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RHOTRIX EIGEN VALUES AND EIGEN VECTORS

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Abstract:

This paper presents a new technique for solving Eigen values and Eigen vectors by the concept of rhotrix. Use of rhotrices in the above mentioned problems reduces the computational cost and time complexity to a greater extent by solving two such problems simultaneously.

1. Introduction:

Mathematical arrays that are in some way between two-dimensional vectors and 2x2 dimensional matrices were suggested by Atanassov and Shannon [1]. As an extension to this idea, Ajibade [2] introduced an array system that lies between 2x2 dimensional matrices and 3x3 dimensional matrices called 'Rhotrix'[3,4]. A rhotrix as given in [1] is of the form

$$R = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle$$

where a,b,d,e,h(R) are all real numbers and h (R) is called the heart of a rhotrix R. The rhotrix R given above is called the base rhotrix. It was also mentioned in [2] that a rhotrix can be extended to n-dimension. The n-dimensional rhotrix is denoted by $R_n = \langle a_{ij}, c_{lk} \rangle$ where, a_{ij} and c_{lk} represent the elements and $|R_n| = \frac{n^2+1}{2}$ (i ,j=1,2,3…..,t)and (l , k=1,2,3,…. t-1). The value of t is given as t=(n+1)/2. An alternative to the multiplication proposed by Sani [4] followed by its generalization in [5]. The method of converting rhotrix to coupled matrix was suggested by Sani [6]. His idea was found useful to solve systems of txt and (t-1) x (t-1) matrix problems simultaneously, $n \in 2z^+ + 1$ and $t = \frac{n^2+1}{2}$. The inverse of rhotrix is given in [2] by Ajibade as, if for any two rhotrices P and Q, we have $P \circ Q = I$, then Q is the inverse of P.

That is, $Q = P^{-1} = \frac{-1}{(h(P))^2} \left\langle \begin{array}{ccc} & a & \\ b & -h(P) & d \\ & e & \end{array} \right\rangle$

And the inverse of rhotrix is given in [4] by Sani as, if $R_3 = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle$ then

$$R_3^{-1} = \left\langle \begin{array}{ccc} & \frac{e}{ae-bd} & \\ \frac{b}{bd-ae} & \frac{1}{h(R)} & \frac{d}{bd-ae} \\ & \frac{a}{ae-bd} & \end{array} \right\rangle$$

Hence R_3 and P_3 are similar

2. Rhotrix Eigen Values and Eigen Vector:

In this Sani.B gave a generalization of the row column multiplication method for n-dimensional rhotrices was presented as an extension of earlier work on row-column multiplication for base level rhotrices of dimension three. The idea was used to introduce the eigenvector , eigenvalue problem in rhotrices.

Definition:

The main rhotrix column vector is represented as $<x^{nj}>$ and the main row vectors represented as $<x^{in}>$ where i, j = 1,2,….t with t = (n+1)/2. Similarly the other columns and which are not the main are defined as follows $<x^{n-1,k}>$ and $<x^{1,n-1}>$ respectively where k, l = 1,2,….t-1.

Theorem:

Let λ be an eigenvalue of a rhotrix R_n with the corresponding eigenvector $\langle x^{nj} \rangle$ then if k is a positive integer, λ^k is an eigenvalue of the rhotrix R_n^k with the corresponding eigenvector $\langle x^{nj} \rangle$

Proof:

$$
\begin{aligned}
R_n^{\,k}\langle x^{nj}\rangle &= R_n^{\,k-1} R_n \langle x^{nj}\rangle \\
&= R_n^{\,k-1}\lambda\langle x^{nj}\rangle \\
&= R_n^{\,k-2} R_n \lambda\langle x^{nj}\rangle \\
&= R_n^{\,k-1}\lambda^2\langle x^{nj}\rangle \\
&\;\;\vdots \\
&= R_n\lambda^{k-1}\langle x^{nj}\rangle \\
&= \lambda^{k-1} R_n\langle x^{nj}\rangle \\
R_n^{\,k}\langle x^{nj}\rangle &= \lambda^{k}\langle x^{nj}\rangle
\end{aligned}
$$

Therefore is an eigenvalue of the rhotix $R_n^{\,k}$ with the corresponding eigenvector $\langle x^{nj}\rangle$

Theorem:

For any given matrix A = $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3*3}$ if

λ'_1 $\lambda'_2$and λ'_3 are the Eigen value of A, then
$\lambda'_1+\lambda'_2+\lambda'_3 = \lambda_1+\lambda_2+\lambda_3$
$\lambda'_1\lambda'_2+\lambda'_2\lambda'_3+\lambda'_3\lambda'_1 = \lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1$
$\lambda'_1\lambda'_2+\lambda'_2\lambda'_3+\lambda'_3\lambda'_1 = \lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1 - (bd+fh)$
$\lambda'_1\lambda'_2\lambda'_3 = \lambda_1\lambda_2\lambda_3 - \{ bdi + afh - (bfg + dhc) \}$
where λ_1, λ_2, λ_3 are Eigen value of its corresponding rhotrix

