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CONSTRUCTION ON INTUITIONISTIC CYCLIC M-FUZZY GROUPS

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Abstract:

In this paper, the notion of cyclic group on a set is well-known. The study and extension of this classical notion of the M-fuzzy set to define concept of Intuitionistic Cyclic M-fuzzy group. By using this Intuitionistic Cyclic M-fuzzy group to investigate some results.

Key Words: M fuzzy sets, M- fuzzy groups, Cyclic M-fuzzy group, Intuitionistic Cyclic M-fuzzy groups.

1. Introduction:

After the introduction of the concept of fuzzy sets by L. A. Zadeh [7], researchers were conducted the generalizations of the notion of fuzzy sets, A. Rosenfeld [3] introduced the concept of fuzzy group and the idea of "Intuitionistic fuzzy set" was first published by K. T. Atanassov [1]. Jianming Zhan and Zhisong Tan [2] introduced the concept of "Intuitionistic M-fuzzy group" with operators. Dr. G. Subbiah[6] introduced the concept of "Cyclic M-fuzzy group" with sufficient conditions. In this paper, we give a sufficient condition for cyclic M fuzzy subgroup to be an Intuitionistic Cycle M-fuzzy group.

2. Preliminaries:

Definition 2.1: Let A and B be fuzzy sets. Then A is a subset of B if $\mu_A(x) \leq \mu_B(x)$ for every x∈ U and it is denoted by A⊆ B or B⊇ A.

Definition 2.2: Two fuzzy sets A and B are called equal if $\mu_A(x)= \mu_B(x)$ for every x∈ U and it is denoted by A= B

Definition 2.3: Let A and B be fuzzy sets. Then the algebraic product of two fuzzy sets A and B is defined by A•B ={ (x, $\mu_{\tilde{A}}$ (x)) / x∈ U, $\mu_{A\bullet B}=\mu_A\bullet\mu_B$}

Definition 2.4: Let A and B be fuzzy sets. Then the Union A∪B and Intersection A∩B are respectively defined by the equations.

A∪ B= (x,$\mu_{A\cap B}$ (x))/ x U, $\mu_{A\cap B}$(x) = max {(μ_A(x),μ_B(x)}and

A∩B= (x,$\mu_{A\cap B}$(x)) / x∈ U,$\mu_{A\cap B}$(x) = min {(μ_A(x), μ_B (x)}

Remark 2.5: These definitions can be generated for countable number of fuzzy sets. If $\tilde{A}_1$, $\tilde{A}_2$,...., are fuzzy sets with membership functions $\mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x)$,....,then the membership functions of X=$\cup\tilde{A}_i$ and Y=$\cap\tilde{A}_i$ are defined as μ_x(x) = max{$\mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x)$,....}and$\mu_y$(x) = max{ $\mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x)$,....},X∈ U respectively.

Definition 2.6: A mapping $\mu : x \rightarrow [0,1]$, where X is an arbitrary non-empty set and is called a fuzzy set in X.

Definition 2.7: Let M and G a set and a group respectively. A mapping $\mu : M \times G \rightarrow [0,1]$ is Called M-fuzzy sets in G. For any M-fuzzy set μ in G and t∈ [0,1] we define the set $U(\mu;t)$={X∈ $G/\ \mu(mx) \geq t,\ m \in M$} which is called an upper cut of 'μ' and can used to the characteristic of μ.

Definition 2.8: let f be a non- fuzzy function from X to Y .The image $f(\tilde{A})$ of a M-fuzzy set $\tilde{A}$ on X is defined by means of the extension principle as

$$f(\tilde{A}) = \{(my), \mu_{f(\tilde{A})}(my)\ /\ y = f(x), x\in X\}$$

$$\text{Where } \mu_{f(\tilde{A})}(my) = \begin{cases} \text{Sup}(mx)\in f^{-1}(my)\ \mu_{\tilde{A}}(mx) & \text{if } f^{-1}(y,q)\neq \varphi \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.9: A M- fuzzy set 'A' is called M- fuzzy group of G if

(MFG1) A(mxy, q) ≥ min{ A(x), A(y)}

(MFG2) A(mx^{-1}) = A(x)

(MFG3) A(me) = 1for all x,y∈G and m∈ M.

Definition 2.10: let G=<a> be a cyclic group. If Ã = {(a^n, μ (a^n)) / n ∈ z } is a fuzzy group, then Ã is called a cyclic fuzzy group generated by (a, μ(a)) and denoted by <a, μ(a)>.

Definition 2.11: Let A = < ma> be a M- cyclic group. If Ã ={ (ma^n) , μ(ma^n)) / n ε z } is a M- fuzzy group, then Ã is called a M- cyclic fuzzy group generated by $\langle (ma), \mu(ma)\rangle$.

3. Properties of M - Fuzzy Groups:

Proposition 3.1:

Let 'A' be a M- fuzzy group of G. Then (i) A(mx) ≤ A(me) for all x ∈G and m ∈ M.

