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N-FUZZY SOFT SUBGROUPS OVER IDEAL STRUCTURES

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Abstract:

In this paper, we introduced the concept of N-fuzzy soft N−fuzzy neutrosopic soft right ideal, N−fuzzy neutrosopic soft (1, 2)−ideal, N−fuzzy neutrosopic soft bi-ideal of a group, N−fuzzy neutrosopic soft subgroupoid and regular groupoid and its properties. An illustrative examples are also given.

Index Terms: Fuzzy Set, Soft Set, Neutrosophic Soft Set, N-Fuzzy Set, Bi-Ideals, Regular Groupoids & Soft Subgroupoids.

1. Introduction:

The concept of Neutrosophic set was introduced by F.Smarandache which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The theory of Neutrosophic set which is the generalization of the classical sets, conventional fuzzy set[zadah], intuitionistic fuzzy set[Atanassov] and interval valued fuzzy set[Rosenfeld] was introduced by F.Smarandache. This concept has recently motivated new approach in several directions such as databases, medical diagonses problems, decision making problem, topology, control theory and so on. The concept of neutrosophic set handles indeterminate data where as fuzzy theory and intuitionistic fuzzy set theory failed when the relations are indeterminate.

2. Preliminaries:

Definition 2.1: Let U be a non-empty set. Then by a fuzzy set on U is meant a function A : U →[0, 1]. A is called the membership function, A(x) is called the membership grade of x in A. We also write A = {(x, A(x)) : x ∈ U }.

Example 2.2: Consider U = {a, b, c, d} and A : U → [0, 1] defined by A(a) = 0, A(b) = 0.7, A(c) = 0.4, A(d) = 1.

Definition 2.3: Let U be the initial universe set and E be the set of parameters. Let P (U) denote the power set of U . Consider a non empty set A, A ⊂ E. A pair (F, A) is called a soft set over U, where F : A →P (U).

Example 2.4: Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . . , h_5} . Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . . , e_8} , where e_1, e_2, . . . , e_8 stand for the attributes "expensive", "beautiful", "wooden", "cheap", "mod- ern" and "in bad", "repair" respectively.

Definition 2.5: A negative fuzzy (N-Fuzzy)neutrosophic soft set $\boldsymbol{\mathcal{A}}$ on the universe of discourse X is defined as $\boldsymbol{\mathcal{A}} = \langle x, \bar{\delta}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x)\rangle,\ x \in \mathcal{X}$, where $\delta, I, \Delta : \boldsymbol{\mathcal{X}} \to [-1, 0]$ and $-1 \le \bar{\delta}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x) \le 0$.

Definition 2.6: A N-Fuzzy neutrosophic soft set $\boldsymbol{\mathcal{A}}$ over the universe $\boldsymbol{\mathcal{X}}$ is said to be null or empty N-Fuzzy neutrosophic soft set if $\delta_{\mathcal{A}}(x) = -1, I_{\mathcal{A}}(x) = -1, \Delta_{\mathcal{A}}(x) = 0$ for all $\in \mathcal{X}$. It is denoted by -1_N.

Definition 2.7: A N-Fuzzy neutrosophic soft set $\boldsymbol{\mathcal{A}}$ over the universe $\boldsymbol{\mathcal{X}}$ is said to be absolute (universe) N-Fuzzy neutrosophic soft set if

$\delta_{\mathcal{A}}(x) = 0, I_{\mathcal{A}}(x) = 0, \Delta_{\mathcal{A}}(x) = -1$ for all $x \in \mathcal{X}$. It is denoted by 0_N.

