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P-FUZZY LEVEL SUBMODULES OF NEAR RINGS

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Abstract:

A Technique of generating of P- fuzzy R- sub module by a given arbitrary P- fuzzy set is provided. It is shown that (i) The sum of two P- fuzzy R- sub module of a module M is the P- fuzzy R- sub module generated by their union and (ii) The set of all P- fuzzy sub module of a given module forms a complete lattice. Consequently it is established that the collection of all P- fuzzy R- sub module, having the same values at zero, of M of the lattice of P- fuzzy R- sub module of M. Interrelationship of these finite range sub lattices is established. Finally it is shown that the lattice of all P- fuzzy R- sub module of M can be embedded into a lattice of P- fuzzy R- sub module of M. Through out this paper , M denote as P- fuzzy R- sub module where R is the commutative near ring with unity. Characterization of P- fuzzy left R- sub modules with respect to t- norm are also given.

Key Words: P-Fuzzy Set, P- Fuzzy R- Sub Module, Level Sub Module, P- Fuzzy Lattice Of R- Sub Modules, Lattice Homomorphism & Modular Lattice

Introduction:

In the Trajectory of stupendous growth of fuzzy set theory. Fuzzy algebra has become an important area of research. In 1971, A. Rosenfeld [12] used the concept of fuzzy set theory due to Zadeh [13] in abstract algebra and opened up a new insight in the field of mathematical science. Since then the study of fuzzy algebraic structures has been pursued in many directions such as semi groups, groups, rings, semi rings, near rings, lattices and so on. Fuzzy sub modules of a module M over a near ring R were first introduced by Negoita and Ralescu [9] Consequently Pan studied fuzzy finitely generated modules and fuzzy quotient modules. Pan [3] and Katsaras and Liu [4] introduced the concept of fuzzy sub modules and fuzzy vector spaces respectively. More recently in the notion of set products is discussed in details and in the lattice theoretical aspect of fuzzy subgroups and fuzzy normal subgroups are explored. The formation of a lattice of sub modules of module is well known feature in classical algebra. However, the same has not been explored in fuzzy setting. In order to initiate such studies, the concept of fuzzy sub module generated by an arbitrary fuzzy set is formulated in this paper. Using this concept we construct various type of lattices of fuzzy sub modules and establish an embedding of the lattice of all submoduls of a module M into the lattice of fuzzy sub modules M. The notion of an intuitionistic P- fuzzy R- subgroups of near rings is given by [10]. The notion of intuionistic P- fuzzy semi primality in a semi group is given Kim .Also Rho.at.al (11) considered the intuionistic P- fuzzy BCK / BCI algebra. In this paper we make some characterization of P- fuzzy left R- sub module and then proved some results on P- fuzzy lattice of R- modules.

Preliminaries:

Definition 2.1: A mapping $\mu : X \rightarrow [0,1]$, where X is an arbitrary non- empty set and is called a fuzzy set in X.

Definition 2.2; A mapping $\mu : M\times P \rightarrow [0,1]$ is called P- fuzzy set in M.

Definition 2.3: Let μ be a P- fuzzy subset of a set S and $t \,\varepsilon\, [0,1]$, Then the set $\mu_t = \{ x \,\varepsilon\, S / \mu(x,p) \geq t \}$, $p \,\varepsilon\, P$ is called a level subset of μ.

Definition 2.4: A t- norm is a function T: $[0, 1] \rightarrow [0, 1] \times [0, 1]$ that satisfies the following conditions for all x, y, z ε [0, 1];

(T1) $T(x,1) = x$,
(T2) $T(x,y) = T(y,x)$,
(T3) $T(x,T(y,z)) = T(T(x,y), z)$,
(T4) $T(x,y) \leq T(x,z)$ whenever $y \leq z$.

Definition 2.5: A P– fuzzy subset μ of M is called a P – fuzzy R- sub module of M if the following conditions are satisfied,

(i) $\mu(x+y , p) \geq T(\mu(x,p) , \mu(y,p)\}$ for all x,y ε M and
(ii) $\mu(rx,p) \geq \mu(x,p)$ for all r ε M and p ε P.

For a P- fuzzy left R – module of μ of M the level subset $\mu_t = \{ x \,\varepsilon\, M / \mu(x,p) \geq t \}$, $t \,\varepsilon\, Im(\mu)$ are sub modules of M called the P- level sub modules of M.

The following result can be easily proved.

