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NEW DIMENSIONS OF SOFT STRUCTURES OVER FUZZY SOFT LATTICE

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Abstract:

In this paper, we introduce a new kind of soft ring called Lattice fuzzy soft intersection action on a ring. We then focused on the concepts of Lattice fuzzy soft pre image, union and intersection of a soft set. Also, we derive its various related properties. We then study and discuss its structural characteristics.

Index Terms: Soft Set, Fuzzy Soft Ring, Non Null Soft Set, Fuzzy Soft Function & Homomorphisms

Introduction:

L.A. Zadeh [33] introduced the notion of a fuzzy subset μ_A set S as a function from X into I = [0, 1]. Rosenfeld [28] applied this concept in group theory and semi group theory, and developed the theory of fuzzy subgroups and fuzzy sub semi], replaced the valuations set [0, 1], by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. In fact it seems in order to obtain a complete analogy of crisp mathematics in terms of fuzzy mathematics, it is necessary to replace the valuation set by a system having more rich algebraic structure. These concepts -groups play a major role in mathematics and fuzzy mathematics. G.S.V Satya Saibaba [29] introduced the concept of L- fuzzy λ-group and L-fuzzy λ-ideal of λ-group .To solve complex problems in economy, engineering, environmental science and social science, the methods in classical mathematics may not be successfully modeled because of various types of uncertainties. There are some mathematical theories for dealing with uncertainties such as; fuzzy set theory [32], soft set theory [21], fuzzy soft set theory [17] and so on. Soft set theory [21] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertainty. At present, research works on the soft set theory and its applications are making progress rapidly. The operations of soft sets are defined in [1, 7, 18, 22]. The algebraic structures of soft sets have been studied by some authors [3, 10, 13, 14, 15, 16, 17]. By embedding the ideas of fuzzy sets, many interesting applications of soft set theory have been expanded [6, 8, 28, 29, 17, 22, 25, 26, 27, 30]. And also algebraic structures of fuzzy soft sets have been studied [4, 33, 18, 20, 31, 23].

2. Preliminaries:

Throughout this paper R denotes a commutative ring and all fuzzy soft sets are considered over R.

2.1 Definition: A pair (f, A), is called a soft set over the lattice L, If f: A → P(L). Here L be the initial universe and E be the set of parameters. Let P(L) denotes the power set of L and I^L denotes the set of all fuzzy sets on L.

2.2 Definition: A pair (f, A is called a fuzzy soft set over L, where f: $A \to I^L$, ie. for each a ϵ A, f_a: L → I is a fuzzy set in L.

2.3 Definition: Let (f, A) be a non-null soft set over a ring R. Then (f, A) is said to be a soft ring over R if and only if f(a) is sub ring of R for each a ϵ A.

2.4 Definition: Let (f, A) be a non Null fuzzy soft set over a ring R.Then (f, A) is called a fuzzy soft ring over R If and only if f(a) = f_a is a fuzzy sub ring of R. for each a ϵ A.

(FSR1) $f_a(x + y) \geq T\{f_a(x), f_a(y)\}$
(FSR2) $f_a(-x) \geq f_a(x)$
(FSR3) $f_a(xy) \geq T\{f_a(x), f_a(y)\}$,for all x, y ϵ R

2.5 Definition: Let (f, A) be a fuzzy soft set over L. The soft set $(f, A)_\alpha = \{(f_a)_\alpha / a \in A\}$ for each α ϵ [0, 1] is called α–level soft set.

2.6 Definition: Let f_a be a fuzzy soft ring in R. Let θ: R → R´ be a map and define

$$f_a^\theta(x) = f_a(\theta x) \text{ by defining } f_a^\theta: R \to [0, 1]$$

2.7 Definition: Let Φ: X → Y and ψ: A →B be two functions, where A and B are parameter sets for the crisp sets X and Y respectively.

Then the pair (Φ, ψ) is called a fuzzy soft function from X to Y.

2.8 Definition: The pre-image of (g, B) under the fuzzy soft function (Φ, ψ) denoted by $(\Phi, \psi)^{-1}$ is the fuzzy soft set defined by $(\Phi, \psi)^{-1}(g, B) = (\Phi^{-1}(g), \psi^{-1}(B))$.