Definition 2.8: The complement of N-Fuzzy neutrosophic soft set $\boldsymbol{\mathcal{A}}$ is denoted by $\mathcal{A}^C$ and is defined as $\mathcal{A}^C = \langle x, \delta_{\mathcal{A}^C}(x), I_{\mathcal{A}^C}(x), \Delta_{\mathcal{A}^C}(x)\rangle$ where

$\delta_{\mathcal{A}^C}(x) = \Delta_{\mathcal{A}}(x), I_{\mathcal{A}^C}(x) = 1 - I_{\mathcal{A}}(x), \Delta_{\mathcal{A}^C}(x) = \delta_{\mathcal{A}}(x)$

The complement of a N-Fuzzy neutrosophic soft set A can also be defined as $0_N - \boldsymbol{\mathcal{A}}$.

Definition 2.9: Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ and Let $\mathcal{B} = (\bar{\delta}_{\mathcal{B}}, \bar{I}_{\mathcal{B}}, \bar{\Delta}_{\mathcal{B}})$ be two NFNSS in $\mathcal{X}$.

$$(\mathcal{A}\bar{\cup}\mathcal{B}) = \left\{x, min\left\{\bar{\delta}_{\mathcal{A}}(x), \bar{\delta}_{\mathcal{B}}(x)\right\}, min\left\{\bar{I}_{\mathcal{A}}(x), \bar{I}_{\mathcal{B}}(x)\right\}, max\left\{\bar{\Delta}_{\mathcal{A}}(x), \bar{\Delta}_{\mathcal{B}}(x)\right\}\right\}$$

Their union and intersection is also a N-Fuzzy neutrosophic soft set in $\mathcal{X}$ defined as for all $x \in \mathcal{X}$.

and

$$(\mathcal{A}\bar{\cap}\mathcal{B}) = \left\{x, max\left\{\bar{\delta}_{\mathcal{A}}(x), \bar{\delta}_{\mathcal{B}}(x)\right\}, max\left\{\bar{I}_{\mathcal{A}}(x), \bar{I}_{\mathcal{B}}(x)\right\}, min\left\{\bar{\Delta}_{\mathcal{A}}(x), \bar{\Delta}_{\mathcal{B}}(x)\right\}\right\}$$

Example 2.10:

Let $\mathcal{X}$ is as in example 1 and

$\mathcal{A} = \{\langle w, -0.9, -0.8, -0.7\rangle, \langle x, -0.5, -0.4, -0.5\rangle, \langle y, -0.7, -0.6, -0.6\rangle, \langle z, -0.6, -0.3, -0.2\rangle\}$

$\boldsymbol{\mathcal{B}} = \{\langle w, -0.8, -0.4, -0.6\rangle, \langle x, -0.9, -0.6, -0.7\rangle, \langle y, -0.7, -0.4, -0.5\rangle, \langle z, -0.8, -0.4, -0.7\rangle\}$

then $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ and $\mathcal{B} = (\bar{\delta}_{\mathcal{B}}, \bar{I}_{\mathcal{B}}, \bar{\Delta}_{\mathcal{B}})$ are N-Fuzzy neutrosophic soft set in X.
Easily we can verify that $\sqsubseteq \mathcal{B}$.

Example 2.11:

Let $\mathcal{X}$ is as in example 2.10 and
$\mathcal{A} = \{\langle w, -0.7, -0.4, -0.5\rangle, \langle x, -0.8, -0.5, -0.7\rangle, \langle y, -0.9, -0.5, -0.6\rangle, \langle z, -0.4, -0.2, -0.3\rangle\}$
$\boldsymbol{\mathcal{B}} = \{\langle w, -0.9, -0.4, -0.5\rangle, \langle x, -0.7, -0.5, -0.4\rangle, \langle y, -0.6, -0.4, -0.5\rangle, \langle z, -0.8, -0.5, -0.6\rangle\}$
then $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ and $\mathcal{B} = (\bar{\delta}_{\mathcal{B}}, \bar{I}_{\mathcal{B}}, \bar{\Delta}_{\mathcal{B}})$ are N-Fuzzy neutrosophic soft set in $\mathcal{X}$.
$\mathcal{A} \bar{\cup} \mathcal{B} = \{\langle w, -0.9, -0.4, -0.5\rangle, \langle x, -0.8, -0.5, -0.4\rangle, \langle y, -0.9, -0.5, -0.5\rangle, \langle z, -0.8, -0.5, -0.3\rangle\}$
$\mathcal{A} \bar{\cap} \mathcal{B} = \{\langle w, -0.7, -0.4, -0.5\rangle, \langle x, -0.7, -0.5, -0.7\rangle, \langle y, -0.6, -0.4, -0.6\rangle, \langle z, -0.4, -0.2, -0.6\rangle\}$
Obviously $\mathcal{A} \bar{\cup} \mathcal{B}$ and $\mathcal{A} \bar{\cap} \mathcal{B}$ are N-Fuzzy neutrosophic soft set in $\mathcal{X}$.

