy neutrosophic soft subgroupoid of G and $x \in$ G.

Case (i): If $x \neq ab$, for $a,b \in$ G, then

$$\left(\bar{\delta}_{\mathcal{A} \circ_{\mathcal{G}} \mathcal{A}}\right)(x) = -10 \leq \left(\bar{\delta}_{\mathcal{A}}\right)(x)$$

$$\left(\bar{I}_{\mathcal{A} \circ_{\mathcal{G}} \mathcal{A}}\right)(x) = -1 \leq \left(\bar{I}_{\mathcal{A}}\right)(x)$$

and

$$\bar{\Delta}_{\mathcal{A} \circ_{\mathcal{G}} \mathcal{A}}\big)(x) = 0 \geq \left(\bar{\Delta}_{\mathcal{A}}\right)(x)$$

Case (ii): If $x = ab$, for $a,b \in$ G, then

$$\begin{aligned}
\left(\bar{\delta}_{\mathcal{A} \circ_{\mathcal{G}} \mathcal{A}}\right)(x) &= \max_{x=ab}\left\{min\left(\left(\bar{\delta}_{\mathcal{A}}\right)(a), \left(\bar{\delta}_{\mathcal{A}}\right)(b)\right)\right\} \\
&\leq \max_{x=ab}\left(\bar{\delta}_{\mathcal{A}}\right)(ab) \\
&\leq \left(\bar{\delta}_{\mathcal{A}}\right)(x), \forall x \in G. \\
\left(\bar{I}_{\mathcal{A} \circ_{\mathcal{G}} \mathcal{A}}\right)(x) &= \max_{x=ab}\left\{min\left(\left(\bar{I}_{\mathcal{A}}\right)(a), \left(\bar{I}_{\mathcal{A}}\right)(b)\right)\right\} \\
&\leq \max_{x=ab}\left(\bar{I}_{\mathcal{A}}\right)(ab) \\
&\leq \left(\bar{I}_{\mathcal{A}}\right)(x), \forall x \in G. \\
\bar{\Delta}_{\mathcal{A} \circ_{\mathcal{G}} \mathcal{A}}\big)(x) &= \min_{x=ab}\left\{max\left(\left(\bar{\Delta}_{\mathcal{A}}\right)(a), \left(\bar{\Delta}_{\mathcal{A}}\right)(b)\right)\right\} \\
&\geq \min_{x=ab}\left(\bar{\Delta}_{\mathcal{A}}\right)(ab) \\
&\geq \left(\bar{\Delta}_{\mathcal{A}}\right)(x), \forall x \in G.
\end{aligned}$$

This implies that $\bar{\delta}_{\mathcal{A}} o_{\mathcal{G}} \bar{\delta}_{\mathcal{A}} \geq \bar{\delta}_{\mathcal{A}}$, $\bar{I}_{\mathcal{A}} o_{\mathcal{G}} \bar{I}_{\mathcal{A}} \geq \bar{I}_{\mathcal{A}}$ and $\bar{\Delta}_{\mathcal{A}} o_{\mathcal{G}} \bar{\Delta}_{\mathcal{A}} \leq \bar{I}_{\mathcal{A}}$.

Conversely, we suppose that $\bar{\delta}_{\mathcal{A}} o_{\mathcal{G}} \bar{\delta}_{\mathcal{A}} \geq \bar{\delta}_{\mathcal{A}}$, $\bar{I}_{\mathcal{A}} o_{\mathcal{G}} \bar{I}_{\mathcal{A}} \geq \bar{I}_{\mathcal{A}}$ and $\bar{\Delta}_{\mathcal{A}} o_{\mathcal{G}} \bar{\Delta}_{\mathcal{A}} \leq \bar{I}_{\mathcal{A}}$.

