International Journal of Applied and Advanced Scientific Research (IJAASR)
Impact Factor: 5.655, ISSN (Online): 2456 - 3080
(www.dvpublication.com) Volume 3, Issue 1, 2018

ANTI MONOTONIC P-FUZZY G – DISTRIBUTIVE LATTICES

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Cite This Article: A. Panneerselvam & A. Jayadevi, "Anti Monotonic P-Fuzzy G – Distributive Lattices", International Journal of Applied and Advanced Scientific Research, Volume 3, Issue 1, Page Number 89-95, 2018.

Abstract:

The main goal of this paper is to study the finite groups whose lattices of fuzzy G-modular are distributive. We obtain a characterization of these lattices which is similar to a well known result of lattice theory.

Key Words: Fuzzy Lattices, Level Set, G-Modular, Anti P-Fuzzy G-Modular Lattices, Fuzzy Filter, Fuzzy Ideal & Lattice Homomorphism.

Introduction:

The theory of fuzzy sets which was introduced by L. A. Zadeh [14] is applied to many mathematical branches. Rosenfeld [13] inspired the fuzzification of algebraic structures and introduced the notion of fuzzy sub groups. P. Das [7] characterized fuzzy sub groups by their level sub groups. In [10] Liu applied the concept of fuzzy sets to the theory of rings and introduced and examined the notion of a fuzzy ideal of a ring. The study of fuzzy sub module was introduced by pan and Golan in [7]. Also Pan [12] studied the fuzzy finitely generated modules and fuzzy quotient modules. The formation of a lattice of sub modules of a module is well known features in classical algebra. However the same has not been explored in fuzzy setting. In order to initiate such studies the concept of fuzzy sub-module generated by an arbitrary fuzzy set is formulate in this note. Using this concept S.K. Bambri and Pratibakumar [5] introduced Lattice of fuzzy sub modules and established an embedding of the lattice of all sub module of a module M into the lattice of fuzzy sub module.

Q. Zhang and Meng [15] considered the sub property to be an assumption and established the more general result that "the lattice I_t (R) of all fuzzy ideals of a ring R with the same tip "t" is modular. On the other hand in [11] the authors have arrived at the false result that the lattice I(R) is distributive. In fact the lattice I(R) of ideals of a ring R is not distributive and has an obvious embedding in I(R). The corrected result has already appeared in several papers [9, 15]. I Jahan in [11], by constructing and employing the technique of strong level subsets, he proved that the lattice (R) of all ideals of a ring R is modular. This proof of the modularity of I(R) is different from the Ajmal's proof of modularity of the lattice of fuzzy normal sub groups of group appeared in [1]. K.C.Gupta and S. Roy in introduce the indirect proof of the above result that the modularity of the quasi Hamiltonians fuzzy sub groups. Hence N. Ajmal and K.V. Thomas in [2] initiated a discussion on the aspect of modularity of the set of fuzzy normal sub groups. A special class of fuzzy normal sub groups of group has been shown to constitute a modular sub lattice of its fuzzy sub group lattice. They first established that the sub lattice (G) is modular. Moreover using the same technique in [3], they demonstrate that whenever the set (G) of all fuzzy quasi normal sub groups of a given group forms a lattice, the lattice is modular. B. Yuan and Wu. Wangning in [4], introduced fuzzy ideals on a distributive lattice. In this paper, we study the characterization of fuzzy G-modular distributive lattice such as ring sums, lattice homomorphism and upper level sets.

Preliminaries:

Definition 2.1:

A non empty set L together with two binary operations V and Λ on L is called a lattice if it satisfies the following identities.

L1: $x \vee y \approx y \vee x,\ x \wedge y \approx y \wedge x$
L2 $x \vee (y \vee z) \approx (x \vee y) \vee z,\ x \wedge (y \wedge z) \approx (x \wedge y) \wedge z$
L3 $x \vee x \approx x,\ x \wedge x \approx x$
L4 $x \approx x \vee (x \wedge y), x \approx x \wedge (x \vee y)$.

The operation Λ is called meet and the operation V is called join. Let L be the set of proposition and let V denote the connective "or" and Λ denote the connective "and". Then L1 to L4 are well known properties from prepositional logic.