3. Some Standard Results in N-Fuzzy Soft Ideals:

Theorem 3.1:

Let G be a regular groupoid. If every bi-ideal of G is a right(left) ideal of G. then every N−fuzzy neutrosopic soft bi-ideal of G is N−fuzzy neutrosopic soft right(left) ideal of G.

Proof:

Assume that every bi-ideal of G is a right ideal of G. Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be any N−fuzzy neutrosopic soft bi-ideal of G and let *x*, *y* ∈ G then *xGx* is a bi-ideal of G and so *xSx* is a right ideal of G. Since G is regular, we have *xy* ∈ *(xGx)G* ⊆ *xGx*, which implies that *xy* = *xwx* for some *w* ∈ G. Since $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ is N-fuzzy neutrosopic soft bi-ideal of G, it follows that

$$\bar{\delta}_{\mathcal{A}}(xy) = \bar{\delta}_{\mathcal{A}}(xwx) \leq \max\{\bar{\delta}_{\mathcal{A}}(x), \bar{\delta}_{\mathcal{A}}(x)\} = \bar{\delta}_{\mathcal{A}}(x)$$
$$\bar{I}_{\mathcal{A}}(xy) = \bar{I}_{\mathcal{A}}(xwx) \leq \max\{\bar{I}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x)\} = \bar{I}_{\mathcal{A}}(x)$$
$$\bar{\Delta}_{\mathcal{A}}(xy) = \bar{\Delta}_{\mathcal{A}}(xwx) \geq \min\{\bar{\Delta}_{\mathcal{A}}(x), \bar{\Delta}_{\mathcal{A}}(y)\} = \bar{\Delta}_{\mathcal{A}}(x).$$

Hence $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ is N−fuzzy neutrosopic soft right ideal of G.

Definition 3.2:

A N−fuzzy neutrosopic soft subgroupoid $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ of G is called N−fuzzy neutrosopic soft (1,2)−ideal of G if

$$\bar{\delta}_{\mathcal{A}}(xw(yz)) \leq max\{\bar{\delta}_{\mathcal{A}}(x), \bar{\delta}_{\mathcal{A}}(y), \bar{\delta}_{\mathcal{A}}(z)\}$$
$$\bar{I}_{\mathcal{A}}(xw(yz)) \leq max\{\bar{I}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(y), \bar{I}_{\mathcal{A}}(z)\}$$
$$\bar{\Delta}_{\mathcal{A}}(xw(yz)) \geq min\{\bar{\Delta}_{\mathcal{A}}(x), \bar{\Delta}_{\mathcal{A}}(y), \bar{\Delta}_{\mathcal{A}}(z)\}$$, for all *x*, *y*, *z*, *w* ∈ G.