Let $a, b \in$ G and $x = ab$. Then

$$\begin{aligned}
\left(\bar{\delta}_{\mathcal{A}}\right)(ab) = \left(\bar{\delta}_{\mathcal{A}}\right)(x) &\geq \left(\bar{\delta}_{\mathcal{A} \circ_{\mathcal{G}} \mathcal{A}}\right)(x) \\
&= \max_{x=uv}\left\{min\left(\left(\bar{\delta}_{\mathcal{A}}\right)(u), \left(\bar{\delta}_{\mathcal{A}}\right)(v)\right)\right\} \\
&\geq min\left\{\left(\bar{\delta}_{\mathcal{A}}\right)(a), \left(\bar{\delta}_{\mathcal{A}}\right)(b)\right\}.
\end{aligned}$$

Similarly we can prove (ii) of definition 1.

$$\begin{aligned}
\bar{\Delta}_{\mathcal{A}}\big)(ab) = \left(\bar{\Delta}_{\mathcal{A}}\right)(x) &\leq \left(\bar{\Delta}_{\mathcal{A} \circ_{\mathcal{G}} \mathcal{A}}\right)(x) \\
&= \min_{x=uv}\left\{max\left(\left(\bar{\Delta}_{\mathcal{A}}\right)(u), \left(\bar{\Delta}_{\mathcal{A}}\right)(v)\right)\right\} \\
&\leq max\left\{\left(\bar{\Delta}_{\mathcal{A}}\right)(a), \left(\bar{\Delta}_{\mathcal{A}}\right)(b)\right\}.
\end{aligned}$$

Hence A is N-Fuzzy neutrosophic soft subgroupoid of G. Similarly, we can prove (ii) and (iii).

Proposition 3.4:

Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be NFNSS in G. Then A is N-Fuzzy neutrosophic soft subgroupoid of G if and only if $M_{\mathcal{A}}(p, q, r)$ is a soft subgroupoid of G, for all $p, q, r \in [-1,0]$ with $p + q + r \geq -1$.

Proof:

Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be NFNSS in G.

Let $x, y \in M_{\mathcal{A}}(p, q, r)$, where $p, q, r \in [-1,0]$ with $p + q + r \geq -1$. Then

$\bar{\delta}_{\mathcal{A}}(x) \geq p, \bar{I}_{\mathcal{A}}(x) \geq q$ and $\bar{\Delta}_{\mathcal{A}}(x) \leq r$ and

$\bar{\delta}_{\mathcal{A}}(y) \geq p, \bar{I}_{\mathcal{A}}(y) \geq q$ and $\overline{\Delta}_{\mathcal{A}}(y) \leq r$. Now

$$\bar{\delta}_{\mathcal{A}}(xy) \leq max\left\{\bar{\delta}_{\mathcal{A}}(x), \bar{\delta}_{\mathcal{A}}(y)\right\} \leq p,$$

$$\bar{I}_{\mathcal{A}}(xy) \leq max\left\{\bar{I}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(y)\right\} \leq q$$

and

$$\overline{\Delta}_{\mathcal{A}}(xy) \geq min\left\{\overline{\Delta}_{\mathcal{A}}(x), \overline{\Delta}_{\mathcal{A}}(y)\right\} \geq r.$$

This implies that $xy \in M_{\mathcal{A}}(p, q, r)$ for all $x, y \in M_{\mathcal{A}}(p, q, r)$.Implies that $M_{\mathcal{A}}(p, q, r)$. $M_{\mathcal{A}}(p, q, r) \subseteq M_{\mathcal{A}}(p, q, r)$.Hence $M_{\mathcal{A}}(p, q, r)$is a soft subgroupoid of G.

Conversely, we suppose that $M_{\mathcal{A}}(p, q, r)$ is a soft subgroupoid of G for all $p, q, r \in [-1,0]$ with $p + q + r \geq -1$. Let $x, y \in$ G such that $\bar{\delta}_{\mathcal{A}}(x) = p_x$, $\bar{I}_{\mathcal{A}}(x) = q_x$ and $\overline{\Delta}_{\mathcal{A}}(x) = r_x$ and $\bar{\delta}_{\mathcal{A}}(y) = p_y$, $\bar{I}_{\mathcal{A}}(y) = q_y$ and $\overline{\Delta}_{\mathcal{A}}(y) = r_y$ with $-1 \leq p_x + q_x + r_x \leq 0$ and may assume that $-1 \leq p_y + q_y + r_y \leq 0$ then $x \in M_{\mathcal{A}}(p_x, q_x, r_x)$ and $y \in M_{\mathcal{A}}(p_y, q_y, r_y)$. We may assume that $p_x \leq p_y$, $q_x \leq q_y$ and $r_x \geq r_y$ then $M_{\mathcal{A}}(p_x, q_x, r_x) \subseteq M_{\mathcal{A}}(p_y, q_y, r_y)$,