Definition 2.2:

A distributive lattice is a lattice which satisfies either of the distributive laws. D1.$x \wedge (y \vee z) \approx (x \wedge y) \vee (x \wedge z)$, D2 $x \vee (y \wedge z) \approx (x \vee y) \wedge (x \vee z)$.One can see a lattice L satisfies D1 if and only if satisfies D2.

Definition 2.3:

Let G be a group and L be a vector distributive lattice then L is called G- modular if for a $\in$G and $l \in$ L, there exist a product a. $l \in$L satisfies the following axioms.

(i) e . l = l, for all $l \in$L

(ii) (a. h) . l = a (h . l)

(iii) a. $(k_1 \ m_1 + k_2 \ m_2) = k_1 (am_1) + k_2(am_2)$ for all $k_1, k_2 \in K$ and $m_1, m_2 \in M$.

Definition 2.4:

A mapping $\mu: X\times P \to [0, 1]$ where X is an arbitrary non-empty set and is called a P-fuzzy set in X.

Definition 2.5:

Let X be a P-fuzzy set and $\mu : X\times P \to G$ be a lattice ordered group of X. Then μ is called P-fuzzy lattice ordered group (PFLG) if

(i) $\mu (x + y, p) \geq \min \{ \mu (x, p), \mu (y, p)\}$

(ii) $\mu (-x, p) \geq \mu (x, p)$

(iii) $\mu (0, p) = 1$, for all $x, y \in G$.

Definition 2.6:

Let μ be a P-fuzzy lattice ordered group of G and $\mu: X\times P \to G$. Then $\mu$is called P-fuzzy lattice if

(i) $\mu (x + y, p) \geq \min \{ \mu (x, p), \mu (y, p)\}$

(ii) $\mu (-x, p) \geq\mu (x, p)$

(iii) $\mu (x \vee y) \geq \min \{ \mu (x), \mu (y)\}$

(iv) $\mu (x\wedge y) \geq \min \{ \mu (x), \mu (y)\}$, for all $x, y \in G$.

Definition 2.7:

Let X be any group and L be a vector distributive lattice extended real valued functions on X. If μ is called a anti P-fuzzy G-modular lattice on L then it satisfies the following conditions.

(i) $\mu (ax + by, p) \leq \max\{ \mu (x, p), \mu (y, p)\}$

(ii) $\mu (gx, p) \leq \mu (x, p)$

(iii) $\mu (x \vee y)_p \wedge (x \ \wedge y)_p \leq \max\{(\mu (x, p), \mu(y, p)\}$, for all $x, y \in L$.

Definition 2.8:

A P-fuzzy sub set μ is called monotonic if $\mu (x,p) \leq \mu (y,p)$ whenever $x \leq y$

Definition 2.9:

Let μ be any P-fuzzy sub set of X. Then the set $\mu_t = \{x \in X / (\mu (x, p) \geq t, t \in [0, 1]\}$ is called a level sub set of μ.

Definition 2.10:

(i) A monotonic fuzzy G-modular lattice is called a fuzzy filter of L.

(ii) A anti-monotonic fuzzy G-modular lattice is called a fuzzy ideal of L.

(iii) A P-fuzzy sub set $\mu$is called a P-fuzzy filter if and only if $\mu (x \wedge y)_p = \mu (x, p) \wedge \mu (y, p)$, for all $x, y \in L$.

(iv) A P-fuzzy subset μ is called a Q-fuzzy ideal if and only if $(\mu \ x \vee y)_p = \mu (x, p) \wedge (\mu \ y, p)$, for all $x, y \in L$.

Definition 2.11:

(i) A P-fuzzy filter is prime if $\mu (x \vee y)_p \leq S\{ \mu (x ,p), \mu (y, p)\}$

(ii) A P-fuzzy ideal is prime if $\mu (x \wedge y)_p \leq S \{ \mu (x, p), \mu (y, p)\}$, holds for all $x, y \in L$.

Definition 2.12:

A P-fuzzy G-modular lattice A is modular if and only if $A (x, z) \geq 0 \to x \vee (y \ \wedge z) = (x \vee y) \wedge \ z$. for all x, y and $z \in X$.