Example 3.3:

Let G = {*a*, *b*, *c*, *d*, *e*} be a groupoid with the following cayley table

·	a	b	c	d	e
a	a	a	a	a	a
b	a	a	a	b	c
c	a	b	c	a	a
d	a	a	a	d	e
e	a	d	e	a	a

Define NFNSS $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ in G by
$\bar{\delta}_{\mathcal{A}}(a)= \bar{\delta}_{\mathcal{A}}(b)= \bar{\delta}_{\mathcal{A}}(c)= -1, \bar{\delta}_{\mathcal{A}}(d)= \bar{\delta}_{\mathcal{A}}(e)= 0$
$\bar{I}_{\mathcal{A}}(a)= \bar{I}_{\mathcal{A}}(b)= \bar{I}_{\mathcal{A}}(c)= -1, \bar{I}_{\mathcal{A}}(d)= \bar{I}_{\mathcal{A}}(e)= 0$
$\bar{\Delta}_{\mathcal{A}}(a)= \bar{\Delta}_{\mathcal{A}}(b)= \bar{\Delta}_{\mathcal{A}}(c)= 0, \bar{\Delta}_{\mathcal{A}}(d)= \bar{\Delta}_{\mathcal{A}}(e)= -1$
By routine calculation, we can check that $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ is N−fuzzy neutrosopic soft (1,2)−ideal of G. Hence $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ can be redefined as follows,

$$\bar{\delta}_{\mathcal{A}}(x) = \begin{cases} 0 & \text{if } x \in Q \\ 1 & \text{otherwise} \end{cases}$$
$$\bar{I}_{\mathcal{A}}(x) = \begin{cases} 0 & \text{if } x \in Q \\ 1 & \text{otherwise} \end{cases}$$
$$\bar{\Delta}_{\mathcal{A}}(x) = \begin{cases} 0 & \text{if } x \in Q \\ 1 & \text{otherwise} \end{cases}$$

Theorem 3.4:

Every N−fuzzy neutrosopic soft bi-ideal is N−fuzzy neutrosopic soft (1, 2)−ideal.

Proof:

Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be N-fuzzy neutrosopic soft bi-ideal of G and let
x, *y*, *z*, *w* ∈ G. Then

$$\begin{aligned}\overline{\delta}_{\mathcal{A}}(xw(yz)) &= \overline{\delta}_{\mathcal{A}}((xwy)z)\\ &\leq max\{\overline{\delta}_{\mathcal{A}}(xwy),\overline{\delta}_{\mathcal{A}}(z)\}\\ &\leq max\left\{max\{\overline{\delta}_{\mathcal{A}}(x),\overline{\delta}_{\mathcal{A}}(y)\},\overline{\delta}_{\mathcal{A}}(z)\right\}\\ &= max\{\overline{\delta}_{\mathcal{A}}(x),\overline{\delta}_{\mathcal{A}}(y),\overline{\delta}_{\mathcal{A}}(z)\}\end{aligned}$$

Similarly we can prove (ii) of definition

$$\begin{aligned}\overline{\Delta}_{\mathcal{A}}(xw(yz)) &= \overline{\Delta}_{\mathcal{A}}((xwy)z)\\ &\geq min\{\overline{\Delta}_{\mathcal{A}}(xwy),\overline{\Delta}_{\mathcal{A}}(z)\}\\ &\geq min\left\{min\{\overline{\Delta}_{\mathcal{A}}(x),\overline{\Delta}_{\mathcal{A}}(y)\},\overline{\Delta}_{\mathcal{A}}(z)\right\}\\ &= min\{\overline{\Delta}_{\mathcal{A}}(x),\overline{\Delta}_{\mathcal{A}}(y),\overline{\Delta}_{\mathcal{A}}(z)\}\end{aligned}$$

Hence $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be N-fuzzy neutrosopic soft (1,2)−ideal of G.

Corollary 3.5:

Every N−fuzzy neutrosopic soft left (right) ideal of G is N−fuzzy neutrosopic soft (1, 2)−ideal of G.

Proof:

To consider then converse of the above theorem, we need to strengthen the condition of a groupoid G.

Theorem 3.6:

If G is a regular groupoid, then every N−fuzzy neutrosopic soft (1, 2)−ideal of G is N−fuzzy neutrosopic soft bi-ideal of G.

