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# ENUMERATIONS OF MAXIMAL CHAINS AND FLAGS OVER FUZZY GROUP STRUCTURES

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**Abstract:**

In this paper, we introduce the new term synonymous to a series, namely a flag. We amongst other things, fix some terminology includes flags, maximal chains, pinned flags and various other terms with respect to finite abelian groups. We also explained how a pinned flag determines a fuzzy group or alternatively how a maximal chains determines a pinned flags. Enumeration of maximal chains of finite abelian groups and some related properties are discussed.

**Key Words:** Key Chains, Flags, Finite Abelian Group Maximal Chains, Equivalent & Pinned Flags

## 1. Introduction:

Some of the past studies of fuzzy groups relied heavily on the usual definition of intersections, union, sum of two fuzzy groups by exploiting the lattice properties of membership values in a simplistic way. On the other hand, one form of representation of fuzzy groups, that proved useful in the literature, is by means of their α – cuts. By refining the chain formed by α – cuts, we get a maximal chain of subgroups which is called flag. The membership values of elements form a decreasing chain of real numbers in the unit interval known as a key chain. A pinned flag is a pair consisting of a flag and keychain. It was observed that in [6] that the pinned flag resulting from operations of intersection and direct sum of fuzzy groups does not for any particular pattern. In section 2, we gather all the preliminaries such as flags, chains, and key chains, pinned flag of fuzzy groups and fix notations. In section 3, given maximal chain of fuzzy groups determines a pinned flags and Enumeration of maximal chains of finite abelian groups

## 2. Preliminaries:

In this section, we discuss the basic concept of flag, maximal chains and pinned flags.

**Definition 2.1:**

A flag is a maximal chain of subgroups of the form $G_o \subset G_1 \subset G_2 \subset \ldots \subset G_n$ in which $G_o = \{0\}$ and $G_n = G_0$ and all the $G_i$'s are called the components of the flag.

We will interchangeably refer to $G_o \subset G_1 \subset G_2 \subset \ldots \subset G_n$ as a flag or a maximal chain.

**Definition 2.2:**

Consider a set of real numbers $\lambda_i$, with i=0,….n in the unit interval I, where the$\lambda_i$'s are not all necessarily distinct. We call a chain of these real numbers as

$$1 = \lambda_0 \geq \lambda_1 \geq \lambda_2 \leq \ldots \geq \lambda_n \geq 0 \qquad (1)$$

A keychain, and all the $\lambda_i$'s are called pins. A keychain always contains $\lambda_0$.

Notice that 1 occupies the first position, and $\lambda_i$ occupies the (i + 1)-$^{th}$ position, for i = 1,2,….n. Hence the length of a keychain is n + 1, and so the n-chain has n+ 1 available position. These positions will play a crucial role in further discussion. Identical key chains, as can be anticipated, have the same length and contain identical pins. For this reason, two key chains are said to be distinct whenever they are either of different lengths or there is a pin which is found in one keychain but not in the other. The combination of these two terms leads us to what we refer as a pinned flag.

**Definition 2.3:**

By a pinned flag on G we mean a flag on G together with a keychain on I written as

$$\{0\}^1 \subset G_1^{\lambda_1} \subset G_2^{\lambda_2} \ldots \subset G_n^{\lambda_n} \text{ , ……. (2)}$$

$G_i^{\lambda i}$, for i = 0,1,2,……n , is called the i$^{th}$ component of pinned flag.

**Example 2.4:** A fuzzy subset $\mu$ associated with a pinned flag in (2) is denoted as follows

$$\mu(x) = \begin{cases} 1 & \text{if } x = 0 \\ \lambda_1 & \text{if } x \in G_1 / \{0\} \\ \lambda_2 & \text{if } x \in G_2 / G_1 \\ \lambda_n & \text{if } x \in G_n / G_{n-1} \end{cases} \qquad (3)$$

**3. Properties of Maximal Chains and Pinned Flag:**

**Proposition 3.1:**

$\mu$ defined by (3) is a fuzzy group.