Lemma 2.6: A P- fuzzy subset μ of M is a P- fuzzy R- sub module of M if and only if each level subset μ_t , $t \,\varepsilon\, Im(\mu)$, is a sub module of M.

Definition 2.7: Let μ be a P- fuzzy subset on M. Define a P- fuzzy subset $<\mu>$ of M as follows. $<\mu>(x,p) = \sup \{ K / x \varepsilon ,<\mu_k> , x \varepsilon M$. $<\mu>$ is called the P- fuzzy subset of M generated by μ. Here $<\mu_k>$ is the sub module of M generated by the level subset μ_k.

Proposition 2.8: Let μ be a P-fuzzy R- sub module of M. Then the P- fuzzy subset$<\mu>$ is a P- fuzzy R- sub module of M generated by. More over $<\mu>$ is the smallest P- fuzzy R- sub module containing μ

Proof: Let x,y ε M and let $\mu(x,p) = t_1$, $\mu(y,p) = t_2$ and $\mu(x+y, p) = t$

L et it possible $t = <\mu>(x+y, p)$

$\leq T\{<\mu>(x,p), <\mu>(y,p)\}$

$T\{t_1,t_2\} = t_1$ (say)

Then $t_1 = <\mu>(x,p) = \sup\{k / x \varepsilon <\mu_k>\} \geq t$, therefore there exist k_1, such that $x \varepsilon <\mu_{k1}>$. Also $t_2 = <\mu>(y,q) = \sup\{k / y\varepsilon <\mu_k>\}$ $t_1 \geq t$ .therefore there exists $k_2 > t$ such that $y \varepsilon <\mu_k>$ without loss of generality, we may assume that k_1 k_2, so that $<\mu_{k1}>C$ $<\mu_{k2}>$.. Then $x,y \varepsilon <\mu_k>$ that is x+y which is a contradiction since $k_2 > t$. therefore $t \geq t_1$.

Consequently, $\mu(x+y,p) \geq T\{<\mu>(x,p), <\mu>(y,p)\}$ -------------- (1)

Now let, if possible, $t_3 = <\mu>(rx,p) \leq <\mu>(x,p) = t_1$ Then $t_1 = <\mu>(x,q) = \sup\{k / x \varepsilon <\mu_k>\} > t_3$, therefore there exists k such that $x \varepsilon <\mu_k>$ and $t_1 > k > t_3$ sothat $rx \varepsilon <\mu_k> C \{\mu_{t1}\}$ which is a contradiction.

Hence $t_3 = <\mu>(rx,p) \geq <\mu>(x,p) = t_1$ ----------------- (2)

Consequently conditions

(1) and (2) yield that $<\mu>$ is a P- fuzzy R- sub module of M. Finally, to show that $<\mu>$ is the smallest P- fuzzy R- sub module containing μ , let us assume that θ to be a P- fuzzy R- sub module of M such that μ C θ and show that $<\mu>$ C θ.

Let it possible, $t = <\mu>(x,p) \geq \theta(x,q)$ for some x ε m , p ε P. Let $\varepsilon > 0$ be given, then $t = \mu_t = \sup\{k / x \varepsilon <\mu_k>\}$ Therefore there exists K such that $x \varepsilon ,\mu_k>$ and $t-\varepsilon \leq k \leq t$ so that $x \varepsilon <\mu_k> C < \mu\ t-\varepsilon >$, for all $\varepsilon > 0$. Now $x = \acute{\alpha}_1 x_1 + \acute{\alpha}_2 x_2 + \ldots\ldots\ldots\ldots \acute{\alpha}_n x_n$, $\acute{\alpha}_i \varepsilon R$, x_i belongs to $t-\varepsilon$. $X_i \varepsilon \mu_{t-\varepsilon}$ implies $\mu(x_i,p) \geq t-\varepsilon$, that is $\theta(x_i,p) \geq t-\varepsilon$ for all $\varepsilon > 0$. Therefore

$\theta(x,p) \geq T\{\theta(x_1,p), \theta(x_2,p), \ldots\ldots\ldots\ldots \theta(x_n,p)\}$

$\geq t-\varepsilon$ for $\varepsilon > 0$

Hence $\theta(x,p) = t$ which is a contradiction to our supposition.

Definition 2.9: Let μ and θ be P- fuzzy R- sub modules of an R- sub module M then the sum of μ and θ is denoted by $\mu + \theta$ is defined as

$(\mu + \theta)(x,p) = \sup\{T(\mu(a,p), \mu(b,p)\}$

$(x_i,p) = (a+b ,p)$ for all $x_i \varepsilon M$

Clearly $\mu + \theta$ is a P- fuzzy subset of M.