Proof:

Assume that a groupoid G is regular and let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be N−fuzzy neutrosopic soft (1,2)−ideal of G. Let $x, y, z, w \in$ G. Since G is regular, we have $xw \in (xGx)G \subseteq xGx$, which implies that $xw = xgx$ for some $g \in$ G. thus

$$\begin{aligned}\overline{\delta}_{\mathcal{A}}(xwy) &= \overline{\delta}_{\mathcal{A}}((xgx)y) = \overline{\delta}_{\mathcal{A}}(xg(xy))\\ &\leq max\{\overline{\delta}_{\mathcal{A}}(x),\overline{\delta}_{\mathcal{A}}(x),\overline{\delta}_{\mathcal{A}}(y)\}\\ &= max\{\overline{\delta}_{\mathcal{A}}(x),\overline{\delta}_{\mathcal{A}}(y)\}\\ \overline{I}_{\mathcal{A}}(xwy) &= \overline{I}_{\mathcal{A}}((xgx)y) = \overline{I}_{\mathcal{A}}(xg(xy))\\ &\leq max\{\overline{I}_{\mathcal{A}}(x),\overline{I}_{\mathcal{A}}(x),\overline{I}_{\mathcal{A}}(y)\}\\ &= max\{\overline{I}_{\mathcal{A}}(x),\overline{I}_{\mathcal{A}}(y)\}\\ \overline{\Delta}_{\mathcal{A}}(xwy) &= \overline{\Delta}_{\mathcal{A}}((xgx)y) = \overline{\Delta}_{\mathcal{A}}(xg(xy))\\ &\geq min\{\overline{\Delta}_{\mathcal{A}}(x),\overline{\Delta}_{\mathcal{A}}(x),\overline{\Delta}_{\mathcal{A}}(y)\}\\ &= min\{\overline{\Delta}_{\mathcal{A}}(x),\overline{\Delta}_{\mathcal{A}}(y)\}\end{aligned}$$

Therefore $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ is N-fuzzy neutrosopic soft bi-ideal of G.

Theorem 3.7: Every N−fuzzy neutrosopic soft bi-ideal of a group G is constant.

Proof. Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}}, \bar{I}_{\mathcal{A}}, \bar{\Delta}_{\mathcal{A}})$ be N-fuzzy neutrosopic soft bi-ideal of a group G and let x be any element of G. Then

$$\begin{aligned}\overline{\delta}_{\mathcal{A}}(x) &= \overline{\delta}_{\mathcal{A}}(exe) \leq max\{\overline{\delta}_{\mathcal{A}}(e),\overline{\delta}_{\mathcal{A}}(e)\}\\ &= \overline{\delta}_{\mathcal{A}}(e) = \overline{\delta}_{\mathcal{A}}(ee)\\ &= \overline{\delta}_{\mathcal{A}}((x^{-1}x)(x^{-1}x)) = \overline{\delta}_{\mathcal{A}}(x(x^{-1}x^{-1})x)\\ &\leq max\{\overline{\delta}_{\mathcal{A}}(x),\overline{\delta}_{\mathcal{A}}(x)\} = \overline{\delta}_{\mathcal{A}}(x)\end{aligned}$$

Similarly we can show (ii) of definition.