2.9 Definition: Let (Φ, ψ): X →Y is a fuzzy soft function, if Φ is a homomorphism from X → Y then (Φ, ψ) is said to be fuzzy soft homomorphism. If Φ is a isomorphism from X → Y and ψ is 1 – 1 mapping from A on to B then (Φ, ψ) is said to be fuzzy soft isomorphism.

3. Some Important Results:

3.1 Proposition:

Let f_a be a fuzzy soft ring of R then

(i) $f_a(x) \le f_a(0)$ for all $x \in R$, the subset

(ii) $Rf_a = \{x \in R / f_a(x) = f_a(0)\}$ is a fuzzy soft ring of R.

Proof:

Let $x \in G$, then

(FSR1) $f_a(x + y) = T\{f_a(x), f_a(y)\} = T\{f_a(x), f_a(-x)\}\ f_a((x) + (-x)) \le f_a(0)$

This implies subdivision (i). To verify subdivision (ii), it follows that $x \in R\ f_a$ and $R\ f_a \ne \Phi$.

Now let x, y $\in$R, then (FSR2) $fa(x + (-y)) \ge T\{f_a(x), f_a(-y)\} = T\{f_a(x), f_a(y)\} = T\{f_a(0), f_a(0)\} = f_a(0)$

But, from subdivision (i), $f_a(x) \le f_a(0)$ for all x, y $\in$ R and so $f_a(x + (-y)) = f_a(0)$. Which then $x + (-y)$ x, y $\in$ R f_a is fuzzy soft ring of R.

(FSR3) Let x, y $\in Rf_a$ and setting y = x, then $f_a(xy) \ge T\{f_a(x), f_a(y)\} = T\{f_a(x), f_a(x)\} = T\{f_a(0), f_a(0)\} = f_a(0)$

Therefore f_a is a fuzzy soft ring over R.

3.2 Corollary:

Let R be a finite ring and f_a be a fuzzy soft ring of R. Consider the subset H of R given by $H = \{x \in R / f_a(x) = f_a(0)$ .Then H is called crisp sub ring of R.

3.3 Proposition:

Let f_a and f_b be two fuzzy soft rings of R then $f_a \cap f_b$ is fuzzy soft ring of R.

Proof:

Let x, y $\in$R

(FSR1)
$$\begin{aligned}(f_a \cap f_b)(x + y) &= T\{f_a(x + y), f_b(x + y)\}\\ &\ge T\{T f_a(x), f_a(y)\}, T\{f_b(x), f_b(y)\}\\ &\ge T\{T\{f_a(x), f_a(y), f_b(x), f_b(y)\}\}\\ &\ge T\{T\{f_a(x), f_b(y)\}, T\{f_a(y), f_b(y)\}\}\\ &\ge T\{(f_a \cap f_b)(x), (f_a \cap f_b)(y)\}\end{aligned}$$

FSR1 satisfied in R

(FSR2) $(f_a \cap f_b)(-x) = T\{f_a(-x), f_b(-x)\} \ge T\{f_a(x), f_b(x)\} \ge \{(f_a \cap f_b)(x)$

(FSR3)
$$\begin{aligned}(f_a \cap f_b)(xy) &= T\{f_a(xy), f_b(xy)\}T\{T\{f_a(x), f_a(y)\}, T\{f_b(x), f_b(y)\}\}\\ &\ge T\{T\{f_a(x), f_a(y)\}, f_b(x), f_b(y)\}\}\\ &\ge T\{T\{f_a(x), f_b(x)\}, T\{f_a(y), f_b(y)\}\}\\ &\ge T\{(f_a \cap f_b)(x), (f_a \cap f_b)(y)\}\end{aligned}$$

Hence $f_a \cap f_b$ is fuzzy soft ring of R

3.4 Proposition:

If f_a is fuzzy soft ring of R then the non-empty level subset $U(f_a; t)$ is fuzzy soft ring for all $t \in I_m(f_a)$.

Proof:

Let f_a be a fuzzy soft ring and $t \in I_m(f_a)$.

Now x, y $\in U(f_a; t)$, we have $f_a(x) \ge t$, $f_a(y) \ge t$.