Proposition 2.10: Let μ and θ be a P- fuzzy R- sub modules of M. Then the sum $\mu + \theta$ i P- fuzzy sub module of M.

Proposition 2.11: Let μ and θ be a P- fuzzy R- sub modules of M such that $\mu(0,q) = \theta(0,p)$ then μ C $\mu + \theta$, θ C $\mu + \theta$.

Proof: Let x ε M, then

$$(\mu + \theta)(x,p) = \sup_{(x,p) = (a+b,p)} \{T(\mu(a,p), \theta(b,p)\}$$

$$\geq T\{\mu(x,p), \theta(x,p)$$

$$= T(\mu(x,p), \mu(0,p)\}$$

$$= \mu(x,p)$$

Similarly $(\mu + \theta)(x,p) \geq \theta(x,p)$.

Lemma 2.12: Proposition (2.10) is not true if $\mu(0,p) \neq \theta(0,p) < \mu(0,q)$ so μ does not contain $\mu + \theta$.

Proposition 2.13: Let μ and θ be a P- fuzzy R- sub module of M such that $\mu(0,p) = \theta(0,p)$ then $\mu + \theta = <\mu + \theta>$.

Proof: Let x ε M and let

$$t_1 = (\mu+\theta)(x,p) = \sup_{(x,p) = (a+b,p)} \{T(\mu(a,p), \mu(b,p)\}$$

Let $\varepsilon > 0$ be given . Then

$t_{1-\varepsilon} < T(\mu(a,p), \theta(b,p)\}$, for some a,b ε M, such that x= a+b , so that $t_{1-\varepsilon} < \mu(a,p)$ and $t_{2-\varepsilon} < \theta(b,p)$

but μ , θ C $<\mu U \theta>$ C $<\mu U\theta>$, therefore

$t_{1-\varepsilon} \leq T\{<\mu U\theta>(a,p), <\mu U\theta>(b,p)\}$

$\leq <\mu U\theta>(a+b,p)$

$= <\mu U\theta>.(x,p)$, for all $\varepsilon > 0$.

Hence $t_1 \leq <\mu+\theta>(x,p) = t$ (say)

Let it possible $(\mu+\theta)(x,p) = t_1 <= <\mu U\theta>(x,e) = \sup\{k / x \varepsilon <(\mu U\theta)>$ therefore there exists k such that $x \varepsilon <(\mu+\theta)k>$ and $t_1 < k < t$

Then $(\mu U\theta)_t$ C $(\mu U\theta)_{t1}$. so that $<(\mu U\theta)_t>$ C $<(\mu U\theta)_k>$ C $<(\mu U\theta)_{t1}>$

Implying $x \varepsilon < (\mu+\theta)_{t1}>$ which is a contradiction. Hence $t_1 = (\mu+\theta)(x,p) = < \mu U\theta>(x,p) = t$.

Corollary 2.14: For any P- fuzzy R- sub module μ of M , $\mu + \mu = \mu$.

Lemma 2.15: Proposition (2.11) is not true if $\mu (0,q) \neq \theta (0,q)$, for let $\mu (0,q) > \theta (0,q)$ then $<\mu U\theta> (0,q) = \mu(0,p) > (\mu U\theta) (0,p)$ so $<\mu U\theta> > (\mu U\theta) (0,p)$ and hence $\mu + \theta \neq <\mu U\theta>$.

Definition 2.16: A P- fuzzy R-sub module 'μ' of a near ring is said to be normal if $\mu(0,p) = 1$

Properties of P- Fuzzy R- Sub Modules of Near Rings:

Proposition 3.1: Let 'μ' be a P- fuzzy R- sub module of a near ring and let μ^* be a P- fuzzy set in R defined by $\mu^*(x,p) = \mu(x,p) +1- \mu(0,q)$ for all x ε R. Then μ^* is a normal P- fuzzy R- sub module of R containing μ.

Proof: For any x,y ε R and q ε P, we have

$$\begin{aligned}\mu^*(x+y,p) &= \mu(x+y,p) +1 - \mu(0,p)\\ &\geq T (\mu(x,p)+1- \mu(0,p) , (\mu(y,p)+1- \mu(0,p))\\ &= T (\mu^*(x,p) , \mu^*(y,p)).\\ \mu^*(rx,p) &= \mu(rx,p) +1 - \mu(0,p)\\ &= \mu(x,p) +1- \mu(0,p)\\ &= \mu(x,q)\end{aligned}$$

Proposition 3.2: Let 'T' be a t- norm. Then every imaginable P- fuzzy left R- sub module 'μ' of a near ring ' R' is a fuzzy left R-sub module of R.