(ii) The subset G_A = { x∈G / A(mx) = A(me)} is a M- fuzzy group of G.

Proof:

Let x be any element of G, then $A(mx,q) = \min\{A(mx,q), A(mx,q)\}$

$= \min\{A(x,), A(x^{-1})\}$

$\leq A(xx^{-1})$ since $A(e)=1$

$= A(me)$ and (i) is proved.

To prove (ii) we have $e \in G_A$, then $G_A \neq \Phi$.

Now let $x,y \in G_A$ and $m \in M$.

$A(mxy^{-1}) \geq \min\{A(x), A(y^{-1})\}$

$= \min\{A(x), A(y)\}$

$=\min\{A(e). A(e)\}$

$= A(e)$

but from (i) $A(mxy^{-1}) \leq A(e)$ for $x,y \in G$ and $m \in M$. Therefore $A(mxy^{-1}) = A(e)$

Which means $mxy^{-1} \in G_A$ and G_A is M- fuzzy group of G.

Corollary: let G be a finite group and A be a M-fuzzy group of G. Consider the subset H of G is given by $H =\{x\in G/ A(mx)=A(e)\}$. Then H is a crisp subgroup of G.

Proposition 3.3:

A set of necessary and sufficient conditions for a M- fuzzy set of a group G to be a M- fuzzy group of G is that $A(mxy^{-1}) \geq \min(A(x), A(y)\}$ for all x,y in G and m in M.

Proof:

Let A be a M- fuzzy group of G. Then

$A(mxy^{-1}) \geq \min\{A(x), A(y^{-1})\}$

$= \min\{A(x), A(y)\}$ for $x,y \in G$ and $m \in M$.

For the converse part suppose that A be M- fuzzy set of the group G of which e is the identity element.

Now $A(myy^{-1}) \geq \min\{A(y), A(y)\}$

$A(e)$ or $A(e) \geq A(y)$

Now $A(mey^{-1}) \geq \min\{A(e), A(y)\}$

or $A(my^{-1}) \geq A(y)$

Also $A(mxy) \geq \min\{A(x), A(y^{-1})\}$

$\geq \min\{A(x), A(y)\}$.

Hence the proof

4. Intuitionistic Cyclic M-Fuzzy Group:

Definition 4.1: Let $\tau = \langle ma \rangle$ be a M-cyclic group if $\tilde{\tau} = \{\langle ma^n, \mu(ma^n), \vartheta(ma^n)\rangle | n \in z\}$ is the Intuitionistic Cyclic M-fuzzy group $\langle ma, \mu(ma^n), \vartheta(ma^n)\rangle$ Where $\mu(ma^n)$ is a membership function, $\vartheta(ma^n)$ is a non-membership function.

Theorem 4.2:

If G is a group, then prove that $\tilde{\tau}^m = \{(ma^n), \mu_{\tilde{\tau}}(ma^n)^m, \vartheta_{\tilde{\tau}}(ma^n)^m / n \in z\}$ is also a Intuitionistic Cycle M- fuzzy group.

Proof:

Let us show that $\tilde{\tau}^m$ satisfies three conditions (MFG1-MFG3) in definition (2.9). We can consider only its membership function because the m^{th} power of $\tilde{\tau}_m$ effects just only the membership function of $\tilde{\tau}_m$.

MFG1: Since τ is a M-fuzzy group and $\mu_{\tilde{\tau}}(ma) \in [0,1]$

$$\mu_{\tilde{\tau}}\left((ma^{n_1}), (ma^{n_2})\right)^m \geq \min\{\mu_{\tilde{\tau}}(ma^{n_1}), \mu_{\tilde{\tau}}(ma^{n_2})\}$$

$$= \min\{\mu_{\tilde{\tau}}(ma^{n_1})^m, \mu_{\tilde{\tau}}(ma^{n_2})^m\}$$

MFG2: $\mu_{\tilde{\tau}}(ma^n) = \mu_{\tilde{\tau}}(ma^{-n})$

Since $\tilde{\tau}$ is M-fuzzy group

$$\mu_{\tilde{\tau}}(ma^n)^m = \left(\mu_{\tilde{\tau}}(ma^{-n})\right)^m$$

MFG3: $\mu_{\tilde{\tau}}(me,q) = 1$

Since $\tilde{\tau}$ is M-fuzzy group

Then $\left(\mu_{\tilde{\tau}}(me,q)\right)^m = 1$

Hence the proof.

Example 4.3:

Let τ be a M-Cyclic group with 12 elements and generated by (ma). Let $\tilde{\tau}$ be a M-fuzzy set of the group τ defined as follows

$$\mu_{\tilde{\tau}}(ma^0) = 1$$

$$\mu_{\tilde{\tau}}(ma^4) = \mu_{\tilde{\tau}}(ma^8) = t_1,$$

$$\mu_{\tilde{\tau}}(ma^2) = \mu_{\tilde{\tau}}(ma^6) = \mu_{\tilde{\tau}}(ma^{16}) = t_2$$

And $\mu_{\tilde{\tau}}(mx) = t_3$ for all other elements X in τ

Where $t_1, t_2, t_3 \in [0,1]$ with $t_1 > t_2 > t_3$.