(FSR1) $f_a(x + y) \ge T\{f_a(x), f_a(y)\} \ge T\{t, t\} \ge t$ So, which implies $x + y \in U(f_a; t)$.

(FSR2) $f_a(-x) \ge f_a(x) \ge t$ Therefore $-x \in U(f_a; t)$.

(FSR3) $f_a(xy) \ge T\{f_a(x), f_a(y)\} \ge T\{t, t\} \ge t$. Therefore x, y $\in U(f_a; t)$.

3.5 Proposition:

If f_a is a fuzzy soft ring of R and defined by $f_a^+(x) = f_a(x) + 1 - f_a(0)$, for all $x \in R$, then $f_a^+(x)$ is normal fuzzy soft ring of R which contains f_a.

Proof:

For any x, y $\in$ R. It follows that $f_a^+(x) = f_a(x) + 1 - f_a(0)$

(FSR1) $f_a^+(x + y) = f_a(x + y) + 1 - f_a(0) \ge T\{\{f_a(x), f_a(y)\} + 1 - f_a(0)\} \ge T\{\{f_a(x) + 1 - f_a(0), f_a(y) + 1 - f_a(0),\} \ge T\{f_a^+(x), f_a^+(y)\}$

(FSR2) $f_a^+(-x) = f_a(-x) + 1 - f_a(0) \ge f_a(x) + 1 - f_a(0) \ge f_a^+(x)$

(FSR3) $f_a^+(x y) = f_a(x y) + 1 - f_a(0) \ge T\{\{f_a(x), f_a(y)\} + 1 - f_a(0)\} \ge T\{\{f_a(x) + 1 - f_a(0), f_a(y) + 1 - f_a(0)\} \ge T\{f_a^+(x), f_a^+(y)\}$

It gives that f_a^+ is normal fuzzy soft ring of R.

3.6 Proposition:

If $\{f_{a_i}\}_{i \in f_a}$ is a family of fuzzy soft rings of R, then $\cap f_{a_i}$ is fuzzy soft ring of R, whose $f_{a_i} = \{(x, \Lambda f_{a_i}(x) / x \Lambda R\}$, where $i \in f_a$.

Proof:

Let x, y $\in$ R, then for $i \in f_a$ It follows that

(FSR1) $\cap f_{a_i}(x + y) = \Lambda f_{a_i}(x + y) \ge T\{f_{a_i}(x), f_{a_i}(y)\} \ge T\{\Lambda f_{a_i}(x), \Lambda f_{a_i}(y)\} \ge T\{\cap f_{a_i}(x), \cap f_{a_i}(y)\}$

(FSR2) $\cap f_{a_i}(-x) = \wedge f_{a_i}(-x) \geq \wedge f_{a_i}(x) \geq \cap f_{a_i}(x)$

(FSR3) $\cap f_{a_i}(xy) = \wedge f_{a_i}(xy) \geq T\{f_{a_i}(x), f_{a_i}(y)\} \geq T\{\wedge f_{a_i}(x), \wedge f_{a_i}(y)$

$\geq T\{\cap f_{a_i}(x), \cap f_{a_i}(y)\}$. Therefore, $\cap f_{a_i}$ is fuzzy soft ring of R.

3.7 Proposition:

If f_a is a fuzzy soft ring of R then f_a^c is a fuzzy soft ring of R.

Proof:

Let x, y ϵ R. then

(FSR1) $f_a^c(x+y) = 1 - f_a(x+y) \geq 1 - T\{f_a(x), f_a(y)\}$

$\leq S\{1 - f_a(x), 1 - f_a(y)\} \leq S\{f_a^c(x), f_a^c(y)\}$

(FSR2) $f_a^c(-x) = 1 - f_a(-x) \leq 1 - f_a(x) \leq f_a^c(x)$

(FSR3) $f_a^c(xy) = 1 - f_a(xy) \leq 1 - T\{f_a(x), f_a(y)\}$

$\leq S\{1 - f_a(x), 1 - f_a(y)\} \leq S\{f_a^c(x), f_a^c(y)\}$

Therefore, f_a^c is fuzzy soft ring of R.

3.8 Proposition:

Let f_a and f_b be two fuzzy soft ring of R then $f_a \cup f_b$ is fuzzy soft ring of R.

