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HALL EFFECT ON MHD FLOW PAST OVER MOVING VERTICAL PLATE WITH MASS DIFFUSION IN POROUS MEDIUM

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Abstract:

In this paper, we examine the motion of unsteady MHD flow past over moving vertical plate with mass diffusion in the presence of Hall current and porous medium. The fluid considered is viscous, electrically conducting, incompressible and absorbing-emitting radiation in a non-scattering medium. The Laplace transform technique has been used to find the solutions for the velocity profiles and Skin friction. The velocity profile and Skin friction have been studied for different parameters like Schmidt number, Hall parameter, magnetic parameter, Mass Grashof number and time. The effects of parameters are shown graphically. The value of the skin-friction for different parameters has been tabulated.

Key Words: MHD Flow, Mass Transfer, Hall Current & Skin Friction

Introduction:

The flow of an electrically conducting fluid has important applications in many branches of engineering and applied sciences such as MHD accelerators and power generation systems, plasma studies, cooling of nuclear reactors, geothermal energy extraction and electromagnetic propulsion. Some researchers study the effect of Hall current in various direction. Effects of Hall current on MHD boundary layer second-order visco elastic fluid flow induced by a continuous surface with heat transfer was analyzed by Zaman et al. [1]. The work on MHD flow past an impulsively started infinite vertical porous plate with heat transfer was done by Rajput and Kumar [2]. The same authors have also studied MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Hall effect on heat and mass transfer flow through porous medium was investigated by Ram et al. [9]. Sulochana [3] has considered Hall effect on unsteady MHD three dimensional flow through a porous medium in a rotating parallel plate channel with effect of inclined magnetic field by The effect of variable properties on the unsteady Hartmann flow with heat transfer considering the Hall effect was examined by Attia [7]. Further, Attia et al. [8] have developed the Hall effect on unsteady MHD Couette flow and heat transfer of a Bingham fluid with suction and injection. Ahmed et al. [13] have discussed unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source. In this present paper we have considered MHD flow through porous media past over moving vertical plate with mass diffusion in the presence of Hall current. The velocity profile has been observed with the help of graphs, and the skin friction has been tabulated.

Mathematical Formulation:

We considered an unsteady, viscous, incompressible and electrically conducting fluid past moving vertical plate with velocity u_0. The x axis is taken along the vertical plate and y normal to it. The plate is electrically non conducting. A uniform magnetic field B is assumed to be applied in the y-direction. Initially the fluid and plate are at the same concentration C_∞ in the stationary condition. A time t>0, the concentration level near the plate is raised linearly with respect to time. Due to the Hall effect there will be two components of the momentum equation, which are as under.

$$\frac{\partial u}{\partial t}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(C-C_\infty)-\frac{\sigma\mu^2 B_0{}^2}{\rho(1+m^2)}(u+mw)-\frac{\upsilon u}{K} \tag{1}$$

$$\frac{\partial w}{\partial t}=\upsilon\frac{\partial^2 w}{\partial y^2}-\frac{\sigma\mu B_0{}^2}{\rho(1+m^2)}(w-mu)-\frac{\upsilon w}{K} \tag{2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2} \tag{3}$$

The boundary conditions taken are as under:

$$\left.\begin{array}{l} t\le 0: u=0, w=0, C=C_\infty \ \forall y \\ t>0: u=u_0, w=0, C=C_\infty+(C_w-C_\infty)\dfrac{u_0{}^2 t}{\upsilon}\ at\, y=0 \\ u\to 0, w\to 0, C\to C_\infty \quad as\, y\to 0 \end{array}\right\} \tag{4}$$

where u is the velocity of the fluid in x- direction, w - the velocity of the fluid in z- direction, m is the Hall parameter, g – acceleration due to gravity, β - volumetric coefficient of concentration expansion, t- time, , C_∞ -is the concentration in the fluid far away from the plate, C is species concentration in the fluid , D is the mass diffusion υ is the kinematic viscosity , ρ is the fluid density and σ electrically conductivity , K is the permeability of the medium, μ is the magnetic permeability. Here $m=\omega_e\tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (3) in dimensionless from, we introduce the following non - dimensional quantities:

$$\bar{u}=\frac{u}{u_0},\bar{w}=\frac{w}{u_0},\bar{y}=\frac{yu_0}{\upsilon},Sc=\frac{\upsilon}{D},\bar{t}=\frac{tu_0^{\ 2}}{\upsilon},Gm=\frac{g\beta\upsilon(C_w-C_\infty)}{u_0^{\ 3}},C=\frac{C-C_\infty}{C_w-C_\infty},\bar{K}=\frac{u_0}{\upsilon^2}K. \tag{5}$$

Here the symbols used are: $\bar{u}$ - dimensionless velocity, $\bar{w}$ - dimensionless velocity, M- magnet field parameter, $\bar{y}$ - dimensionless coordinate axis normal to the plate, Sc - Schmidt number, Gm - mass Grashof number.

The dimensionless forms of Equation (1), (2), and (3) are as follows

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+Gm\bar{C}-\frac{M}{1+m^2}(\bar{u}+m\bar{w})-\frac{\bar{u}}{\bar{K}} \tag{6}$$

$$\frac{\partial\bar{w}}{\partial\bar{t}}=\frac{\partial^2\bar{w}}{\partial\bar{y}^2}-\frac{M}{1+m^2}(\bar{w}-u\bar{m})-\frac{\bar{w}}{\bar{K}} \tag{7}$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{Sc}\frac{\partial^2\bar{C}}{\partial\bar{y}^2} \tag{8}$$

The boundary conditions become:

$$\left.\begin{array}{l}\bar{t}\le 0,\bar{u}\le 0,\bar{C}=0,\vec{w}=0\ ,\forall\ \bar{y}\\ \bar{t}>0,\bar{u}=1,\bar{w}=0,\bar{C}=\bar{t}\ at\ \bar{y}=0\\ \bar{u}\to 0,\bar{C}\to 0,\bar{w}\to 0\ as\ \bar{y}\to 0\end{array}\right\} \tag{9}$$

Dropping the bars and combining the equations (6) and (7) with $q=u+iw,$ we get

$$\frac{\partial q}{\partial t}=\frac{\partial^2 q}{\partial y^2}-\frac{M}{1+m^2}q(1-mi)+GmC-\frac{q}{K} \tag{10}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{11}$$

The Final boundary conditions become:

$$\left.\begin{array}{l}t\le 0,q=0,C=0,w=0\ \forall\ y\\ t>0,q=1,C=t,w=0\quad at\ y=0\\ q\to 0,C\to 0,w\to 0\ as\quad y\to\infty\end{array}\right\} \tag{12}$$

The dimensionless equations (10) and (11) with the boundary conditions (12), are solved by the Laplace transform method. The solutions obtained are as under:

$$q=\frac{1}{2}e^{-\sqrt{a}\,y}(1+Erf\,[\frac{2\sqrt{at}-y}{2\sqrt{t}}]+e^{2\sqrt{a}\,y}Erfc\,[\frac{2\sqrt{at}+y}{2\sqrt{t}}])+\frac{1}{4a^2}y\,Gm\{A_1(1-a)+\sqrt{a}\,e^{-\sqrt{a}y}(1-e^{2\sqrt{a}y}$$

$$+A_5+e^{2\sqrt{a}y}A_6)+A_2(1-Sc)-ASc\}+\frac{1}{2a^2\sqrt{\pi}}yGm\{a(2e^{\frac{-y^2Sc}{4t}}\sqrt{t}+\sqrt{\pi}y)(-1+Erf\,[\frac{y\sqrt{Sc}}{2\sqrt{t}}])\sqrt{Sc})+\frac{A_7}{y}$$

$$(\sqrt{Sc}-\frac{1}{\sqrt{Sc}})(-1+a+Sc)A_4\}\sqrt{Sc}$$

$$C=t\left\{(1+\frac{y^2S_c}{2t})erfc[\frac{\sqrt{S_c}}{2\sqrt{t}}]-\frac{y\sqrt{S_c}}{\sqrt{\pi\sqrt{t}}}e^{-\frac{y^2}{4t}}S_c\right\},$$