**Proof:**

For any $a,b \in G$ , there exist indices i, j with the property $1 \le i \le j \le n$ such that $a \in G_i / G_{i-1}$ *and* $b \in G_j / G_{j-1}$. Now a, b are elements of $G_j$, and hence their sum a + b is also in $G_j$ so that $\mu$ (a + b) $\ge \lambda_j$. *But* $\lambda_j \ge \lambda_i \wedge \lambda_j$ and by the way i, j have been defined. It follows that $\mu(a+b) \ge \mu(a) \wedge \mu(b)$. It is easy to show the second part, for if $a \in G$ there is an index I such that $a \in G_i / G_{i-1}$. Now since $-a \in G_i$, $\mu(a) = \lambda_i \ge \mu(-a)$ for some i. Also $\mu(0) \ge \mu(a)$ for all a $a \in G$. This completes the proof that $\mu$ is a fuzzy group. The converse to proposition 3.1 also holds. That is given any fuzzy subgroup $\mu$ of G, and then $\mu$ can be decomposed into a pinned – flag representing $\mu$ as in key chain.

**Proposition 3.2:**

Let $\mu$ be a fuzzy group of G. Then $\mu$ can be decomposed into a pinned flag $\{0\}^1 \subset G_1^{\lambda_1} \subset G_2^{\lambda_2} \ldots . \subset G_n^{\lambda_n}$ such that a key chain holds.Since $\lambda_n$ may or may not be equal to 0, it follows that the support of $\mu$ is strictly contained in G or is equal to G respectively.

**Definition 3.3:**

Let $\mu$ and $\vee$ be any fuzzy sets on $I^x$…. Equivalence on $I^x$ is defined as follows. $\mu \sim \vee$ if and only if for all $x, y \in x, \mu(x) > \mu(y)$ if and only if $\vee(x) > \vee(y)$ and μ(x) = 0 if and only if $\vee(x)$ = 0. The second condition, namely, μ(x) = 0 if and only if $\vee(x)$ = 0 implies that the supports of $\mu$ and $\vee$ are equal.

If fuzzy groups are equivalent, what can be said about their pinned flags? Normally, one would expect that their pinned flags have the same length and that their corresponding components are identical. Indeed this is the case as stated in the following proposition.

**Proposition 3.4:**

Let $\mu$ and $\vee$ be two fuzzy groups. Suppose $X_0^1 \subset X_1^{\lambda_1} \subset X_2^{\lambda_2} \ldots . \subset X_m^{\lambda_m}$ *and* $Y_0^1 \subset Y_1^{\beta_1} Y_2^{\beta_2} \ldots . \subset Y_n^{\beta_n}$ are the corresponding pinned flags to $\mu$ and $\vee$ respectively. Then $\mu \sim \vee$ on G if and only if the following are true.

1. n =m
2. $X_i = Y_i$, for i = 0. 1…… n and for distinct $\lambda_i s$ and distinct $\beta_i s$
3. $\lambda_i > \lambda_j$ if and only if $\beta_i > \beta_j$ for $1 \le i, j \le n$.

**Proof:**

$(\Rightarrow)$ Suppose $\mu \sim \vee$.

1. Define a function $f : \mu(X) \to \vee(X)$ *by* $f(\mu[x]) = \vee(x)$ *for* $x \in X$, where $\mu(X)$, $\vee(X)$ are subsets of I. We need to show that f is well defined and objective. To do this let $x, y \in X$ . Now $\mu(x) > \mu(y)$ if and only if. $\vee(x) > \vee(y)$, since $\mu \sim \vee$. Similarly, $\mu(x) < \mu(y)$ if and only if $\vee(x) < \vee(y)$. Thus $\mu(x) = \mu(y)$ if and only if $\vee(x) = \vee(y)$, that is $f(\mu(x)) = f(\mu(y))$. Hence f is well defined and is one-to-one. Lastly, by the way f has been defined, there exists a $\vee(x) \in \vee(X)$ such that $f(\mu(x)) = \vee(x)$ for all $\mu(x) \in \mu(X)$ and for all x∈X. Hence f surjective. Now f is well defined and bijective. Thus $|\mu(X)| = |\vee(X)|$. Therefore, n = m.