Which implies that $x,y \in M_{\mathcal{A}}(p_y, q_y, r_y)$.Since $M_{\mathcal{A}}(p_y, q_y, r_y)$is a neutrosophic soft subgroupoid of G implies $xy \in M_{\mathcal{A}}(p_y, q_y, r_y)$. Then

$$\bar{\delta}_{\mathcal{A}}(xy) \leq p_y = max\{p_x, p_y\} = max\left\{\bar{\delta}_{\mathcal{A}}(x), \bar{\delta}_{\mathcal{A}}(y)\right\}$$

$$\bar{I}_{\mathcal{A}}(xy) \leq q_y = max\{q_x, q_y\} = max\left\{\bar{I}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(y)\right\},$$

and

$$\overline{\Delta}_{\mathcal{A}}(xy) \geq r_y = min\{r_x, r_y\} = min\left\{\overline{\Delta}_{\mathcal{A}}(x), \overline{\Delta}_{\mathcal{A}}(y)\right\}. \text{for all } x,y \in G.$$

Hence $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}})$ is N-Fuzzy neutrosophic soft subgroupoid of G.

Proposition 3.5:

Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}})$ be NFNSS in G. then A is N−fuzzy neutrosophic soft left(right) ideal of G if and only if $M_{\mathcal{A}}(p, q, r)$ is soft left(right) ideal of G for all $p, q, r \in [-1,0]$ with $p + q + r \leq -1$.

Proof:

Straight forward.

Proposition 3.6:

A non-empty subset C of G is a soft subgroupoid of G if and only if $\overline{\Psi}_C = (\bar{\delta}_{\Psi_C}, \bar{I}_{\Psi_C}, \overline{\Delta}_{\Psi_C})$ is N-fuzzy neutrosopic soft subgroupoid of G.

Proof:

Let C be a neutrosopic soft subgroupoid of G. Then $CGC \subseteq C$.

Let $x, y \in$ G then we have the following cases.

Case (i): If $x,y \in C$ then $xy \in C$ and hence

$\bar{\delta}_{\Psi_C}(x) = \bar{\delta}_{\Psi_C}(y) = \bar{\delta}_{\Psi_C}(xy) = -1$ implies that $\bar{\delta}_{\Psi_C}(xy) = \max\left\{\bar{\delta}_{\Psi_C}(x), \bar{\delta}_{\Psi_C}(y)\right\}$. Also that $\bar{I}_{\Psi_C}(x) = \bar{I}_{\Psi_C}(y) = \bar{I}_{\Psi_C}(xy) = -1$ implies that $\bar{I}_{\Psi_C}(xy) = \max\left\{\bar{I}_{\Psi_C}(x), \bar{I}_{\Psi_C}(y)\right\}$. Also that $\overline{\Delta}_{\Psi_C}(x) = \overline{\Delta}_{\Psi_C}(y) = \overline{\Delta}_{\Psi_C}(xy) = -1$ implies that $\overline{\Delta}_{\Psi_C}(xy) = \min\left\{\overline{\Delta}_{\Psi_C}(x), \overline{\Delta}_{\Psi_C}(y)\right\}$.