Definition 2.13:

Let μbe a anti P-fuzzy G-modular lattice. If μ is distributive then

(i) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge y)$ and

(ii) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ for all x, y and $Z \in X$.

Definition 2.14:

Let L and L' be lattices. A mapping $f: L \to L'$ is said to be lattice homomorphism if

(ii) $f(x + y) = f(x) + f(y)$ and

(iii) $f(xy) = f(x) . f(y)$, for all $x, y \in L$.

Definition 2.15:

We say that a Anti P-fuzzy G-modular distributive lattice $M = (L, \mu_M)$ in L satisfies the imaginable property if $I_m (\mu_M)$ is subset of T.

Definition 2.16:

Let λ and μ be two anti P-fuzzy G-modular distributive lattice of L .Then the sum of λ and μ are denoted by $\lambda + \mu$ and is defined as

$$(\lambda+\mu)(z,p) = \inf_{z = x + y} \{\max\{\lambda(x, p), \mu(y, p)\}\} \quad \text{where } x, y \in L.$$

Clearly $\lambda + \mu$ is a anti P-fuzzy G modular distributive lattice in L.

Definition 2.17:

Let λ and μ be two anti P-fuzzy G-modular distributive lattice of L.

Then the ring sum of λ and μ is defined as

$$(\lambda \circledR \mu)(z,p) = \inf\{(\mu_1 + \mu_2)(z,p) / x = f^{-1}(\lambda, p), y = f^{-1}(\mu,p)\}$$

$$Z \in (\lambda \circledR \mu) \text{ for all } x, y \text{ and } z \in L$$

Characterization of Anti Q-Fuzzy G-Modular Distributive Lattices:

Throughout this paper G be a finite groups and L = <L, +, · > denotes a lattice. That is of maps from L into <[0, 1]; V, Λ >, where [0, 1] is the set of reals between 0 and 1. and

(i) $x \vee y = \max(x, y)$, if $y \le x$ then $\mu(y) \ge \mu(x)$ and

(ii) $x \wedge y = \min(x, y)$, if $x \le y$ then $\mu(x) \ge \mu(y)$

Proposition 3.1:

Let M be a G-modular lattice and N be a anti P-fuzzy G-sub modular lattice of M has a anti P-fuzzy G-modular distributive lattice. Then M/N and N has anti P-fuzzy G-modular distributive lattice.

Proof:

Let μ be a fuzzy sub modular lattice of M. Then V: μ_N is anti P-fuzzy G-modular lattice on N.

Define $\mu : \dfrac{M}{N} \times P \to [0, 1]$ by

$$\mu(x + N, p) = \mu(x, p), \text{ for all } x \in G \text{ and } n \in \frac{M}{N}$$

(1) $$\begin{aligned}\mu(a(x + N) + b(y + N), p) &= \mu((ax + by) + N, p)\\ &= \mu(ax + by, p)\\ &\le \max\{\mu(x, p) + \mu(y, p)\}\\ &\le \max\{\mu(x + N, p) + \mu(y + N, p)\}\end{aligned}$$

(2) $$\begin{aligned}\mu(g(x + N, p)) &= \mu(gx + N, p) = \mu(gx, p)\\ &\le \mu(x, p)\\ &\le \mu(x + N, p)\end{aligned}$$

(3) $$\begin{aligned}\mu(x \vee y)p \wedge \mu(x \wedge y))p + N &= \max\{\mu((x \vee y)p + N, \mu(x \wedge y)p + N\}\\ &\le \max\ \{\max\{\mu(x, p), \mu(y, p)) + N\}, \{\max\{\mu(x, p), \mu(y, p)) + N\}\}\\ &\le \max\{\mu(x, p) + N, \mu(y, p) + N\}\\ &\le \max\{\mu(x + N, p), \mu(y + N, p)\}\\ &\le \max\{\mu(x, p), \mu(y, p)\}, \text{ for all } x, y \in G\end{aligned}$$

Therefore $\dfrac{M}{N}$ and N are anti P-fuzzy G-modular distributive lattice.