It is clear that $\tilde{\tau}$ is a M-fuzzy group of τ thus $\tilde{\tau} = \{(ma^k), \mu_{\tilde{\tau}}(ma^k), \vartheta_{\tilde{\tau}}(ma^k)/k \in z\}$ is a Intuitionistic Cyclic M-fuzzy group generated by $((ma), \mu_{\tilde{\tau}}(ma))$.

Proposition 4.4:

If $\tilde{\tau}^i$ and $\tilde{\tau}^j$ are M-cyclic fuzzy groups then $\tilde{\tau}^i \cup \tilde{\tau}^j$ is also Intuitionistic Cycle M-fuzzy group if $i < j$.

Proof:

Since $i < j$ then we've $\mu_{\tilde{\tau}^i} > \mu_{\tilde{\tau}^j}$

MFG1:

$$= \max\{\mu_{\tilde{\tau}^i}(ma^n a^m), \mu_{\tilde{\tau}^j}(ma^n a^m)\}$$
$$= \max\{\{\mu_{\tilde{\tau}}(ma^n a^m)^i, \mu_{\tilde{\tau}}(ma^n a^m)^j\}$$
$$= \left(\mu_{\tilde{\tau}}(ma^n ma^m)\right)^i$$
$$> \min\{\mu_{\tilde{\tau}^i}(ma^n), \mu_{\tilde{\tau}^j}(ma^m)\}$$
$$> \min\left\{\max\left\{\left(\mu_{\tilde{\tau}^i}(ma^n), \mu_{\tilde{\tau}^j}(ma^m)\right)\right\}, \max\left\{\left(\mu_{\tilde{\tau}^i}(ma^n), \mu_{\tilde{\tau}^j}(ma^m)\right)\right\}\right\}$$
$$= \min\left\{\max\left\{\left(\mu_{\tilde{\tau}^i}(ma^n), \mu_{\tilde{\tau}^j}(ma^n)\right)\right\}, \max\left\{\left(\mu_{\tilde{\tau}^i}(ma^m), \mu_{\tilde{\tau}^j}(ma^m)\right)\right\}\right\}$$
$$\geq \min\{\mu_{\tilde{\tau}^i \cup \tilde{\tau}^j}(ma^n), \mu_{\tilde{\tau}^i \cup \tilde{\tau}^j}(ma^m)\}$$

MFG1 is satisfied.

MFG2:

$$\mu_{\tilde{\tau}^i \cup \tilde{\tau}^j}(ma^{-n})$$
$$= \max\left\{\mu_{\tilde{\tau}_i}(ma^{-n}), \mu_{\tilde{\tau}_j}(ma^{-n})\right\}$$
$$= \max\{\mu_{\tilde{\tau}}(ma^{-n})^i, \mu_{\tilde{\tau}}(ma^{-n})^j\}$$
$$= \max\{\mu_{\tilde{\tau}}(ma^n)^i, \mu_{\tilde{\tau}}(ma^n)^j\}$$
$$= \max\left\{\mu_{\tilde{\tau}_i}(ma^n), \mu_{\tilde{\tau}_j}(ma^n)\right\}$$
$$= \mu_{\tilde{\tau}_i} \cup \mu_{\tilde{\tau}_j}(ma^n)$$

MFG2 is satisfied

MFG3:

$$\mu_{\tilde{\tau}_i} \cup \mu_{\tilde{\tau}_j}(me)$$
$$= \max\left\{\mu_{\tilde{\tau}_i}(me), \mu_{\tilde{\tau}_j}(me)\right\}$$
$$= \max\{1,1\} = 1.$$

Hence $\tilde{\tau}^i \cup \tilde{\tau}^j$ is Intuitionistic Cyclic M- fuzzy group.

Proposition 4.5:

If $\tilde{A}_i$ and $\tilde{A}_j$ are M-cyclic fuzzy groups then $\tilde{A}_i \cap \tilde{A}_j$ is also Intuitionistic cyclic M-fuzzy group

Proof:

This theorem may be proved similarly to proposition.4.4.

Remark:

Since a cyclic Intuitionistic M- fuzzy group is an abelian group , It is clear that $\mu(mxy) = \mu(myx)$ for $x, y \in A, m \in M$.

Conclusion:

In this paper, we extend Intuitionistic Cyclic M Fuzzy group to M Fuzzy set to define the concept of Cyclic M- fuzzy groups. We give a sufficient for M – Fuzzy subset to be Intuitionistic Cyclic M fuzzy group family and investigate its structure and properties with application.

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