Case (ii) : If either $x \notin C$ or $y \notin C$ then either

$\bar{\delta}_{\Psi_C}(x) = 0$, $\bar{I}_{\Psi_C}(x) = 0$ and $\overline{\Delta}_{\Psi_C}(x) = -1$ or $\bar{\delta}_{\Psi_C}(y) = 0$, $\bar{I}_{\Psi_C}(y) = 0$ and $\overline{\Delta}_{\Psi_C}(y) = -1$

This implies that, max $\{\bar{\delta}_{\Psi_C}(x), \bar{\delta}_{\Psi_C}(x)\} = 0$ but $\bar{\delta}_{\Psi_C}(xy) \leq 0$ implies that $\bar{\delta}_{\Psi_C}(xy) = \max\left\{\bar{\delta}_{\Psi_C}(x), \bar{\delta}_{\Psi_C}(y)\right\}$.

Similarly we can prove (ii) of definition. Also either $\overline{\Delta}_{\Psi_C}(x) = -1$ or $\overline{\Delta}_{\Psi_C}(y) = -1$ which implies that $\min\left\{\overline{\Delta}_{\Psi_C}(x), \overline{\Delta}_{\Psi_C}(y)\right\} = -1$,but $\overline{\Delta}_{\Psi_C}(xy) \geq -1$ implies that $\overline{\Delta}_{\Psi_C}(xy) = \min\left\{\overline{\Delta}_{\Psi_C}(x), \overline{\Delta}_{\Psi_C}(y)\right\}$ for all $x,y \in G$.

Case (iii): When $x \notin C$ and $y \notin C$ gives the same result as in case (ii). Hence $\overline{\Psi}_C = (\bar{\delta}_{\Psi_C}, \bar{I}_{\Psi_C}, \overline{\Delta}_{\Psi_C})$ is N-fuzzy neutrosopic soft subgroupoid of G

Conversely, we suppose that $\overline{\Psi}_C = (\bar{\delta}_{\Psi_C}, \bar{I}_{\Psi_C}, \overline{\Delta}_{\Psi_C})$ is N-fuzzy neutrosopic soft subgroupoid of G. Let $x,y \in C$ then $xy \in CGC$. By definition of $\overline{\Psi}_C$, $\bar{\delta}_{\Psi_C}(x) = \bar{\delta}_{\Psi_C}(y) = -1$, implies that max $\left\{\bar{\delta}_{\Psi_C}(x), \bar{\delta}_{\Psi_C}(y)\right\} = -1$. Similarly we can prove (ii) of definition. Also we can show that $\overline{\Delta}_{\Psi_C}(xy) = 0$.This gives that $xy \in C$ implies that $CGC \subseteq C$. Hence C is a neutrosopic soft subgroupoid of G.

Proposition 3.7:

A non-empty subset C of G is a neutrosopic soft left(right) ideal of G if and only if $\overline{\Psi}_C = (\bar{\delta}_{\Psi_C}, \bar{I}_{\Psi_C}, \overline{\Delta}_{\Psi_C})$ is N - fuzzy neutrosopic soft left(right) ideal of C.

Proof:

Straight forward.

Definition 3.8: A N−fuzzy neutrosopic soft subgroupoid $\mathcal{A} = (\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}})$ of G is called N−fuzzy neutrosopic soft bi-ideal of G if

$$(NFNSBI_1)\ \overline{\delta}_{\mathcal{A}}(xwy) \leq max\{\overline{\delta}_{\mathcal{A}}(x), \overline{\delta}_{\mathcal{A}}(y)\}$$

$$(NFNSBI_2)\ \overline{I}_{\mathcal{A}}(xwy) \leq max\{\overline{I}_{\mathcal{A}}(x), \overline{I}_{\mathcal{A}}(y)\}$$

$$(NFNSBI_3)\ \overline{\Delta}_{\mathcal{A}}(xwy) \geq min\{\overline{\Delta}_{\mathcal{A}}(x), \overline{\Delta}_{\mathcal{A}}(y)\}$$

Example 3.9:

Let G = {*a*, *b*, *c*, *d*, *e*} be a subgroupoid with the following cayley table.