Proof:

Let x, y ϵR, then

(FSR1) $(f_a \cup f_b)(x+y) = S\{f_a(x+y), f_b(x+y)\}$

$\geq S\{T\{f_a(x), f_b(y)\}, T\{\{f_b(x), f_b(y)\}\}$

$\geq T\{S\{f_a(x), f_a(y), f_b(x), f_b(y)\}\}$

$\geq T\{S\{f_a(x), f_b(x)\}, S\{f_a(y), f_b(y)\}\}$

$\geq T\{(f_a \cup f_b)(x), (f_a \cup f_b)(y)\}$

(FSR2) $(f_a \cup f_b)(-x) = S\{f_a(-x), f_b(-x)\} \geq S\{f_a(x), f_b(x)\}$

$\geq (f_a \cup f_b)(x)$

(FSR3) $(f_a \cup f_b)(xy) = S\{f_a(xy), f_b(xy)\}$

$\geq S\{T\{f_a(x), f_a(y)\}, T\{f_b(x), f_b(y)\}\}$

$\geq T\{S(f_a(x), f_a(y), f_b(x), f_b(y)\}$

$\geq T\{S\{f_a(x), f_b(x)\}, S\{f_a(y), f_b(y)\}\}$

$\geq T\{(f_a \cup f_b)(x), (f_a \cup f_b)(y)\}$

Therefore, $f_a \cup f_b$ is fuzzy soft ring of R.

3.9 Proposition:

Let f_a and f_b be two fuzzy soft rings of R then $f_a \times f_b$ is fuzzy soft ring of R.

Proof:

Let x, y ϵ R. then

(FSR1) $(f_a \times f_b)(x+y) = T\{f_a(x+y), f_b(x+y)\}$

$\geq T\{T\{f_a(x), f_a(y)\}, T\{\{f_b(x), f_b(y)\}\}$

$\geq T\{T\{f_a(x), f_a(y), f_b(x), f_b(y)\}\}$

$\geq T\{T\{f_a(x), f_b(x)\}, T\{f_a(y), f_b(y)\}\}$

$\geq T\{(f_a \times f_b)(x), (f_a \times f_b)(y)\}$

(FSR2) $(f_a \times f_b)(-x) = T\{f_a(-x), f_b(-x)\} \geq T\{f_a(x), f_b(x)\} \geq (f_a \times f_b)(x)$

(FSR3) $(f_a \times f_b)(xy) = T\{f_a(xy), f_b(xy)\}$

$\geq T\{T\{f_a(x), f_a(y)\}, T\{\{f_b(x), f_b(y)\}\}$

$\geq T\{T\{f_a(x), f_a(y), f_b(x), f_b(y)\}\}$

$\geq T\{T\{f_a(x), f_b(x)\}, T\{f_a(y), f_b(y)\}\}$

$\geq T\{(f_a \times f_b)(x), (f_a \times f_b)(y)\}$

Therefore, $f_a \times f_b$ is fuzzy soft ring of R.

3.10 Proposition:

If $f_{a_1}, f_{a_2}, \dots, f_{a_n}$ be fuzzy soft ring of the rings $R_1, R_2, \dots, R_n$ respectively then $f_{a_1} \times f_{a_2} \times \dots \times f_{a_n}$ is fuzzy soft ring of $R_1 \times R_2 \times \dots \times R_n$.

Proof:

Let $X = (x_1, x_2, \dots, x_n)$, $Y = (y_1, y_2, \dots, y_n) \in R$

(FSR1) $(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(X + Y) = (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})((x_1, x_2, \dots, x_n) + (y_1, y_2, \dots, y_n))$

$= (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(x_1 + y_1, x_2 + y_2, \dots, x_n + y_n)$

$= T\{ f_{a_1}(x_1 + y_1), f_{a_2}(x_2 + y_2), \dots, f_{a_n}(x_n + y_n)\}$

$\geq T\{ T\{f_{a_1}(x_1), f_{a_1}(y_1)\}, T\{f_{a_2}(x_2), f_{a_2}(y_2)\}, \dots, T\{f_{a_n}(x_n), f_{a_n}(y_n)\}\}$