Proof: Assume 'μ' is imaginable P- fuzzy left R- sub module of 'R', then we have

$\mu (x+y , p) \geq T \{ \mu(x,p), \mu(y,p) \}$and $\mu (rx, p) \geq \mu (x,p)$ for all x,y in R.

Since 'μ' is imaginable, we have

$$\begin{aligned}\min \{ \mu(x,p) , \mu(y,p) \} &= T \{ \min \{ \mu(x,p) , \mu(y,p), \min \{ \mu(x,p) , \mu(y,p) \} \}\\ &\leq T (\mu(x,p) , \mu(y,p))\\ &\leq \min \{ \mu(x,p) , \mu(y,p) \}\end{aligned}$$

And so $T(\mu(x,p) , \mu(y,p)) = \min \{ \mu(x,p) , \mu(y,p) \}$. It follows that

$\mu(x+y, p) \geq T(\mu(x,p) , \mu(y,p))= \min \{ \mu(x,p) , \mu(y,p) \}$ for all x,y ε R. Hence 'μ' is a fuzzy left R-sub module of R.

Proposition 3.3: If 'μ' is a P- fuzzy left R- sub module of a near ring 'R' and 'ϴ' is an endomorphism of R, then $\mu_{[\Theta]}$ is a P- fuzzy left R- sub module of 'R'.

Proof: For any x,y ε S, we have

$$\begin{aligned}\text{(i) } \mu_{[\Theta]} (x+y , p) &= \mu (\Theta(x+y , p))\\ &= \mu (\Theta(x,p) , \Theta(y,p))\\ &\geq T \{ \mu(\Theta(x,p)) , \mu(\Theta(y,p)) \}\\ &= T \{ \mu_{[\Theta]} (x,p) , \mu_{[\Theta]} (y,p) \}\\ \text{(ii) } \mu_{[\Theta]} (rx , p) &= \mu(\Theta (rx, p)\\ &\geq \mu (\Theta(x,p))\\ &\geq \mu_{[\Theta]} (x,p)\end{aligned}$$

. Hence $\mu_{[\Theta]}$ is a P- fuzzy left R- sub module of R.

Proposition 3.4: An onto homomorphism of a P- fuzzy left R- sub module of near ring'R' is P- fuzzy left R- sub module.

Proof: Let $f : R \rightarrow R^1$ be an onto homomorphism of near rings and let 'ξ' be a P- fuzzy left R- subgroup of R^1 and 'μ' be the pre image of 'ξ' under 'f', then we have

$$\begin{aligned}\text{(i) } \mu(x+y , p) &= \xi (f(x+y , p))\\ &= \xi (f(x,p) , f(y,p))\\ &\geq T (\xi (f(x,p)) , \xi(f(y,p)))\\ &\geq T (\mu(x,p) , \mu(y,p))\\ \text{(ii) } \mu(rx,p) &= \xi (f (rx,p))\\ &\geq \xi (f(x,p))\\ &\geq \mu(x,p).\end{aligned}$$

Proposition 3.5: An onto homomorphism image of a fuzzy left R- sub module with the sup property is a fuzzy left R- sub module.

Proof: Let $f: R\rightarrow R^1$ be an onto homomorphism of near rings and let 'μ' be a sup property of fuzzy left R- sub module of 'R'.

Let $x^1, y^1 \; \varepsilon \; R^1$, and $x_0 \; \varepsilon \; f^{-1}(x^1)$, $y_0 \; \varepsilon \; f^{-1}(y^1)$ be such that

$$\mu(x_0, p) = \sup_{(h,p)\varepsilon f^{-1}(x^1)} \mu(h,p), \qquad \mu(y_0,p) = \sup_{(h,p)\varepsilon f^{-1}(y^1)} \mu(h,p)$$

respectively, then we can deduce that

$$\begin{aligned}\text{(i) } \mu^f (x^1+y^1, p) &= \sup_{(z,p) \varepsilon f^{-1}(x^1+y^1,p)} \mu(z,p)\\ &\geq \min \{ \mu(x_0,p) , \mu(y_0,p)\\ &= \min \{ \sup_{(h,p)\varepsilon f^{-1}(x^1,p)} \mu(h ,p) , \sup_{(h,p)\varepsilon f^{-1}(y^1,p)} \mu(h,p) \}\end{aligned}$$

$= \min \{ \mu^f(x^1,p) , \mu^f(y^1,p) \}$

(ii) $\mu^f(rx,p) = \sup_{(z,p)\varepsilon f^{-1}(r^1x^1, p)} \mu(z,qp$

$\geq \mu(y_0 , p)$

$= \sup_{(h,p)\varepsilon f^{-1}(y^1,p)} \mu(h,p)$

$= \mu^f(y^1,p)$. Hence 'μf' is a fuzzy left R- sub module of R^1.