Skin Friction:

The dimensionless skin friction at the plate y=0 is obtained as

$$\left(\frac{dq}{dy}\right)_{y=0}=-\frac{1}{2}\sqrt{a}(1+erf\,[\sqrt{at}\,]+erfc\,[\sqrt{at}\,])-\frac{e^{at}}{\sqrt{\pi t}}+\frac{1}{a^2\sqrt{\pi}}Gm\sqrt{Sc}\{-\frac{(1-Sc)}{\sqrt{t}}+2a\sqrt{t}$$

$$+e^{\frac{at}{Sc-1}}(1-Sc)(\frac{e^{-\frac{at}{Sc-1}}}{\sqrt{t}}+\sqrt{\pi}erf\,[\sqrt{\frac{at}{Sc-1}}]\sqrt{\frac{at}{Sc-1}})\}+\frac{1}{a^2}Gm\,(a^{3/2}t+Sc\sqrt{a}-\frac{1}{2}\sqrt{a}Erf\,[\sqrt{at}\,]$$

$$-\frac{e^{at}}{\sqrt{\pi t}}+(at+Sc)(-\sqrt{a}+\frac{e^{-at}}{\sqrt{\pi t}}+\sqrt{a}erf\,[\sqrt{at}\,])+(1-Sc)e^{-\frac{at}{Sc-1}}(\frac{e^{-\frac{atSc}{Sc-1}}}{\sqrt{t\pi}}+erf\,[\sqrt{\frac{atSc}{Sc-1}}]\sqrt{\frac{atSc}{Sc-1}})).$$

Separating $\left(\frac{dq}{dy}\right)_{y=0}$ into real and imaginary part, the dimensionless skin friction component $\tau_x = \left(\frac{du}{dy}\right)_{y=0}$ and $\tau_z = \left(\frac{dw}{dy}\right)_{y=0}$ can be computed. The numerical values of τ_x and τ_y, for different parameters are given in table-1.

Discussion and Results:

The numerical value of velocity and skin friction are computed for different parameters like, mass Grashof number *Gm*, magnetic field parameter *M*, Hall parameter *m*, Schmidt number *Sc* and time *t.* The value of main parameters considered are *Gm*=10, 20, 30, magnetic parameter *M*= 1, 3, 5, Hall parameter m= 0.5, 2, 5, Schmidt number *Sc* = 2.01, 3, 4, Permeability of the medium *K*= 0.5, 1.0, 1.5, and t=0.4, 0.5 and 0.6. Figure (2), (3) and (6) shows that primary velocity increased when *M*, *Gm* and *t* is increased. Figure (1), (4) and (5) shows that primary velocity decreased when *m*, *K* and *Sc* is increased. Figure (7), (9), (10) and (12) shows that the secondary velocity increased when *m* , *M*, *Sc*, and *t* are increased. Figure (8) and (11) shows that secondary velocity decreased when *Gm* and *K* are increased.