$$\begin{aligned}\overline{I}_{\mathcal{A}}(x) &= \overline{I}_{\mathcal{A}}(exe) \leq max\{\overline{I}_{\mathcal{A}}(e),\overline{I}_{\mathcal{A}}(e)\}\\ &= \overline{I}_{\mathcal{A}}(e) = \overline{I}_{\mathcal{A}}(ee)\\ &= \overline{I}_{\mathcal{A}}((x^{-1}x)(x^{-1}x)) = \overline{I}_{\mathcal{A}}(x(x^{-1}x^{-1})x)\\ &\leq max\{\overline{I}_{\mathcal{A}}(x),\overline{I}_{\mathcal{A}}(x)\} = \overline{I}_{\mathcal{A}}(x)\\ \overline{\Delta}_{\mathcal{A}}(x) &= \overline{\Delta}_{\mathcal{A}}(exe) \geq min\{\overline{\Delta}_{\mathcal{A}}(e),\overline{\Delta}_{\mathcal{A}}(e)\}\\ &= \overline{\Delta}_{\mathcal{A}}(e) = \overline{\Delta}_{\mathcal{A}}(ee)\\ &= \overline{\Delta}_{\mathcal{A}}((x^{-1}x)(x^{-1}x)) = \overline{\Delta}_{\mathcal{A}}(x(x^{-1}x^{-1})x)\\ &\geq min\{\overline{\Delta}_{\mathcal{A}}(x),\overline{\Delta}_{\mathcal{A}}(x)\} = \overline{\Delta}_{\mathcal{A}}(x)\end{aligned}$$

Where e is the identity of G it follows that $\bar{\delta}_{\mathcal{A}}(x)= \bar{\delta}_{\mathcal{A}}(e)$, $\bar{I}_{\mathcal{A}}(x)= \bar{I}_{\mathcal{A}}(e)$, $\bar{\Delta}_{\mathcal{A}}(x)= \bar{\Delta}_{\mathcal{A}}(e)$ which means that $\mathcal{A} = (\bar{\delta}_{\mathcal{A}},\bar{I}_{\mathcal{A}},\bar{\Delta}_{\mathcal{A}})$ is constant.

For NFNSS $\mathcal{A} = (\bar{\delta}_{\mathcal{A}},\bar{I}_{\mathcal{A}},\bar{\Delta}_{\mathcal{A}})$ of G by $\mathcal{A}(x)$ we mean that $(\bar{\delta}_{\mathcal{A}},\bar{I}_{\mathcal{A}},\bar{\Delta}_{\mathcal{A}})$ for all $x \in$G.

Denote by M[x] the principle bi-ideal of G generated by $x \in$ G (i.e) M[x] = $\{x\} \cup \{x^2\} \cup xGx$.

Theorem 3.8:

Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}},\bar{I}_{\mathcal{A}},\bar{\Delta}_{\mathcal{A}})$ *be* $N-$fuzzy neutrosopic soft bi-ideal of G. If G is (2,2)− regular, then A(x) = A(x^2) for all x ∈ G.

Proof:

Let $x \in$ G then there exists $a \in$ G such that $x = x^2ax^2$. Hence

$$\begin{aligned}
\bar{\delta}_{\mathcal{A}} &= \bar{\delta}_{\mathcal{A}}(x^2ax^2) \\
&\leq max\{\bar{\delta}_{\mathcal{A}}(x^2),\bar{\delta}_{\mathcal{A}}(x^2)\} \\
&= \bar{\delta}_{\mathcal{A}}(x^2) \leq max\{\bar{\delta}_{\mathcal{A}}(x),\bar{\delta}_{\mathcal{A}}(x)\} = \bar{\delta}_{\mathcal{A}}(x) \\
\bar{I}_{\mathcal{A}} &= \bar{I}_{\mathcal{A}}(x^2ax^2) \\
&\leq max\{\bar{I}_{\mathcal{A}}(x^2),\bar{I}_{\mathcal{A}}(x^2)\} \\
&= \bar{I}_{\mathcal{A}}(x^2) \leq max\{\bar{I}_{\mathcal{A}}(x),\bar{I}_{\mathcal{A}}(x)\} = \bar{I}_{\mathcal{A}}(x)
\end{aligned}$$

$$\begin{aligned}
\bar{\Delta}_{\mathcal{A}} &= \bar{\Delta}_{\mathcal{A}}(x^2ax^2) \\
&\geq \min\{\bar{\Delta}_{\mathcal{A}}(x^2),\bar{\Delta}_{\mathcal{A}}(x^2)\} \\
&= \bar{\Delta}_{\mathcal{A}}(x^2) \geq \min\{\bar{\Delta}_{\mathcal{A}}(x),\bar{\Delta}_{\mathcal{A}}(x)\} = \bar{\Delta}_{\mathcal{A}}(x)
\end{aligned}$$