$\geq T\{ T\{f_{a_1}(x_1), f_{a_2}(x_2) \dots, f_{a_n}(x_n), f_{a_1}(y_1), f_{a_2}(y_2) \dots, f_{a_n}(y_n)\}\}$

$\geq T\{ T\{f_{a_1}(x_1), f_{a_2}(x_2) \dots, f_{a_n}(x_n)\}, T\{f_{a_1}(y_1), f_{a_2}(y_2) \dots, f_{a_n}(y_n)\}\}$

$\geq T\{ (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(x_1, x_2, \dots, x_n), (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(y_1, y_2, \dots, y_n)\}$

FSR1 satisfied in $R_1 \times R_2 \times \dots \times R_n$.

(FSR2) $(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(-X) = (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(-(x_1, x_2, \dots, x_n))$

$= (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(-x_1, -x_2, \dots, -x_n)$

$= T\{ f_{a_1}(-x_1), f_{a_2}(-x_2), \dots, f_{a_n}(-x_n)\}$

$\geq T\{f_{a_1}(x_1), f_{a_2}(x_2), \dots, f_{a_n}(x_n)\}$

$\geq (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(x_1, x_2, \dots, x_n)$

$\geq (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(X)$

(FSR3) $(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(xy) = (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})((x_1, x_2, \dots, x_n)\ (y_1, y_2, \dots, y_n))$

$= (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(x_1 y_1, x_2 y_2, \dots, x_n y_n)$

$\geq T\{ f_{a_1}(x_1 y_1), f_{a_2}(x_2 y_2), \dots, f_{a_n}(x_n y_n)\}$

$\geq T\{ T\{f_{a_1}(x_1), f_{a_1}(y_1)\}, T\{f_{a_2}(x_2), f_{a_2}(y_2)\}, \dots, T\{f_{a_n}(x_n), f_{a_n}(y_n)\}\}$

$\geq T\{ T\{f_{a_1}(x_1), f_{a_2}(x_2) \dots, f_{a_n}(x_n), f_{a_1}(y_1), f_{a_2}(y_2) \dots, f_{a_n}(y_n)\}\}$

$\geq T\{ T\{f_{a_1}(x_1), f_{a_2}(x_2) \dots, f_{a_n}(x_n)\}, T\{f_{a_1}(y_1), f_{a_2}(y_2) \dots, f_{a_n}(y_n)\}\}$

$\geq T\{ (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(x_1, x_2, \dots, x_n), (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(y_1, y_2, \dots, y_n)\}$

$\geq T\{ (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(X), (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(Y)\}$

Therefore, $(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n}$ is fuzzy soft ring of $R_1 \times R_2 \times \dots \times R_n$.

3.11 Proposition:

Let R and R´ be two ring and $\theta: R \to R'$ be a soft homomorphism. If f_b is a fuzzy soft ring of R then the pre-image $\theta^{-1}(f_b)$ is fuzzy soft ring of R.

Proof:

Assume that f_b is fuzzy soft ring of R´. Let x, y ϵ R.

(FSR1) $\mu_{\theta^{-1}(f_b)}(x + y) = \mu_{f_b}\theta(x + y) = \mu_{f_b}(\theta x + \theta y) \geq T\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\}$

$\geq T\{ \mu_{\theta^{-1}(f_b)}(x), \mu_{\theta^{-1}(f_b)}(y)$

(FSR2) $\mu_{\theta^{-1}(f_b)}(-x) = \mu_{f_b}\theta(-x) \geq \mu_{f_b}\theta(x) \geq \mu_{\theta^{-1}(f_b)}(x)$

(FSR3) $\mu_{\theta^{-1}(f_b)}(x\,y) = \mu_{f_b}\theta(x\,y) = \mu_{f_b}(\theta x)(\theta y)) \geq T\{\mu_{f_b}\theta(x), \mu_{f_b}\theta(y)\} \geq T\{ \mu_{\theta^{-1}(f_b)}(x), \mu_{\theta^{-1}(f_b)}(y)\}$.

3.12 Proposition:

Let $\theta: R \to R'$ be an epimorphism and f_b be fuzzy soft set in R´. If $\theta^{-1}(f_b)$ is fuzzy soft ring of R´ then f_b is fuzzy soft ring of R.