Proof:

Characteristic equation of A is $|A-\lambda I| = 0$

$$\Rightarrow \begin{vmatrix} a-\lambda & b & c \\ d & e-\lambda & f \\ g & h & i-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ (a-λ)[(e-λ)(i-λ)-hf]-b[di-dλ -gf]+c[dh-ge+gλ] = 0

Simplifying this we get

$\lambda^3 - (a+e+i)\lambda^2 + (ai+ei-gc-hf+ae-bd)\lambda -$
$(aei-ahf-bdi+bdf+cdh-cge) = 0$ → (1)

λ'_1, $\lambda'_2$and λ'_3 are the roots of equation (1).

Therefore, $\lambda'_1+\lambda'_2+\lambda'_3 = a+e+i$ → (2)

$\lambda'_1\lambda'_2+\lambda'_2\lambda'_3+\lambda'_3\lambda'_1 = ai+ei-gc-hf+ae-bd$ →(3)

$\lambda'_1\lambda'_2\lambda'_3 = aei-ahf-bdi+bgf+cdh-cge$ →(4)

The corresponding rhotrix R_3 is $\left\langle \begin{matrix} & a & \\ g & e & c \\ & i & \end{matrix} \right\rangle$ and the coupled matrix as reported in is C = $\begin{bmatrix} a & 0 & c \\ 0 & e & 0 \\ g & 0 & i \end{bmatrix}_{3*3}$

We considered the characteristic roots (Eigen values) of R_3 and C will be same.

$$|C-\lambda I| = 0$$

$\Rightarrow$ $\lambda^3 - (a+e+i)\lambda^2 + (ei+ai-gc-ae)\lambda - (aei- gce) = 0$ →(5)

λ_1,λ_2 , λ_3 are the roots of equation (5)

$\lambda_1+\lambda_2+\lambda_3 = a+e+i$ →(6)

$\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1 = ai+ei-gc+ae$ →(7)

$\lambda_1 \lambda_2 \lambda_3 = aei-gce$ →(8)

Theorem is followed from the equations (2), (3), (4) and (6), (7), (8) respectively.

Numerical Illustration 1:

Consider A = $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. The corresponding rhotrix is

$$R_3 = \left\langle \begin{array}{ccc} & 3 & \\ 1 & 5 & 1 \\ & 3 & \end{array} \right\rangle$$

Finding the given Eigen value of R_3 is equivalent to finding $\lambda \in R$
Such that

$$\begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$\Leftrightarrow \lambda^2 - 6\lambda - 8 = 0$
$\Leftrightarrow (\lambda-4)(\lambda-2) = 0$
$\Leftrightarrow \lambda = 2,4$

Eigen values of R_3 are $\lambda = 2,4,5$, considering the third Eigen value as 5, being the sum of Eigen values equal to sum of diagonal elements of rhotrix. Using above theorem

$\lambda'_1+\lambda'_2+\lambda'_3 = 11$,
$\lambda'_1 \lambda'_2 \lambda'_3 = 36$,
$\lambda'_1 \lambda'_2 + \lambda'_2 \lambda'_3 + \lambda'_3 \lambda'_1 = 36$
Solving this we get
$\lambda'_1=2, \lambda'_2= 3, \lambda'_3 = 6$

Numerical illustration 2:

Consider a rhotrix

$$R_5 = \left\langle \begin{array}{ccccc} & & 1 & & \\ & 1 & 1 & 1 & \\ 3 & 2 & 5 & 4 & 3 \\ & 1 & 3 & 1 & \\ & & 1 & & \end{array} \right\rangle$$

Now the major matrix is

$$A=\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

The eigen values of A are -2,3,6

The eigen vectors corresponding to λ_1=-2 is (-2,0,2), λ_2= 3 is (-1,1,-1) and λ_3=6 is (1,2,1).

Again for the minor matrix

$$C=\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

The eigen values of C are 5,-1.

The eigen vectors corresponding to 5and -1 is(1,1) and (-2,1).

Finding

$$R_5 \langle x^{5j} \rangle = \left\langle \begin{array}{ccccc} & & 4 & & \\ & 0 & 5 & -3 & \\ -4 & 5 & 3 & 2 & 6 \\ & -3 & -1 & 12 & \\ & & 6 & & \end{array} \right\rangle$$

$$\lambda \langle x^{5j} \rangle = \left\langle \begin{array}{ccccc} & & 4 & & \\ & 0 & 5 & -3 & \\ -4 & 5 & 3 & 2 & 6 \\ & -3 & -1 & 12 & \\ & & 6 & & \end{array} \right\rangle$$

Therefore $R_5 \langle x^{5j} \rangle = \lambda \langle x^{5j} \rangle$, j = 1,2,3.

Similarly, $R_5 \langle x^{j5} \rangle = \langle x^{j5} \rangle \lambda$, j = 1,2,3.

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