We prove by induction on n. Let n =0, then $Y_0 = X_0$ each set is the empty as each set is the empty subgroup. Assume that $Y_k = X_k$ for $k \ge 0$. We need to show that $Y_{k+1} = X_{k+1}$. To do this we show that $Y_{k+1} \subset X_{k+1}$ and $X_{k+1} \subset Y_{k+1}$. To show the first inclusion, let an element $g \in Y_{k+1}$ and show that $g \in X_{k+1}$. Let $g \in Y_{k+1}$. By assumption, if $g \in Y_k$ than $g \in X_k$ a subset of $X_{k+1}$. If not, that is if $g \notin Y_k$, then there is a $\beta_{k+1}$ such that $\vee(g) = \beta_{k+1}$. Such a g must be an element of $X_{k+1}$. To verify our claim, suppose not, that is suppose $g \notin X_{k+1}$. Then we have that $\mu(g) < \lambda_{k+1}$. Now select a $x \in X_{k+1}$. Such that $x \notin X_k = Y_k$. It follows that $\mu(g) < \mu(x)$ which by equivalence implies that $\vee(g) < \vee(x)$. This implies that $\beta_{k+1} = \vee(g) < \vee(x) = \alpha$, say, where $\beta_k > \alpha$

$>\beta_{k+1}$. From this we learn that x is in the α -cut of $\vee$, and so $x \in \vee^{a} \supseteq Y_k = X_k$. This is a contradiction. Hence $Y_{k+1} \subset X_{k+1}$. The reverse inclusion can be shown is a similar manner. Therefore, by the principle of mathematical induction, (2) holds.

By equivalence, supp $\mu$ =supp $\vee$. Now since $\mu(x) > \mu(y)$ if and only if $\vee(x) > \vee(y)$ and since $X_i = Y_i$, for i = 0.1.., n and for distinct $\lambda_i s$ and distinct $\beta_i s$ by 2, then 3 follows.

$(\Leftarrow)$ Let $\mu$ and $\vee$ be two fuzzy groups satisfying 1, 2 and 3. We need to show that $\mu \sim \vee$. From 1 and 3, supp $\mu$ =supp $\vee$(since $\lambda_k$ =0 if and only if $\beta_k = 0 \ for 1 \le k \le n, and\ m = n)$. Let x, y ∈ supp $\mu$, and suppose that $\mu(x) > \mu(y)$. Then there exist $\lambda_i, \lambda_j$ for some $1 \le i, j \le n$ for which $\mu(x) = \lambda_i$ and $\mu(y) = \lambda_j$. According to 1, n = m and condition 3 says that $\lambda_i > \lambda_j$ if and only if $\beta_i > \beta_j$ for $1 \le i, j \le n$. From these two conditions conclusion is $\vee(x) > \vee(y)$. We have shown that $\mu(x) > \mu(y)$ if and only if $\vee(x) > \vee(y)$. Thus $\mu \sim \vee$.

The following example illustrates how we classify the fuzzy groups of a given group.

**Example 3.5:**

Let $G = Z_{16}$. $Define\ \mu : G \to I\ as\ follows$

$$\mu(x) = \begin{cases} 1 & if\ x = 0 \\ \frac{1}{2} & if\ x \in Z_2 / \{0\} \\ \frac{1}{4} & if\ x \in Z_4 / Z_2 \\ \frac{1}{8} & if\ x \in Z_8 / Z_4 \\ \frac{1}{16} & if\ x \in Z_{16} / Z_8 \end{cases} \quad (4)$$

$\mu$ can be represented by

$$\{0\} \subset Z_2^{1/2} \subset Z_4^{1/4} \subset Z_8^{1/8} \subset Z_{16}^{1/16} \quad (5)$$

Where 1, ½ , etc are the weights in each subgroup.

All fuzzy groups $\vee$ that are equivalent to $\mu$ can be represented by

$$\vee : \{0\}^1 \subset Z_2^{\lambda} \subset Z_4^{\beta} \subset Z_8^{\gamma} \subset Z_{16}^{\delta} \quad (6)$$

Such that

$$1 > \lambda > \beta > \gamma > \delta > 0. \quad (7)$$

We simply denote the keychain defined in 2.2 by $1\lambda_1\lambda_2 \ldots \lambda_{n-1}\lambda_n$ in the descending order, where the last entry may or may not be zero.