Proposition 3.6: Let 'T' be a continuous t-norm and let 'f' be a homomorphism on a near ring 'R'. If 'μ' is P-fuzzy left R- sub module of R, then μ^f is a P- fuzzy left R- sub module of f(R).

Proof: Let $A_1 = f^{-1}(y_1, p)$, $A_2 = f^{-1}(y_2,p)$ and $A_{12} = f^{-1}(y_1\text{-}y_2 , p)$ where $y_1,y_2 \varepsilon f(R)$, $p\varepsilon P$.
Consider the set

$A_1\text{-} A_2 = \{ x \varepsilon S / (x,p) = (a_1,p) - (a_2,p) \}$ for some $(a_1,p) \varepsilon A_1$ and $(a_2,p) \varepsilon A_2.$

If $(x,p) \varepsilon A_1\text{-}A_2$, then $(x,p) = (x_1,p) - (x_2,p)$ for some $(x_1,p) \varepsilon A_1$ and $(x_2,p) \varepsilon A_2$ so that we have

$f (x,p) = f(x_1,p) - f(x_2,p)$
$= y_1\text{-} y2$

(ie) $(x,p) \varepsilon f^{-1}((y_1,p) – (y_2,p)) = f^{-1}(y_1\text{-}y_2, p) = A_{12}$. Thus $A_1\text{-}A_2 \subset A_{12}$.

It follows that

(i) $\mu^f(y_1+y_2, p) = \sup \{ \mu(x,p)/ (x,p) \varepsilon f^{-1}{}_{((}y_1,p)\text{-} (y_2,p))\}$
$= \sup\{ \mu(x,p) / (x,p) \varepsilon A_{12} \}$
$\geq \sup \{ \mu(x,p)/ (x,p) \varepsilon A_1\text{-}A_2\}$
$\geq \sup \{ \mu((x_1,p)\text{-} (x_2,p)) / (x_1,p) \varepsilon A_1$ and $(x_2,p) \varepsilon A_2\}$
$\geq \sup \{ T(\mu(x_1,p) , \mu(x_2,p))/ (x_1,p) \varepsilon A_1$ and $(x_2,p) \varepsilon A_2\}$

Since 'T' is continuous. For every $\varepsilon > 0$, we see that if

$Sup \{ \mu(x_1,p) / (x_1,p) \varepsilon A_1\} - (x_1^*, p) \} \leq \delta$ and
$Sup \{ \mu(x_2,p) / (x_2,p) \varepsilon A_2\} - (x_2^*,p) \} \leq \delta$

$T\{sup\{\mu(x_1,p) / (x_1,p) \varepsilon A_1\} , \sup \{ \mu(x_2,p) / (x_2,p) \varepsilon A_2 \} - T ((x_1^*,p), (x_2^*,p) \leq \varepsilon$

Choose $(a_1,p) \varepsilon A_1$ and $(a_2,p) \varepsilon A_2$ such that

$Sup \{ \mu(x_1,p) / (x_1.p) \varepsilon A_1 \} - \mu(a_1,p) \leq \delta$ and
$Sup \{ \mu(x_2,p) / (x_2,p) \varepsilon A_2\} - \mu(a_2,p) \leq \delta$. Then we have

$T\{sup\{ \mu(x_1,p) / (x_1,p) \varepsilon A_1\}, \sup \{ \mu(x_2,p) / (x_2,p) \varepsilon A_2 \} – T(\mu(a_1,p), \mu(a_2,p) \leq \varepsilon$ consequently, we have $\mu^f(y_1\text{-}y_2, p) \geq \sup\{ T(\mu(x_1,p), \mu(x_2,p)) / (x_1,p) \varepsilon A_1 ,(x_2,p) \varepsilon A_2\}$
$\geq T (\sup\{\mu(x_1,p) / (x_1,p) \varepsilon A_1\}, \sup\{\mu(x_2,p) / (x_2,p)\varepsilon A_2\}$
$\geq T (\mu^f(y_1,p) , \mu^f(y_2,p) \}$

Similarly we can show $\mu f(rx,p) \geq \mu^f(y,p)$. Hence '$\mu^f$' is a P- fuzzy left R- sub module of 'f(R)'.