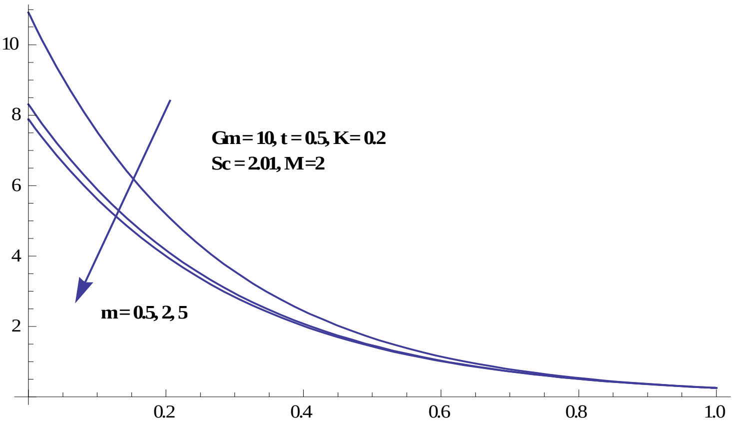

Figure 1: Velocity profile *u* for different values of *m*

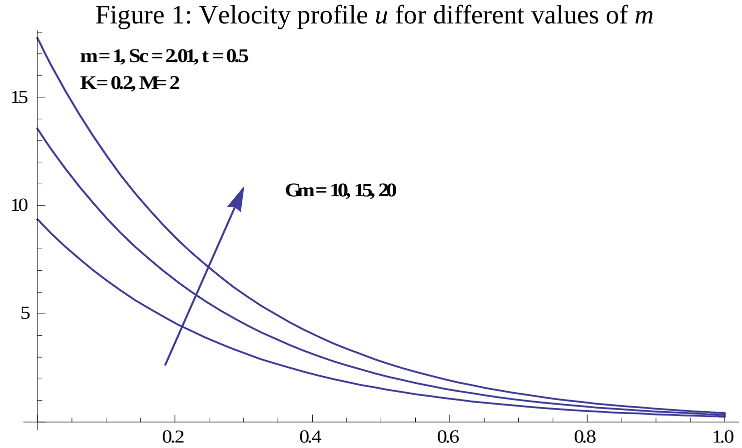

Figure 2: Velocity profile *u* for different values of *Gm*

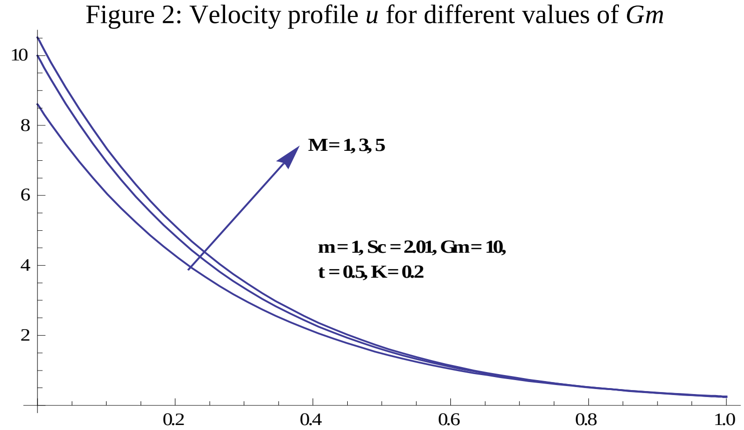

Figure 3: Velocity profile *u* for different values of *M*