Proposition 3.2:

Let f: L → L* be a G-modular distributive lattice homomorphism where L and L* are G-modular distributive lattices. If V is anti P-fuzzy G –modular distributive lattice on L*, then $f^{-1}(v)$ is anti P- fuzzy G-modular distributive lattice on L.

Proof:

Since V is anti P-fuzzy G-modular distributive lattice on L* and f: L →L* be a G-modular lattice homomorphism. For any a, b ∈ K and x, y ∈ L, we have

(1) $$\begin{aligned}f^{-1}(V)(ax + by, p) &= V(f(ax + by, p))\\ &= V(f(ax, p) + f(by, p))\\ &\le V\{(f(x, p) + f(y, p)\}\\ &\le \max\{V f(x, p), Vf(y, p)\}\\ &\le \max\{f^{-1}(V)(x, p), f^{-1}(V)(y, p)\}\end{aligned}$$

(2) $$\begin{aligned}f^{-1}(V)(gm, p) &= Vf(gm, p)\\ &\le Vf(m, p)\\ &\le f^{-1}(V)(m, p)\end{aligned}$$

(3) $$\begin{aligned}f^{-1}(V)(x \vee y)p \wedge f^{-1}(V)(x \wedge y)p &= \max\{f^{-1}(V)((x \vee y)p, f^{-1}(V)(x \wedge y)p\}\\ &= \max\{V f(x \vee y, p), V f(x \wedge y, p)\}\\ &= \max\{V(f(x, p) + f(y, p)), V(f(x, p) . f(y, p))\}\end{aligned}$$

(Since V is homomorphism

$\leq \max \{\max\{Vf(x, p), Vf(y, p)\}, \{\max\{Vf(x, p), Vf(y, p)\}\}$

$\leq \max \{Vf(x, p), Vf(y, p)\}$

$\leq \max \{ f^{-1}(v)(x, p), f^{-1}(v)(y, p)\}$

Therefore f^{-1} (V) is anti P- fuzzy G-modular distributive lattice on L.

Preposition 3.3:

Let S be a t-norm. Then every imaginable anti P-fuzzy G-modular distributive lattice of L is anti P-fuzzy G-modular distributive lattice.

Proof:

Let M = (L, μ_M) be an imaginable anti P-fuzzy G-modular distributive lattice of L under S-norms. Then

(i) $\mu_M (ax + by, p) \leq S \{ \mu_M (x, p), \mu_M (y, p)\}$

(ii $\mu_M (gm, p) \leq \mu_M (m, p)$,

(iii) $\mu_M (x \vee y)p \wedge \mu_M (x \wedge y)p \leq S\{ \mu_M (x, p), \mu_M (y, p)\}$ for all a, b $\in$G, x, y and m $\in$M.

We have

$\max \{ \mu_M(x, p), \mu_M(y, p)\} = S \{\max\{ \mu_M (x, p), \mu_M (y, p)\}, \max \{ \mu_M (x, p), \mu_M (y, p)\}$

$\geq S \{\mu_M (x, p), \mu_M (y, p)\}$

$\geq \max\{ \mu_M (x, p), \mu_M (y, p)\}$

It follows that $\mu_M (ax + by, p) \leq S \{ \mu_M (x, p), \mu_M (y, p)\}$

Therefore M = (L, μ_M) is an imaginable anti P-fuzzy G-modular distributive lattice of L.

Proposition: 3.4:

If L is a complete lattice, then the intersection of a family of anti P- fuzzy G-modular distributive lattice is a fuzzy G-modular distributive lattice.