Consider the pinned flag

$$\{0\}^1 \subset G_1^{\lambda_1} \subset G_2^{\lambda_2} \subset G_3^{\lambda_3}\ with\ 1 \ge \lambda_1 \ge \lambda_2 \ge \lambda_3 \ge 0. \quad (8)$$

Corresponding to the flag there are $15=2^4-1$ distinct equivalent classes of fuzzy groups. The key chains for such fuzzy groups arise from all the possibilities in (8) and can be written out thus 1111, 111 $\lambda_1$, 1110, 11 $\lambda_1$ $\lambda_1$, 11 $\lambda_1$ $\lambda_2$,…1000. We choose to denote the number of fuzzy groups in the form $15 = 2^4-1 = 2^r$, r = 0,1,2,3 and make on assumption that $1 > \lambda_1 > \lambda_2 > \lambda_3 > 0$ . The number of fuzzy groups whose support is strictly $G_3$ is one more than the number of fuzzy groups whose support is properly contained in $G_3$. Hence we can write the number of fuzzy groups as

$$15 = 2^4 - 1 = \sum_{r=0}^{3} 2^r = 2^4/2 + 2^4/2 - 1. \quad (9)$$

The number of fuzzy groups whose support is $G_i$, for i =0, 1, 2, 3, correspond to the components (8, 4, 2, 1) of the partition of 15.

If a flag is extended by one component, the effect on the resulting number of distinct equivalent classes of fuzzy groups can be explained in the following way. Each fuzzy group of $G_{i-1}$ whose pins are all different

from zero yields three fuzzy subgroups of $G_i$. Each of the remaining key chains gives rise to one fuzzy subgroup of $G_n$ by simply attaching 0 to the keychain.

**Lemma 4.1:**

The group $G = Z_p \times Z_p$ has p + 1 maximal chains.

**Proof:**

The group $Z_p \times Z_p$ is a non cyclic group of order $p^2$ and is abelian. The group has subgroups of order 1, p, p2, where the subgroup of order 1 is the trivial subgroup while that of order $p^2$ is the whole group. A group of order p cannot have a proper subgroup. so it is a cyclic group whose generator is any element different from the identity. The subgroups of order p are all cyclic and disjoint, with their generators being (1,0), (1,1)…(1,p-1), (0,1). The whole group is described by the linear combination of the subgroups of order p. Hence the group $Z_p \times Z_p$ has maximal chains of the form $0 \subset \langle a \rangle \subset G.$ From this, it follows that $Z_p \times Z_p$ has p + 1 maximal chains of subgroups.

Secondly for other values of n, $m \in z^+$, we state the following propositions and prove them.

**Lemma 4.2:**

There are $2p^2+3p+1$ maximal chains in the subgroup lattice of $G = \in Z_{p^2} \times Z_{p^2}$

**Proof:**

Since $Z_{p^2} \times Z_{p^2}$ is a non-cyclic group of order $p^4$, it has subgroups of order k, where k is a divisor of $p^4$. Hence each maximal chain of $Z_{p^2} \times Z_{p^2}$ is a chain of the form

$$\{0\} \subset \langle a \rangle \subset \langle b \rangle \subset \langle c \rangle \subset G,$$

Where a, b, and c are cyclic of order p, $p^2$,$p^3$ respectively. Now $\langle a \rangle$ is representative of the subgroup generators of the form 0p;p(ip), for i = 0,1…..p-1 whose intersection contains the identity alone. It is easy to see that there are only p + 1 such generators. The subgroup generators of the form $\langle b \rangle$ fall into different slots as shown on the table below. Again it is not difficult to see that there are p + 1 such slot each with a membership of p + 1. It should be noted that in this count one group, namely one generated by $\langle 0p, p0 \rangle$, has been counted more than once. The equation (p+1) (p+1) - p= $p^2$ + p + 1 fixes the over count. This means that $\langle b \rangle$ is comprised of $p^2$+p cyclic and one non-cyclic subgroups.

The intersection. $\cap \langle b \rangle_j / \langle 0p, p0 \rangle$, j =0,1,….p, is empty. With the exception of the non-cyclic $\langle 0p, p0 \rangle$ which contains all the subgroups $\langle a \rangle$ and is itself contained in all $\langle c \rangle$, the each of rest of the subgroups in each slot is contained in only one subgroups of $\langle c \rangle$ respectively. By counting, there are p+1 subgroup generators comprising $\langle c \rangle$ which is itself contained in G. Collectively, the result is (p+1) (p+1) (p+1)) maximal chains of subgroups as claimed in the proposition.

Alternatively, the maximal chains of the group G can view as extensions of the maximal chains of the previous groups in this manner. The maximal chains of G are extensions of $Z_{p2} \times Z_p$ or $Z_p \times Z_p$ or $Z_p$. Now $Z_{p2} \times Z_p$ contributes 2p+1 maximal chains to G. These are the chains that go through, <op.po>.