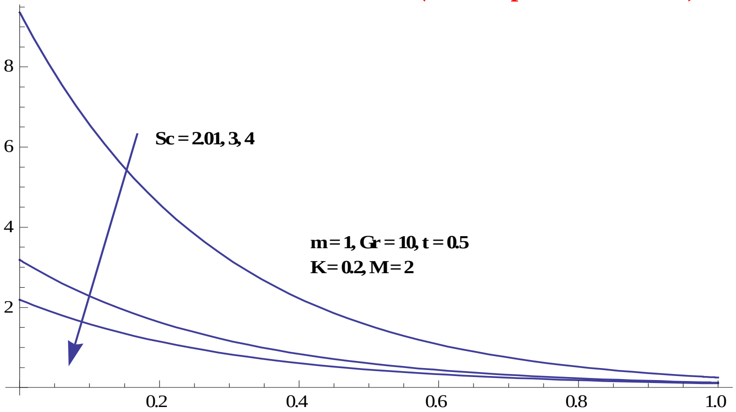

Figure 4: Velocity profile u for different values of Sc

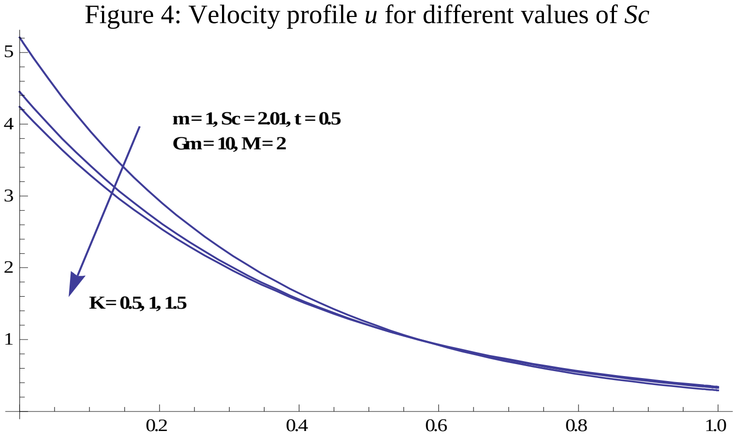

Figure 5: Velocity profile u for different values of K

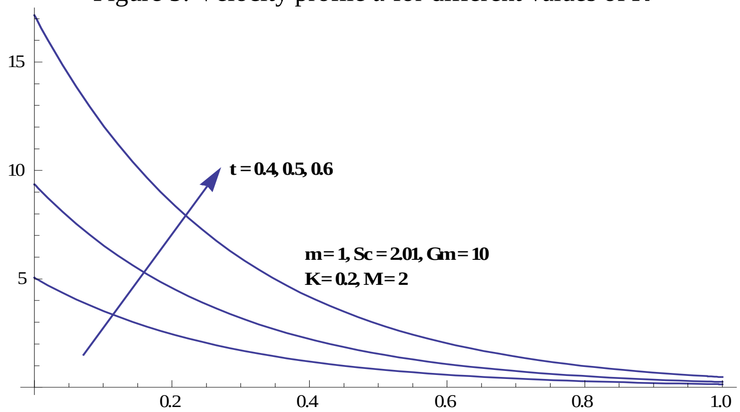

Figure 6: Velocity profile u for different values of t.