Proof:

Let x, y ϵ R. Then there exist a, b ϵ R such that θ (a) = x, θ (b) = y. It follows that.

(FSR1) $\mu_{\theta(f_b)}(x + y) = \mu_{f_b}\theta(x + y) = \mu_{f_b}(\theta x + \theta y) \geq T\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\}$

$\geq T\{ \mu_{\theta(f_b)}(x), \mu_{\theta(f_b)}(y)\}$

(FSR2) $\mu_{\theta(f_b)}(-x) = \mu_{f_b}\theta(-x) \geq \mu_{f_b}\theta(x) \geq \mu_{\theta(f_b)}(x)$

(FSR3) $\mu_{\theta(f_b)}(x\,y) = \mu_{f_b}\theta(x\,y) = \mu_{f_b}(\theta x)(\theta y))$

$\geq T\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\}$

$\geq T\{\mu_{f_b}\theta(x), \mu_{f_b}\theta(y)\}$

$\geq T\{\mu_{\theta(f_b)}(x), \mu_{\theta(f_b)}(y)\}$

Therefore $\theta(f_b)$ is fuzzy soft ring of R.

4. Fuzzy Soft Homomorphisms:

4.1Proposition:

If f_a is fuzzy soft ring of R and θ:R $\rightarrow$ R′ be a soft homomorphism of R then the fuzzy soft set f_a^θ = $\{(x, f_a^\theta(x)) / x \in R\}$ is fuzzy soft ring of R.

Proof:

Let x, y ϵ R.

(FSR1) $f_a^\theta(x+y) = f_a\,\theta(x+y) = f_a(\theta x + \theta y) \geq T\{f_a(\theta x), f_a(\theta y)\} \geq T\{f_a^\theta(x), f_a^\theta(y)\}$

(FSR2) $f_a^\theta(-x) = f_a(\theta(-x)) \geq f_a(\theta x)) \geq f_a^\theta(x)$

(FSR3) $f_a^\theta(x\,y) = f_a(\theta(x\,y)) = f_a((\theta x)(\theta y)) \geq T\{f_a(\theta x), f_a(\theta y)\} \geq T\{f_a^\theta(x), f_a^\theta(y)\}$

Therefore, f_a^θ is fuzzy soft ring of R.

4.2 Proposition:

Let f_a be a fuzzy soft set over L, then f_a is fuzzy soft ring over L if and only if for all a ϵ A and for arbitrary $\alpha \in [0, 1]$ with $(f_a) \neq 0$, then α– level soft set (f_a) is fuzzy soft ring over L.

Proof:

Let f_a be fuzzy soft ring over L. Then for each a ϵ A, f_a is a fuzzy sub ring of L. For arbitrary $\alpha \in [0, 1]$, $(f_a) \neq 0$. Let x, y ϵ (f_a). Then $f_a(x) \geq \alpha$ and $f_a(y) \geq \alpha$.

(FSR1) $(f_a)_\alpha(x+y) \geq T\{(f_a)(x), (f_a)(y)\} \geq T\{f_a(x), f_a(y)\} \geq T\{\alpha, \alpha\} \geq \alpha$

Therefore, $x + y \in (f_a)_\alpha$

(FSR2) $(f_a)_\alpha(-x) \geq \{(f_a)_\alpha(x) \geq (f_a)_\alpha(x) \geq \alpha$.

Therefore, $-x \in (f_a)_\alpha$

(FSR3) $(f_a)_\alpha(x\,y) \geq T\{(f_a)_\alpha(x), (f_a)(y)\} \geq T\{f_a(x), f_a(y)\} \geq T\{\alpha, \alpha\} \geq \alpha$

Therefore, $xy \in (f_a)_\alpha$ Therefore (f_a) is a fuzzy soft ring of R.

4.3 Proposition:

Every imaginable fuzzy soft ring μ of R is fuzzy soft ring of R.

Proof:

Assume that μ is imaginable fuzzy soft ring of R. Then we have $\mu(x+y) \geq T\{\mu(x), \mu(y)\}$ and $\mu(-x) \geq \mu(x)$, $\mu(xy) \geq T\{\mu(x), \mu(y)\}$ for all x, y ϵ R.Since μ is imaginable, we have.