Proof:

Let $\{ M_j : j \in J\}$ be a family of anti P-fuzzy G-modular distributive lattice and let $M = \cup M_j$

We have $\mu_M (x, p) = \sup \mu_{M_j} (x, p)$ and $\mu_M (y, p) = \sup \mu_{M_j} (y, p)$

(1) $\mu_M (ax + by, p) = \sup \mu_{M_j} (ax + by, p)$

$\leq \sup\{\max \{ \mu_{M_j} (x, p), \mu_{M_j} (y, p)\}\}$

$\leq \max \{\sup \mu_{M_j} (x, p), \sup \mu_{M_j} (y, p)\}$

$\leq \max \{ \mu_M (x, p), \mu_M (y, p)\}$

(2) $\mu_M (gx, p) = \sup \mu_{M_j} (gx, p)$

$\leq \sup \mu_{M_j} (x, p)$

$\leq \mu_M (x, p)$

(3) $\mu_M (x \vee y)p \wedge \mu_M (x \wedge y)p = \max \{ \mu_M ((x \vee y, p), \mu_M (x \wedge y, p) \}$

$= \max \{ \sup \mu_{M_j} (x \vee y, p), \sup \mu_{M_j} (x \wedge y, p) \}$

$= \max \{\sup \max\{ \mu_{M_j} (x, p), \mu_{M_j} (y, p)\}, \mathrm{Sup} \max\{ \mu_{M_j} (x, p), \mu_{M_j} (y, p)\}\}$

$\leq \max\{\sup \mu_{M_j} (x, p), \sup \mu_{M_j} (y, p)\}$

$\leq T \{\mu_M(x, p), \mu_M(y, p)\}$, for all x, y $\in$L.

Therefore $\cup \mu_{M_j}$ is anti P-fuzzy G-modular distributive lattice.

Proposition 3.5:

Any finite dimensional G-modular distributive lattice over a sub lattice L is anti P- fuzzy G-modular distributive lattice over L.

Proof:

Let μ:M $\rightarrow$ [0, 1] be a map and M be a finite n-dimensional anti P-fuzzy G-modular distributive lattice over L.

For all $\alpha, \beta \in$ L and x, y $\in$ M

Such that $\mu^n (\alpha x, p) = \mu (n(\alpha x, p) = \mu^n(x, p)$

(1) $\mu^n (\alpha x + \beta y, p) = \mu (n(\alpha x + \beta y, p))$

$= n\,\mu\,(\alpha\, x + \beta\, y, p)$
$\leq n\,\mu\,\{\max\{\mu(\alpha x, p) + \mu(\beta y, p)\}$
$\leq \max\{ n\,\mu\,(\alpha x, p)\,, n\,\mu\,(\beta y, p)\}$
$\leq \max\{ \mu^{n}(\alpha x, p)\,,\ \mu^{n}(\beta y, p)\}$
$\leq \max\{ \mu^{n}(x, p)\,,\ \mu^{n}(y, p)\}$

(2) $\mu^{n}\,(gx, p)$ $= \mu\,(n(gx, p))$
$= n\,\mu\,(gx, p)$
$\leq n\,\mu\,(x, p)$
$\leq \mu^{n}\,(x, p)$

(3) $\mu^{n}\,(x \vee y)p\ \ \Lambda \mu^{n}\,(x\ \ \Lambda y)p$ $= \max\{ \mu^{n}\,((x \vee y, p), \mu^{n}\,(x\, \Lambda\, y, p)\}$
$= \max\{ \mu\,(n\,(x \vee y, p), \mu\,(n\,(x\, \Lambda\, y, p)\}$
$= \max\{n\,\mu\,\{(x \vee y, p)\,, n\,\mu\,(x\, \Lambda\, y, p)\}$
$\leq \max\{n\{\max\{ \mu\,(x, p), \mu\,(y, p)\}, n\{\max\{ \mu\,(x, p), \mu\,(y, p)\}\}$
$\leq \max\{n\,\mu\,(x, p), n\,\mu\,(y, p)\}$
$\leq \max\{ \mu^{n}(x, p), \mu^{n}(y, p)\}$, for all $x, y \in M$.

Therefore, n-dimensional anti P-fuzzy G-modular distributive lattice is anti P- fuzzy G-modular distributive lattice.

Proposition 3.6:

If M and N are anti P-fuzzy G-modular distributive lattices over a fuzzy sub-lattice L then L = M ® N is anti P- fuzzy G-modular distributive lattice on L. Here ® is referred as a ring sum.