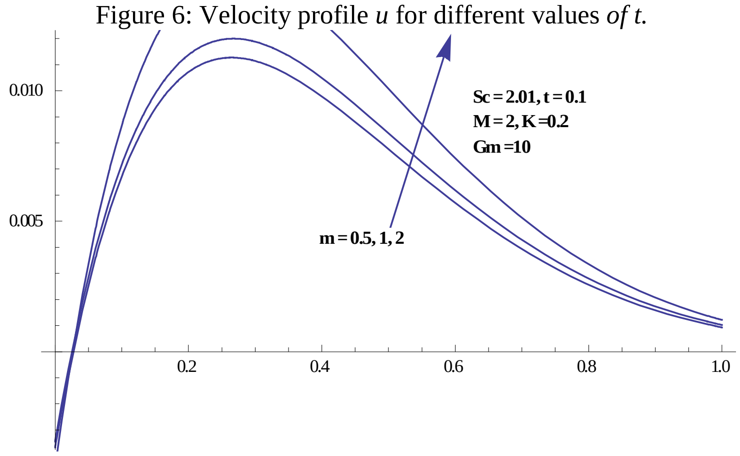

Figure 7: Velocity profile w for different values of m

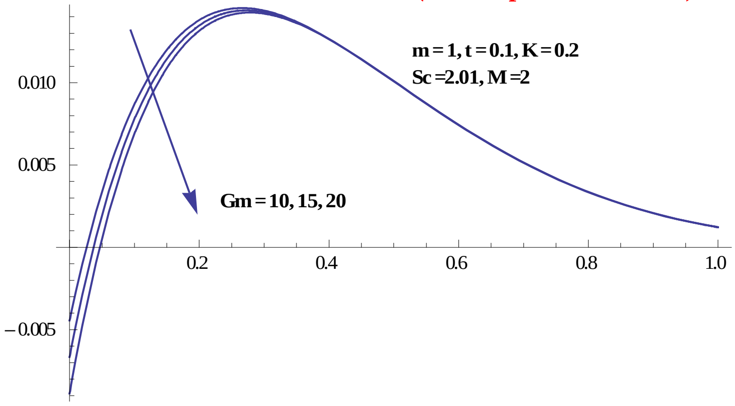

Figure 8: Velocity profile *w* for different values of *Gm*

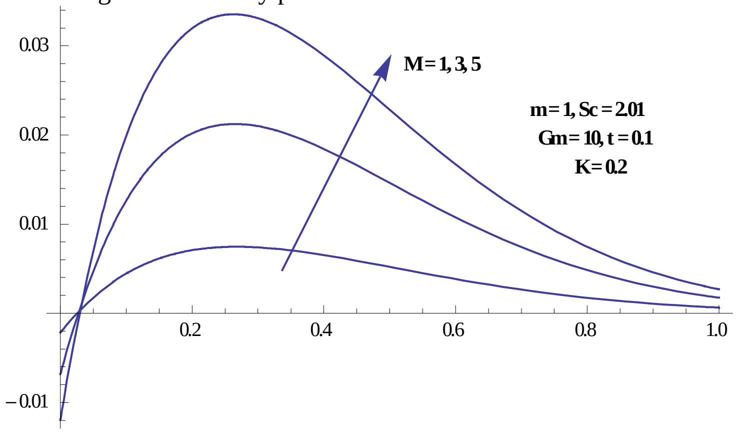

Figure 9: Velocity profile *w* for different values of *M*

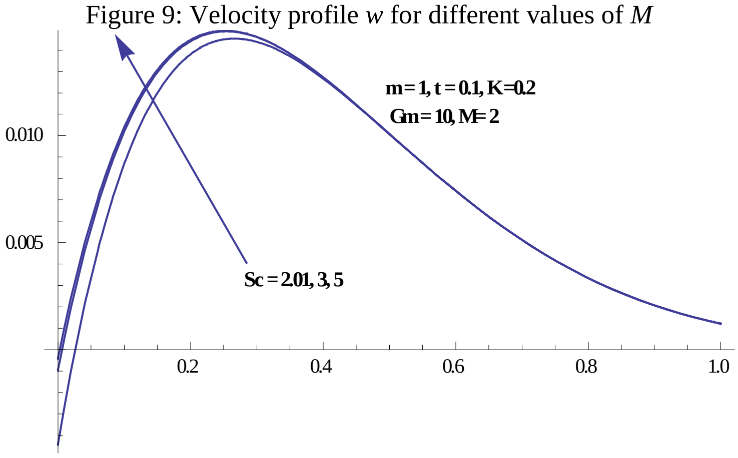

Figure 10: Velocity profile w for different values of *Sc*.