$\min\{\mu(x), \mu(y)\} = T\{\min\{\mu(x), \mu(y)\}, \min\{\mu(x), \mu(y)\}\} \geq T\{\mu(x), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$

and so$T\{\mu(x), \mu(y)\} = \min\{\mu(x), \mu(y)\}$

It follows that $\mu(x+y) \geq T\{\mu(x), \mu(y)\}$ for all x, y ϵ R.Hence μ is fuzzy soft ring of R.

4.4 Proposition:

If μ is fuzzy soft ring of R and θ is endomorphism of R then $\mu^{[\theta]}$ is fuzzy soft ring of R.

Proof:

For any x, y ϵ R, we have

(FSR1) $\mu^{[\theta]}(x+y)) = \mu(\theta(x+y)) = \mu(\theta x + \theta y) \geq T\{\mu(\theta x), \mu(\theta y)\} \geq T\{\mu^{[\theta]}(x), \mu^{[\theta]}(y)\}$

(FSR2) $\mu^{[\theta]}(-x) = \mu(\theta(-x) \geq \mu(\theta x) \geq \mu^{[\theta]}(x)$

(FSR3) $\mu^{[\theta]}(xy) = \mu\,\theta(x\,y) = \mu((\theta x)(\theta y))\ T\{\mu(\theta x), \mu(\theta y)\} \geq T\{\mu^{[\theta]}(x), \mu^{[\theta]}(y)\}$

Therefore, $\mu^{[\theta]}$ is fuzzy soft ring of R.

4.5 Proposition:

Let T be a continuous t-norms and let f be a soft homomorphism on R. If μ is fuzzy soft ring of R then μ^f is fuzzy soft ring of f(R).

Proof:

Let $A_1 = f^{-1}(y_1)$ and $A_2 = f^{-1}(y_2)$ and

$A_{12} = f^{-1}(y_1 + y_2)$. Where y_1, y_2 f(R).

Consider the set.

$A_1 + A_2 = \{x \in R / x = a_1 + a_2\}$, for some $a_1 \in A_1$ and $a_2 \in A_2$

If $x \in A_1 + A_2$ then $x = x_1 + x_2$ for some $x_1 \in A_1$ and $x_2 \in A_2$. So that we have

$f(x) = f(x_1) + f(x_2) = y_1 + y_2$Therefore, $x \in f^{-1}(y_1 + y_2) = A_{12}$.Thus $A_1 + A_2 \in A_{12}$

It follows that

(FSR1) $\mu^f(y_1 + y_2) = \sup\{\mu(x) / x \in f^{-1}(y_1 + y_2)\} = \sup\{\mu(x) / x \in A_{12}\}$

$= \sup\{\mu(x) / x \in A_1 + A_2\}$

$= \sup\{\mu(x_1 + x_2) / x_1 \in A_1, x_2 \in A_2\}$

Since T is continuous, therefore for every $\epsilon > 0$, we see that if

$\text{Sup}\{\{\mu(x_1) / x_1 \in A_1\} + x_1^* \leq \delta$ and $\text{Sup}\{\{\mu(x_2) / x_2 \in A_2\} + x_2^* \leq \delta$.

$T\{\text{Sup}\{\{\mu(x_1) / x_1 \in A_1\}, \text{Sup}\{\{\mu(x_2) / x_2 \in A_2\}\} + T(x_1^*, x_2^*) \leq \epsilon$.

Choose $a_1 \in A_1$ and $a_2 \in A_2$ such that$\text{Sup}\{\{\mu(x_1) / x_1 \in A_1\} + \mu(a_1) \leq \delta$.and

$\text{Sup}\{\{\mu(x_2) / x_2 \in A_2\} + \mu(a_2) \leq \delta$.

$T\{\text{Sup}\{\{\mu(x_1) / x_1 \in A_1\}, \text{Sup}\{\{\mu(x_2) / x_2 \in A_2\}\} + T(\mu(a_1), \mu(a_2)) \leq \epsilon$.