·	*a*	*b*	*c*	*d*	*e*
a	*a*	*a*	*a*	*a*	*a*
b	*a*	*a*	*a*	*a*	*a*
c	*a*	*a*	*c*	*c*	*e*
d	*a*	*a*	*c*	*d*	*e*
e	*a*	*a*	*c*	*c*	*c*

Define NFNSS $\mathcal{A} = (\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}})$ in G by
$\overline{\delta}_{\mathcal{A}}(a)$=-0.6, $\overline{\delta}_{\mathcal{A}}(b)$= -0.5, $\overline{\delta}_{\mathcal{A}}(c)$= -0.4, $\overline{\delta}_{\mathcal{A}}(d)$= $\overline{\delta}_{\mathcal{A}}(e)$= -0.3
$\overline{I}_{\mathcal{A}}(a)$=-0.4, $\overline{I}_{\mathcal{A}}(b)$= -0.3, $\overline{I}_{\mathcal{A}}(c)$= $\overline{I}_{\mathcal{A}}(d)$=-0.2, $\overline{I}_{\mathcal{A}}(e)$= -0.1
$\overline{\Delta}_{\mathcal{A}}(a)$= $\overline{\Delta}_{\mathcal{A}}(b)$= -0.3, $\overline{\Delta}_{\mathcal{A}}(c)$= -0.2,$\overline{\Delta}_{\mathcal{A}}(d)$=-0.1, $\overline{\Delta}_{\mathcal{A}}(e)$= -0.1
By routine calculation, we can check thatfuzzy neutrosopic soft bi-ideal of G.

Proposition 3.10:

Every N−fuzzy neutrosopic soft left(right) ideal of G is N−fuzzy neutrosopic soft bi-ideal of G.

Proof:

Let $\mathcal{A} = (\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}})$ be N-fuzzy neutrosopic soft left ideal of G and *x*, *y*, *w* ∈ G. Then

$$\overline{\delta}_{\mathcal{A}}(xwy) = \overline{\delta}_{\mathcal{A}}((xw)y) \leq \overline{\delta}_{\mathcal{A}}(y) \leq \max\{\overline{\delta}_{\mathcal{A}}(x), \overline{\delta}_{\mathcal{A}}(y)\}$$
$$\overline{I}_{\mathcal{A}}(xwy) = \overline{I}_{\mathcal{A}}((xw)y) \leq \overline{I}_{\mathcal{A}}(y) \leq \max\{\overline{I}_{\mathcal{A}}(x), \overline{I}_{\mathcal{A}}(y)\}$$
$$\overline{\Delta}_{\mathcal{A}}(xwy) = \overline{\Delta}_{\mathcal{A}}((xw)y) \geq \overline{\Delta}_{\mathcal{A}}(y) \geq \min\{\overline{\Delta}_{\mathcal{A}}(x), \overline{\Delta}_{\mathcal{A}}(y)\}$$

Thus $\mathcal{A} = (\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}})$ is N-fuzzy neutrosopic soft bi-ideal of G. The right case is proved in an analogous way.

Conclusion:

We have studied various soft structures over subgroupoids and ideals in this article. Furthur researchers may continue this studies in the field of modules and near-ring theory.

References:

1. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems,(1986), 20pp 87-96.
2. H. Aktas and N. Cagman, Soft sets and Soft groups, Inf. sci 177(13)(2007),2726-2735.
3. S. Broumi and F. Smarandache, Intuitionstic Neutrosophic Soft Set, Journal of Information and Computing Science, England, UK, ISSN:1745-7659, Vol.8,N0.2(2013),pp 130-140.
4. P. K. Maji, Neutrosophic soft set, Annals of fuzzy mathematics and Informatics", Vol.5, N0.1, ISSN: 2093-9310, ISSN: 2287-623.
5. D. A. Molodtsov, Soft set theory, First results, Comput. Math. Appli. 37(1999), 19-31.
6. A. Rosenfield, Fuzzy groups, J. Math. Anal. Appl.3 5(1971), 512-517.
7. F. Smarandache, A unifying Field in logics, Neutrosophy; Neutrosophic probability set and logic, Rehoboth; American Research press. (1999).
8. F. Smarandache, Neutrosophic set, a generalization of the intuitionistic fuzzy sets, Inter. J. Pure. Appl. Math, 24(2005), 287-297.
9. L. A. Zadeh, Fuzzy sets Information and control (1965), 8 pp 338-353.