Next we consider the chains that go though $\langle 0p, p0 \rangle$ as such chains are the extensions $Z_p \times Z_p$. In this effect we obtain p ways to do this, with the path through <op.po> having been considered. Hence it has p(p+1) such extensions. Finally $Z_p$ contributes p(p) maximal chains not having been considered before, as can be checked. Thus the total number of maximal chains of G is (2p+1)+p(p+1)+p(p) = $2p^2$+3p+1.as required. This completes the proof.

| Level | Subgroups | range |
|---|---|---|
| 0 | 1 | |
| $\langle a \rangle \equiv 0p; p(\alpha p)$ | p+1 | α=0,1….p-1 |
| $\langle b \rangle \equiv 01; \langle 0p, p0 \rangle; p\beta$<br>$\langle b \rangle \equiv \langle 0p, p0 \rangle; 1(0+\alpha p)$<br>$\langle b \rangle \equiv \langle 0p, p0 \rangle; 1(0+\alpha p)$<br>: : : | $p^2$+p+1 | $\beta = 1....p-1$<br>α = 0,1….p-1 |

| | | |
|---|---|---|
| $\langle b \rangle \equiv \langle 0p,\ p0 \rangle;\ 1((p-1)+\alpha p)$ | | |
| $\langle c \rangle \quad \langle 01,\ p0 \rangle; \langle 0p, p0 \rangle; \langle 1\delta, 1(\delta + p) \rangle$ | p +1 | $\delta = 1 \ldots p-1$ |

Subgroups of $Z_{p^2} \times Z_{p^2}$

**Proposition 4.3:**

Let G = $Z_{p^n} \times Z_p$ . Then the number of maximal chains of G is np +1.

**Proof:**

We prove by induction on n. When n=1 the result resembles Lemma 5.2.1. Assume that $Z_{p^k} \times Z_p$ has kp+1 maximal chains. Now $Z_{p^{k+1}} \times Z_p$ have two sets of maximal subgroups. They are $Z_{p^k} \times Z_p$ and $Z_{p^{k+1}} \times Z_p \subset \langle b \rangle \subset Z_{p^{k+1}} \times Z_p$, where $\langle b \rangle$ has subgroup generators of the form (10), (1,1),…(1,p-1) as illustrated in the diagram. Hence $Z_{p^{k+1}} \times Z_p \subset \langle b \rangle \subset Z_{p^{k+1}} \times Z_p$ contributes p maximal chains. Thus $Z_{p^{k+1}} \times Z_p$ has kp+1+p=(k+1)+p maximal chains. This completes the induction.

Where A = $\langle p^{n-1}, p-1 \rangle;\ B = \langle p^{n-1} 2 \rangle;\ C = \langle p^{n-1} 1 \rangle;$

D = $\langle p^{n-2}, p-1 \rangle;\ E = \langle p^{n-2} 2 \rangle;\ F = \langle p^{n-2} 1 \rangle$ .

**Proposition 4.4:**

G = $Z_{p^n} \times Z_{p^2}$ has [C(n,2)+C(n-1,1)]$p^2$+C(n+1,1)p+1 maximal chains.

**Proof:**

We prove by induction on n. Let C(a,b) = 0 for a<b.

For n =2, G has $2p^2$+3p+1 maximal chains, in agreement with Lemma 4.2.

These maximal chains are of the form $0 \subset \langle a \rangle \subset \langle b \rangle \subset \langle c \rangle \subset G$

Where $\langle a \rangle, \langle b \rangle, \langle c \rangle$ have the form explained in Table 2.