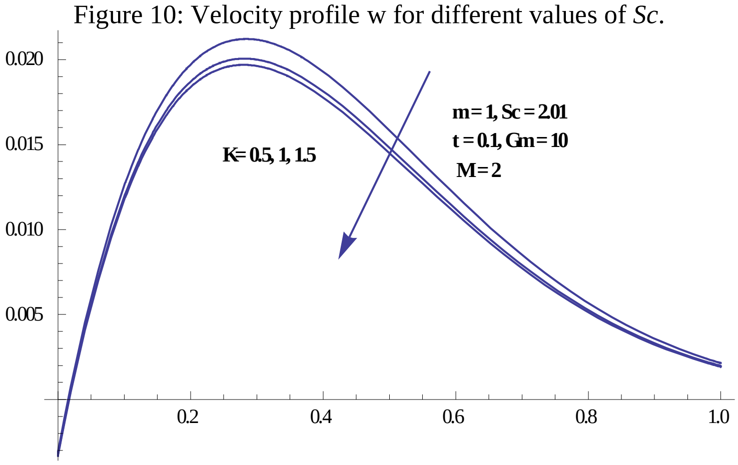

Figure 11: Velocity profile w for different values of *K*

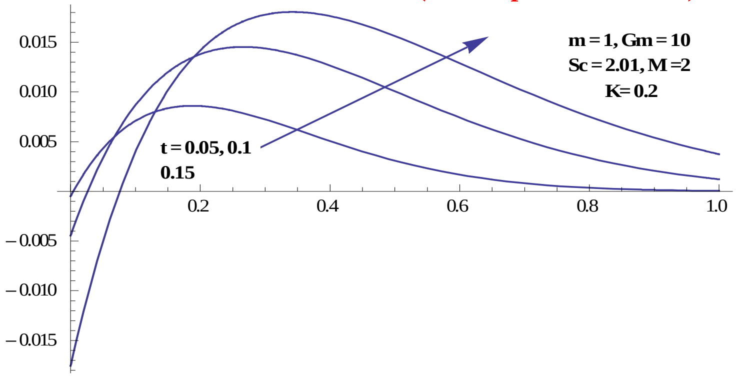

Figure 12: Velocity profile w for different values of *t*

Table 1: The Skin Friction

m	*Gm*	*M*	*Sc*	*K*	*t*	τ_x	τ_z
0.5	10	2	2.01	0.2	0.5	-40.477	09.997
02	10	2	2.01	0.2	0.5	-28.546	06.628
05	10	2	2.01	0.2	0.5	-26.682	02.892
01	15	2	2.01	0.2	0.5	-48.690	15.006
01	20	2	2.01	0.2	0.5	-64.099	19.941
01	10	1	2.01	0.2	0.5	-29.929	04.319
01	10	3	2.01	0.2	0.5	-35.964	17.453
01	10	5	2.01	0.2	0.5	-37.177	37.591
01	10	2	2.01	0.2	0.5	-33.280	10.071
01	10	2	3.00	0.2	0.5	-10.641	01.366
01	10	2	4.00	0.2	0.5	-7.1641	0.6461
01	10	2	2.01	0.5	0.5	-14.836	3.8867
01	10	2	2.01	1.0	0.5	-11.561	02.956
01	10	2	2.01	1.5	0.5	-10.647	02.716
01	10	2	2.01	0.2	0.4	-18.174	03.964
01	10	2	2.01	0.2	0.6	-59.909	23.834

Conclusion:

The influence of Hall effect on unsteady MHD flow past a moving vertical plate with variable mass diffusion in presence of porous medium has been studied in this paper and solutions for the model have been derived by using Laplace -transform technique. The conclusions of the study are summarized as follows:

- ✓ Primary velocity increases with the increase in mass Grashof number, Hall parameter, permeability of the medium and time.
- ✓ Primary velocity decreases with the increase in magnetic field parameter and Schmidt number.
- ✓ Secondary velocity increases with increase in mass Grashof number, time, Hall parameter, Schmidt number and magnetic field parameter.
- ✓ Secondary velocity decreases with the increase, permeability of the medium.
- ✓ The values of τ_x decreases with increase in mass Grashof number, magnetic parameter and time, and it increases when Schmidt number, Hall parameter, permeability of the medium are increased.
- ✓ The values of τ_z increases with increase in mass Grashof number, time, permeability of the medium and magnetic field parameter, and it decreases when Schmidt number is increased.

Appendix:

$$A_1 = \frac{1}{y}(2e^{-\sqrt{a}\,y}(1+e^{2\sqrt{a}\,y}+A_5-e^{2\sqrt{a}\,y}A_6)),$$

$$A_2 = \frac{1}{y}2A_{11}(-1-A_{10}+A_8+A_{10}A_9),$$

$$A_3 = \sqrt{\pi}(A_{10}A_{13}-1-A_{10}-A_{12}),$$

$$A_8 = Erf[\frac{1}{2\sqrt{t}}(y-2t\sqrt{\frac{aSc}{-1+Sc}})],$$

$$A_9 = Erf[\frac{1}{2\sqrt{t}}(y + 2t\sqrt{\frac{aSc}{-1+Sc}})],$$

$$A_{10} = \exp(2y\sqrt{\frac{aSc}{-1+Sc}}),$$

$$A_{11} = e^{\frac{at}{-1+Sc} - y\sqrt{\frac{aSc}{-1+Sc}}},$$

$$A_{12} = Erf[\frac{2t\sqrt{\frac{a}{-1+Sc}} - y\sqrt{Sc}}{2t},$$

$$A_{13} = Erf[\frac{2t\sqrt{\frac{a}{-1+Sc}} + y\sqrt{Sc}}{2t},$$

$$A_4 = \frac{2\sqrt{\pi}(-1 + Erf[\frac{y\sqrt{Sc}}{2\sqrt{t}}])}{y\sqrt{Sc}},$$