Lattice Valued P- Fuzzy R- Sub Modules of Near Rings:

Let L be the set of all P- fuzzy R- sub modules of M. Then clearly the intersection of an arbitrary family of P- fuzzy R- sub modules of M is a fuzzy sub module and proposition (2.11) shows that the existence of the least P-fuzzy sub module containing the union of an arbitrary family of P- fuzzy R- sub modules of M. These facts give rise to the following.

Proposition 4.1: L is a lattice under the usual ordering of P- fuzzy set inclusion. More over L is a complete lattice, Next, let 't' be an arbitrary but fixed real number in [0,1] and let us denote by L_t, the subset of all P-fuzzy R- sub modules θ of M such that $\theta (0,q) = t$.

Proposition 4.2: L_t is a complete lattice of L.

Let L_t denote the set of all P- fuzzy R- sub modules θ of M such that $I_m \theta = \{ \theta(x,p) / x \varepsilon M \}$ is finite (that is has finite range).

Proposition 4.3: L_t is a sub lattice of L. Let L_t denote the set of all P- fuzzy R- sub modules θ of M such that $\theta (0,p) = t$ and $I_m\theta$ is finite

Proposition 4.4: Lf_t is a sub lattice of L.

Proof: Since $Lf_t = L_f \cap L_t$ and intersection of sub lattices is a sub lattice, L_{ft} is a sub lattice of L. finally , we intend to demonstrate an embedding of well known lattice of all sub modules of M into the lattice L of all P-fuzzy R- sub modules defined as follows.

$$L_2 = \{\theta / \theta \varepsilon M \text{ such that } I_m\theta = (r,t) , r <t \}$$

Proposition 4.5: L_2 is a sub lattice of L_{ft} and hence L.

Proof; It is obvious.

Proposition 4.6: L(M), the lattice of all sub modules of M can be embedded in L_2

Proof: Let A ε L(M). Define $\theta: MxP \to [0,1]$

$\theta (x,p) = 1$ if $x \varepsilon A$
$= 0$ if x does not belong to A.

Then θ is a P –fuzzy R- sub module of M (2.6) such that $I_m\theta (0,t) , 0 < t < 1$.

Therefore $\theta \varepsilon L_2$. Define σ: $L(M) \times P \rightarrow L_2$, such that σ (A,q) = θ then σ is clearly well defined. Let σ (A,p) = σ (B,p) . Then $\theta = \mu$ implies A = B. so σ is one- one.

Let A, B ε L(M) . Let σ (A,p) = θ , σ (B,p) = μ then

σ (A,p) + σ (B,p) = $\theta+\mu$

= < $\theta U\mu$ > by

= σ (A+B, p) and σ (A$\cap$B, p) = $\theta \cap \mu$ = σ (A,p) $\cap$ σ (B,p). Thus σ is a lattice homomorphism.

Hence σ is lattice embedding.

Remark 4.7: The above result justifies our modification of Pan's definition (3) of P- fuzzy R- sub modules wherein he assumes that the P- fuzzy R- sub modules necessarily assumes value 1 at zero of given module.

Since P- fuzzy R- sub modules are P- fuzzy normal subgroups. The following result follows from the corresponding theorem in (2).

Proposition 4.8: L_t is a modular lattice of L for each t ε [0,1].

Corollary 4.9: The set L(M) of all sub modules of M forms a modular lattice.

Proof: Let L_χ denote the set of characteristic functions of all the sub modules of M. It is easy to verify that L_χ is a sub lattice of L_t for t =1 . More over L(M) is isomorphic to L_χ under the map A$\rightarrow \chi_A$ are $\chi_{A+B} = \chi_A + \chi_B$.since sub lattice of a modular lattice is a modular lattice. L_t is modular and hence, L(M) is modular.

Conclusion:

N. Ajmal and K. V. Thomas (2) introduced the concept of lattice of fuzzy subgroups and Osman Kazanci, Sultan Yamark and Serife Yimaz (10) introduced notion of the intuitionist P- fuzzy R-subgroups of near rings. In this paper we investigate the notion of Lattice valued P- fuzzy left R- sub modules of near ring w.r.to t-norm and characterization of them.

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