Assume that G= $Z_{p^k} \times Z_{p^2}$ has [C(k,2)+C(k-1,1)]$p^2$+C(k+1,1)p+1 maximal chains, where $k \geq 2.$ We need to show that the result is true for n = k +1. Every maximal chain of $Z_{p^{k+1}} \times Z_{p^2}$ can be considered as an extension of maximal chains of $Z_{p^k} \times Z_{p^2}$ or $Z_{p^k} \times Z_p$ or $Z_{p^k}$ (by using double lines to mark the diagram that has been extended). We first determine the number of maximal chains of $Z_{p^{k+1}} \times Z_{p^2}$ which are extensions of G = $Z_{p^k} \times Z_{p^2}$ . By assumption there are [C(k,2)+C(k-1,1)]$p^2$+C(k+1,1)p+1such maximal chains. Such chains only go through the generator 1. (For ease of reference digits to represent the nodes in question are used). We next consider those which are extensions of $Z_{p^{k+1}} \times Z_p$. Clearly these are the chains that pass through generator 2. Such chains give rise to p chains $Z_{p^{k+1}} \times Z_{p^2}$ as the path through 1 has already been counted. Thus the number of this extension is p(kp+1). Finally we consider the extensions of $Z_{p^k}$ . The extensions through the nodes 3,4,5 have not been considered, and they constitute the extensions of $Z_{p^k}$ . Each of these p nodes rise to p maximal chains of $Z_{p^{k+1}} \times Z_{p^2}$ with those paths through 1 and 2 having been accounted for already. This exhausts all the possible extensions. Thus the total number of maximal chains of $Z_{p^{k+1}} \times Z_{p^2}$ is

[C(k,2)+C(k-1,1)]$p^2$+C(k+1,1)p+1+p(kp+1)+p(p) (10)
= [C(k,2)+C(k-1,1)+k+1]$p^2$+C(k+1,1)+1]p+1
= [C(k+1,2)+C(k,1)]$p^2$+C(k+2,1)p+1, as required.

This completes the induction. Note that the number of maximal chains in Proposition 4.4 can also be counted using the formula $\sum_{r=2}^{n} p(rp+1) + (2p+1).$ The two formulas are identical as shown below:

$[C(n,2)+C(n-1,1)]p^2+C(n+1,1)p+1 = \sum_{r=2}^{n} p(rp+1)+(2p+1).$ for all natural numbers n such that $n \geq 2.$

To prove this, let n =2. Then

$$[C(2,2)+C(1,1)]p^2+C(3,1)p+1 = \sum_{r=2}^{2} p(rp+1)+(2p+1)$$

Assume that the statement is true for n=k>2, i.e

$$[C(k,2)+C(k-1,1)]p^2+C(k+1,1)p+1=\sum_{r=2}^{n} p(rp+1)+(2p+1)$$

We show that the statement is true for n=k+1. Now,

$$\sum_{r=2}^{k+1} p(rp+1)+(2p+1) = \sum_{r=2}^{k} p(rp+1)+(2p+1)+p[k+1)p+1]$$

$$= [C(k,2)+C(k-1,1)]p^2+C(k+1,1)p+1+(k+1)p^2+p$$

$$= [C(k+1,2)+C(k,1)]p^2+C(k+2,1)p+1.$$

By the Principle of Mathematical induction, the statement holds for all $n \in N$.

We state this result as a corollary.

**Corollary 4.5:**

G = $Z_{p^n} \times Z_{p^2}$ has $\sum_{r=2}^{n} p(rp+1)+(2p+1)$ maximal chains rε N

This can be proved by similar argument as in proposition 4.4.

For m =3 and any n, it follows that

**Proposition 4.6:**

Let G = $Z_{p^n} \times Z_{p^3}$ where p is a prime. Then the number of maximal chains of G is $[C(n,3)+2C(n-1,2)+2C(n-2,1]p^3+[C(n+1,2)+C(n,1)]p^2+C(n+2,1)p+1$.

**Proof:**

The proof follows by the same argument given in Propositions 4.3 and 4.4 above. In fact the results appear in the following sequence, with the +1 term in all expressions on the right column not shown

| Group | Number of Maximal Chains |
|---|---|
| pnp1 | C(n,1)p |
| pnp2 | $[C(n,2)+C(n-1,1)]p^2+p(n+1)p$ |
| pnp3 | $[C(n,3)+2C(n-1,2)+2C(n-2,1)]p^3+p(n+1)p^2$ |
| : | : |
| pnpm | $\left\{\sum_{j=0}^{m-1}[(n-j,\ m-j)]*Tm\right\}\ p^m + P(n+1)p(m-1)$ *with* $p(p+m-1)p=(n+m-1)p$ |

**Conclusion:**

The Partition Problem of finite abelian groups introduced by [8] and Equivalence of fuzzy subgroups by [4] & [6] . In this paper, we investigate the notion of maximal chains and its enumeration with respect to finite abelian groups and characterization of them.

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