It follows that $\bar{\delta}_{\mathcal{A}}(x) = \bar{\delta}_{\mathcal{A}}(x^2)$, $\bar{I}_{\mathcal{A}}(x) = \bar{I}_{\mathcal{A}}(x^2)$, $\bar{\Delta}_{\mathcal{A}}(x) = \bar{\Delta}_{\mathcal{A}}(x^2)$ so that A(x) = A(x^2) .

Proposition 3.9:

Let $\mathcal{A} = (\bar{\delta}_{\mathcal{A}},\bar{I}_{\mathcal{A}},\bar{\Delta}_{\mathcal{A}})$ be N-fuzzy neutrosopic soft bi-ideal of G. If G is a semi lattice of groups, then A(x) = A(x^2) and A(xy) = A(yx) for all $x,y \in$ G.

Proof:

Since G is a semi lattice of groups. S is a union of groups. Hence S is (2,2)−regular. It follows that from theorem-10 that A(x) = A(x^2) for all $x \in$ G. Let $x,y \in$ G noticing that G is a semi lattice of groups if and only if the set of all bi-ideals of G is a semi lattice under the multiplication subsets. We have

$(xy)^3= (xyx)(yxy) \in$ M[xyx]M[yxy]=M[yxy](M[xyx]^2) ⊆ $M[yxy]G\, M[xyx] \subseteq yxy\, G\, xyx \subseteq yx\, G\, yx$

It follows that there exists $w \in$ G such that $(xy)^3 = (yx)w(yx)$. Hence

$$\begin{aligned}
\bar{\delta}_{\mathcal{A}}(xy) &= \bar{\delta}_{\mathcal{A}}((xy)^3) = \bar{\delta}_{\mathcal{A}}((yx)w(yx)) \\
&\leq max\{\bar{\delta}_{\mathcal{A}}(yx),\bar{\delta}_{\mathcal{A}}(yx)\} = \bar{\delta}_{\mathcal{A}}(yx) \\
\bar{I}_{\mathcal{A}}(xy) &= \bar{I}_{\mathcal{A}}((xy)^3) = \bar{I}_{\mathcal{A}}((yx)w(yx)) \\
&\leq max\{\bar{I}_{\mathcal{A}}(yx),\bar{I}_{\mathcal{A}}(yx)\} = \bar{I}_{\mathcal{A}}(yx) \\
\bar{\Delta}_{\mathcal{A}}(xy) &= \bar{\Delta}_{\mathcal{A}}((xy)^3) = \bar{\Delta}_{\mathcal{A}}((yx)w(yx)) \\
&\geq min\{\bar{\Delta}_{\mathcal{A}}(yx),\bar{\Delta}_{\mathcal{A}}(yx)\} = \bar{\Delta}_{\mathcal{A}}(yx).
\end{aligned}$$

Similarly we can prove that $\bar{\delta}_{\mathcal{A}}(yx) \leq \bar{\delta}_{\mathcal{A}}(xy)$, $\bar{I}_{\mathcal{A}}(yx) \leq \bar{I}_{\mathcal{A}}(xy)$, $\bar{\Delta}_{\mathcal{A}}(yx) \leq \bar{\Delta}_{\mathcal{A}}(xy)$. Therefore A($xy$) = A($yx$) for all $x,y \in$ G.

Conclusion:

S. Broumi and F. Smarandache, Intuitionstic Neutrosophic Soft Set has been extended in this paper. Various ideals is proved with suitable exampls.

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