Proof:

Let M and N be two anti P-fuzzy G-modular distributive lattices of $\mu 1$ and μ_2 respectively. Then we define the ring sum of M. and N as

$$(M ® N)\,(z, p) = \inf_{z \in M ® N} \{(\mu_1 + \mu_2)\,(z, p) / x = f^{-1}(\mu_1,p),\ y = f^{-1}(\mu_2,p)\}$$

Now

(1) $(M ® N)\,(\alpha x + \beta y, p)$ $= (\mu_1 + \mu_2)(\alpha x + \beta y, p)$
$\leq \max\{((\mu_1 + \mu_2)\,(x, p), (\mu_1 + \mu_2)\,(y, p)\}$
$\leq \max\{\inf_{z \in M ® N} \mu_1\,(z, p), \inf_{z \in M ® N} \mu_2\,(z, p)\}$
$\leq \max\{(M ® N)\,(z, p), M ® N\,(z, p)\}$
$\leq \max\{(M ® N)\,(x, p), M ® N\,(y, p)\}$

(2) $(M ® N)\,(gz, p)$ $= (\mu_1 + \mu_2)\,(gz, p)$
$\leq (\mu_1 + \mu_2)\,(z, p)$
$\leq \inf_{z \in M ® N}\{\mu_1 + \mu_2\}\,(z, p)$
$\leq (M ® N)\,(z, p)$ (3)

$(M ® N)(x \vee y)p\ \Lambda\ (M ® N)(x\ \Lambda\ y)p$
$= \max\{(M ® N)\,(x \vee y, p),(M ® N)(x \Lambda y, p)\}$
$= \max\{ (\mu_1 + \mu_2)\ (x \vee y, p), (\mu_1 + \mu_2)\ (x\, \Lambda\, y, p)\}$
$\leq \max\{\max\{(\mu_1 + \mu_2)\,(x, p), (\mu_1 + \mu_2)\,(y, p)\},\max\{(\mu_1 + \mu_2)\,(x, p), (\mu_1 + \mu_2)\,(y, p)\}\}$
$\leq \max\{(\mu_1 + \mu_2)\,(x, p), (\mu_1 + \mu_2)\,(y, p)\}$
$\leq \max\{ \inf_{z \in M ® N} \mu_1\,(z, p)\ ,\ \inf_{z \in M ® N} \mu_2\,(z, p)$
$\leq \max\{(M ® N)\,(x, p), M ® N\,(y, p)\}$

Therefore, M ® N is anti P- fuzzy G-modular distributive lattices.

Proposition 3.7:

Let M be a G-modular distributive lattices over a sub lattice L and $M = \sum_{i=1}^{n} M_i$, where M_i's are G sub modular distributive lattice of M. If V_i are anti P-fuzzy G-modular distributive lattice of M_i, then $V: M \rightarrow [0, 1]$ is anti P-fuzzy G-modular distributive lattice in L.

Proof:

Since V_i is anti P- fuzzy G-modular distributive lattice on M_i for every x, y $\in$ M; g $\in$ L and $\alpha, \beta \in$ L. We have

(1) $V(\alpha x + \beta y, p) = V(\sum(\alpha M_i + \beta M_i', p))$
$= \Lambda (V_i (\alpha M_i + \beta M_i', p))$, where i = 1, 2 … n.
$= V_j (\alpha M_j + \beta M_j', p))$, for some j
$\leq \max (V_j(M_j, p), V_j(M_j', p))$
$\leq \max (V(x, p), V(y, p)\}$

(2) $V(gx, p) = V(\sum gM_i, p)$
$= \Lambda (V_i (gM_i, p)$, where i = 1, 2 … n.
$= V_j (gM_j, p)$, for some j
$\leq V_j(M_j, p)$
$\leq V(x, p)$

(3) $V (x \vee y)p \wedge V (x \wedge y)p = \max \{ V ((x \vee y, p), V (x \wedge y, p) \}$
$= \max \{V\sum (x \vee y, p)M_i, V \sum (x \wedge y, p) M_i'$ for i = 1, 2, … n
$= \max \{V (x \vee y, p)M_i , V (x \wedge y, p) M_i'\}$
$\leq \max\{\max \{V(x, p), V(y, p)\} M_i,\{\max \{V(x, p), V(y, p)\} M_i'\}\max \{ V(x, p)Mi, V(y, p) M_i'\}$
$\leq \max \{V_iM_j, V_j M_j'\}$, for some j.
$\leq \max \{V(x, p), V(y, p)\}$.