Consequently, we have

$\mu^f(y_1 + y_2) \geq \sup\{T\{\mu(x_1), \mu(x_2) / x_1 \in A_1, x_2 \in A_2\}\}$

$\geq \{\sup\{\mu(x_1) / x_1 \in A_1\}, \sup\{\mu(x_2) / x_2 \in A_2\}\}$

$\geq T\{\mu^f(y_1), \mu^f(y_2)\}$

Similarly, we can show that, $\mu^f(-x) \geq \mu^f(x)$, and $\mu^f(xy) \geq T\{\mu^f(x), \mu^f(y)\}$

Hence μ^f is fuzzy soft ring of f(R).

4.6 Proposition: Onto homomorphic image of fuzzy soft ring with sup-property is fuzzy soft ring of R.

Proof:

Let $f: R \rightarrow R'$ be an onto homomorphism of rings and μ be a sup property of fuzzy soft ring of R. Let $x', y' \in R'$ and $x_0\, f^{-1}(x')$ and $y_0 \in f^{-1}(y')$ be such that

$\mu(x_0) = \sup_{h \in f^{-1}(x')} \mu(h)$ and $\mu(y_0) = \sup_{h \in f^{-1}(y')} \mu(h)$

respectively, then we can deduce that

(FSR1) $\mu^f(x' + y') = \sup_{z \in f^{-1}(x' + y')} \mu(z)$

$\geq T\{\mu(x_0), \mu(y_0)\}$

$= T\{\sup_{h \in f^{-1}(x')} \mu(h), \sup_{h \in f^{-1}(y')} \mu(h)\}$

$\geq T\{\mu^f(x'), \mu^f(y')\}$

(FSR2) $\mu^f(-x') = \sup_{z \in f^{-1}(-x')} \mu(z)$

$\geq \mu(x_0)$

$= \sup_{h \in f^{-1}(x')} \mu(h)$

$\geq \mu^f(x')$

(FSR3) $\mu^f(x'y') = \sup_{z \in f^{-1}(x'y')} \mu(z)$

$\geq T\{\mu(x_0), \mu(y_0)\}$

$= T\{\sup_{h \in f^{-1}(x')} \mu(h), \sup_{h \in f^{-1}(y')} \mu(h)\}$

$\geq T\{\mu^f(x'), \mu^f(y')\}$.Therefore μ^f is fuzzy soft ring of R´.

4.7 Proposition:

Let f_a be a fuzzy soft ring over R and (Φ, ψ) be a fuzzy soft homomorphism from R to R´. Then (Φ, ψ) fa is fuzzy soft ring over R´.

Proof:

Let $k \in (\psi)f_a$ and $y_1, y_2 \in Y$. If $\Phi^{-1}(y_1) = \Phi$ or $\Phi^{-1}(y_2) = \Phi$.

The proof is straight forward.

4.8 Proposition:

Let g_b be a fuzzy soft ring over R´ and (Φ, ψ) be a fuzzy soft homomorphism from R to R´, then $(\phi, \psi)^{-1} g_b$ is fuzzy soft ring over R.

Proof:

Let $a \in \psi^{-1}(B)$ and $x_1, x_2 \in X$.

(FSR1) $\Phi^{-1}(g_a)(x_1 + x_2) = g_{(a)}(\Phi(x_1 . x_2))$
$= g_{(a)}(\Phi(x_1) . \Phi(x_2)) \geq T\{g_{(a)}\Phi(x_1) . g_{(a)}\Phi(x_2)\}$
$\geq T\{\Phi^{-1}(g_a)(x_1), \Phi^{-1}(g_a)(x_2)\}$

and similarly, we have

$\Phi^{-1}(g_a)(-x) \geq \Phi^{-1}(g_a)(x)$ and $\Phi^{-1}(g_a)(x_1 x_2) \geq \{\Phi^{-1}(g_a)(x_1), \Phi^{-1}(g_a)(x_2)$

Therefore, $(\phi, \psi)^{-1} g_b$ is fuzzy soft ring over R.

Conclusion:

Here, we define of fuzzy soft intersection ring R that as alternative definition to soft rings. We then focused on the concepts of fuzzy soft homomorphism, union, product of two soft sets and study their properties. To extend over work, further research could be done in other algebraic structures such as fields, modules as in the case of fuzzy soft intersection ring R.

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