$$A_5 = Erf[\frac{2\sqrt{at} - y}{2\sqrt{t}}],\ A_6 = Erf[\frac{2\sqrt{at} + y}{2\sqrt{t}}],$$

$$A_7 = \exp(\frac{at}{-1+Sc} - y\sqrt{\frac{a}{-1+Sc}}\sqrt{Sc}),$$

$$\eta = \frac{y}{2\sqrt{t}},\quad a = \frac{M(1-im)}{1+m^2} + \frac{1}{K}.$$

References:

1. Haider Zaman, Murad Ali Shah, Farhan Khan, Qaiser Javed, Effects of Hall Current on MHD Boundary Layer Second-Order Viscoelastic Fluid FlowInduced by a Continuous Surface with HeatTransfer. American Journal of Computational Mathematics, 4, 143-152. (2014).
2. U.S. Rajput and Surendra kumar, MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion, Applied Mathematical sciences, vol.5,.no 3,149-157, (2011).
3. P. Sulochana Hall Effects on Unsteady MHD Three Dimensional Flow through a Porous Medium in a Rotating Parallel Plate Channel with Effect of Inclined Magnetic Field, American Journal of Computational Mathematics, 4, 396-405, (2014).
4. Ahmed N, Kalita H and. Barua D P., International Journal of Engineering, Science and Technology , Vol. 2, No. 6, pp. 59-74, (2010).
5. R.K. Deka*, Hall effect on MHD flow past an accelerated plate Theoret. Appl. Mech. Vol.35, no. 4, pp. 333-346, Belgrade (2008).
6. Attia, H.A. and katb. N.A, MHD flow between two parallel plate with Heat Transfer. Acta Mechanica, 117, 215-220, (1996).
7. H.A. Attia, The effect of variable properties on the unsteady Hartmann flow with heat transfer considering the Hall effect, Appl. Math. Model. 27 (7) (2003) 551–563.
8. Hazem Ali Attia, Mohamed Eissa Sayed- Ahmed, the Hall effect on unsteady MHD Couette flow and heat transfer of a Bingham fluid with suction and injection, applied Mathematical Modelling 28, 1027-1045, (2004).
9. P.C. Ram, C.B. Singh, U. Singh, Hall effects on heat and mass transfer flow through porous medium, Astrophysics Space Science, 10, 45-51, (1984).
10. R.B. Hetnarski, algorithm for generating some inverse Laplace transforms of exponential form, ZAMP, 26, 249-253,(1975).
11. Crammer K .P. and Pai.S.I. Magneto fluid Dynamics for engineers and Applied physicist. MC Graw. Hill Book, (1973).
12. POP, I.: The effect of Hall currents on hydro magnetic, flow near an accelerated plate. J. Math. Phys. Sci.5, 375-379, (1971).
13. N. Ahmed, H. Kalita and D. P. Barua ,Unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source. International Journal of Engineering, Science and Technology, Vol. 2, No. 6, pp. 59-74, (2010).

14. Debnath L. Ray, S.C. and Chatterjee, A.K, Effect of Hall current on unsteady hydro magnetic flow past a porous plate in a Rotating fluid system. Zeitschrift für Angewandte Mathematik and Mechanik, 59, 469-471, (1979).
15. Prasada Rao, D.R.V., Krishna, D.V. and Debnath. L., Hall Effects on Free and Forced Convective Flow in Rotating Channel. Acta Mechanica, 43, 49-59, (1982).
16. A. Raptis and C. Perdikis, Flow of a viscous fluid through a porous medium bounded by a vertical surface, Int. J. Eng. Sci. 2, 1327–1330, (1983).
17. Abo. Eldahab, EM, The hall effect on magnetohydrodynamic free convective flow at stretching surface with a uniform free stream, physics scripta, 63; 29-35, (2001).