Therefore V is anti P- fuzzy G-modular distributive lattice on L.

Proposition 3.8:

Let λ and μ be anti P-fuzzy G-modular distributive lattices of L, Then $\lambda + \mu$ is the smallest anti P-fuzzy G-modular distributive lattice of L.

Proof:

For any x, y, h, f $\in$ L, we have

(1) $\max \{(\lambda + \mu)p\, x, (\lambda + \mu)p\, y\} = \max \{\inf_{x = a + b} \max \{(\lambda(a,p), \mu (b,p)\}, \inf_{y = c + d} \max \{(\lambda(c,p), \mu(d,p)\}\}$

$= \max \{\inf_{x = a + b,\ y = c + d} (\max \{(\lambda(a,p), \mu(b,p)\}, \max \{(\lambda(c,p), \mu(d,p))\}$

$= \max \{\inf_{x = a + b,\ y = c + d} \max \{(\lambda(a,p), \mu(b,p), \lambda(c,p), \mu(d,p)\}\}$

$\geq \max \{\inf_{x = a + b,\ y = c + d} \max \{(\lambda (a+(b–c–b),p), \mu(b+(c–d–c),p)\}\}$

$\geq \max \{\inf_{x + y = h + f} \max \{(\lambda (h,p), \mu(f,p)\}\}$

$\geq (\lambda + \mu)p(x + y)$

Therefore $(\lambda + \mu (x + y) \leq S \{(\lambda + \mu)p(x), (\lambda + \mu)p(y)\}$

(2) $(\lambda + \mu)(gx,p) = \inf_{x = a + b} \max \{(\lambda(a,p), \mu(b,p)\}$

$= \inf_{g_x = g_a + g_b} \max \{(\lambda(a,p), \mu(b,p)\}$

$\leq \inf_{X = G_A + G_A} \max \{(\lambda(A,p), \mu(B,p)\}$

$\leq (\lambda + \mu)(x,p)$

(3) $(\lambda + \mu)(x \vee y)p \wedge (\lambda + \mu)(x \wedge y)p) = \max \{(\lambda + \mu)(x \vee y)p, (\lambda + \mu)(x \wedge y)p \}$

$= S \{\max \{ (\lambda + \mu)(x,p), (\lambda + \mu)(y,p)\}, \max \{ (\lambda + \mu)(x,p), (\lambda + \mu)(y,p)\}\}$

$= S \{\max \{\inf_{x = a + b} \max \{\lambda (x,p), \mu (x,p), \inf_{y = c + d} \max \{\lambda(y,p), \mu(y,p\}\},$

$= \max \{\inf \max \{\lambda (x,p), \mu (x,p)\}$

$$\inf_{x = a + b} \max \{\lambda(y,q), \mu(y,q)\}\}\}$$

$$\le S \{\inf_{y = c + d} \max \{\lambda(y,p), \mu(y,p)\},$$

$$\inf_{y = c + d} \max \{\lambda (x,p), \mu(x,p)\}\}$$

$$\le S \{\inf_{x = a + b} \max \{\lambda (x,p), \mu(x,p)\},$$

$$\inf_{x = a + b} \max \{\lambda(y,p), \mu(y,p)\}\}$$

$$\le S \{(\lambda + \mu) (x,p), (\lambda + \mu) (y,p)\}$$

Therefore $(\lambda + \mu) (x \vee y)p \wedge (\lambda + \mu) (x \wedge y)p \le S \{(\lambda + \mu) (x,p), (\lambda + \mu) (y,p)\}$

Hence $(\lambda + \mu)$ is anti P- fuzzy G-modular distributive lattice on L.

Conclusions:

I. Jahan [2008] introduce the concept of modularity of Ajmal [1995] for the lattices of fuzzy ideals of a ring. In this paper, we study the characterization of anti P-fuzzy G-modular distributive lattices. One can obtain the similar results in soft modules and soft lattices by using a suitable mathematical tool.

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