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CUBIC IDEALS OF PO-Γ-SEMIGROUPS

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Abstract:

In this paper, we introduce the concept of cubic ideals of po-Γ-semigroups, which is generalized concept of fuzzy ideals of po-Γ-semigroups in order to obtain some characterization theorems. Operator po-semigroups of a po-Γ-semigroup have been made to work by obtaining various relationships between cubic ideals of a po-Γ-semigroup and that of its operator semigroups. We also investigated some of its properties with examples.

Key Words: Semigroup, Γ-Semigroup, po-Γ-Semigroup, Ideal, Bi-Ideal, Interior Ideal, Cubic Ideal, Cubic Bi-Ideal, Cubic Interior Ideal, Left Operator & Right Operator.

1. Introduction:

The important concept of fuzzy set has been introduced by Lofti.A. Zadeh [25]. Since then the work on fuzzy sets appeared showing its importance in various branches of mathematics. In 1971 Rosenfeld [18] introduced the concept of fuzzy group. R. Biswas [1] introduced the notion of fuzzy subgroup and antifuzzy subgroups. Semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation. The formal study of semigroups began in the early 20^{th} century. Kuroki [12, 13, 14] characterized several class of semigroups in terms of fuzzy left ideals, fuzzy right ideals, fuzzy bi-ideals. In [23, 24] X.Y. Xie introduced the ideal extensions of fuzzy ideals in semigroups. The idea of fuzzy bi-ideals in semigroups has been introduced by W.J. Lie [16]. In 1984 the notion of Γ-semigroups was introduced by M.K. Sen [20]. In 1986 M.K. Sen and N.K. Saha [21] modified the definition of sen's Γ-semigroup. This newly defined Γ-semigroup is known as one sided Γ-semigroup. Γ-semigroup has been analyzed by a lot of mathematicians, for instance by Chattopadhyay [2], T.K. Dutta and N.C. Adhikari [6, 7, 8]. They defined different properties of po-Γ-semigroup by defining operator po-semigroups of such type of po-Γ-semigroups. In 1996 Y.I. Kwon and S.K. Lee [15] introduced po-Γ-semigroups (partially ordered Γ-semigroups). Jun et al. [10] introduced the new concept called cubic sets. These structures encompass interval-valued fuzzy set and fuzzy set. Also Jun et al. [9] introduced the notion of cubic subgroups. V. Chinnadurai et al. [4] introduced cubic ideals of Γ-semigroups. The purpose of this paper to introduce the notion of cubic ideals of po- Γ-semigroups. We investigate some basic results, examples and properties.

2. Preliminaries:

Now we recall some known concepts related to cubic ideals of po-Γ-semigroups from the literature, which will be needed in the sequel.

Definition 2.1: [5] Let S and Γ be two non-empty sets. Then S is called a Γ-semigroup if there exist a mapping from $S \times \Gamma \times S \to S$ written as $(a, \alpha, b) \to a\alpha b$ satisfying the identity $(a\alpha b)\beta c = a\alpha(b\beta c), for\ all\ a, b, c \in S\ and\ \alpha, \beta \in \Gamma$.

Definition 2.2: [3] Let S be a semigroup. By a subsemigroup of S, we mean a non-empty subset A of S such that $A^2 \subseteq A$.

Definition 2.3: [3] Let S be a Γ-semigroup. By Γ-subsemigroup, we mean a non-empty subset A of S such that if $A\Gamma A \subseteq A$.

Definition 2.4: [7] A Γ-semigroup S is called po-Γ-semigroup if for any $a, b, c \in S$ and $\alpha \in \Gamma$, $a \leq b$ implies $a\alpha c \leq b\alpha c$ and $c\alpha a \leq c\alpha b$.

Definition 2.5: [7] Let S be a po-Γ-semigroup. A non-empty subset A of S is said to be right (left) ideal of S if

i) $A\Gamma S \subseteq A$ ($S\Gamma A \subseteq A$)

ii) if $x \in A$ and $y \in S$, such that $y \leq x$ then $y \in A$.

Definition 2.6: [22] Let S be a po-Γ-semigroup. A Γ-subsemigroup A of S is said to be bi-ideal of S if

$i)\ A\Gamma S\Gamma A \subseteq A$

$ii)\ if\ x \in A$ and $y \in S$, such that $y \leq x$ then $y \in A$.

Definition 2.7: [17] Let S be a po-Γ-semigroup. A Γ-subsemigroup A of S is said to be interior ideal of S if

$i)\ S\Gamma A\Gamma S \subseteq A$

$ii)\ if\ x \in A$ and $y \in S$, such that $y \leq x$ then $y \in A$.

Definition 2.8: [5] A fuzzy subset μ of a po-Γ-semigroup S is called a fuzzy Γ- subsemigroup of S, if $\mu(x\gamma y) \geq \min\{\mu(x), \mu(y)\}$. $\forall\ x, y \in S\ and\ \forall\ \gamma \in \Gamma$.

Definition 2.9: [19] A fuzzy subset μ of a po-Γ-semigroup S is called a fuzzy right (left) ideal of S, if

$i)\ x \leq y \Rightarrow \mu(x) \geq \mu(y)$

$ii)\mu(x\alpha y) \geq \mu(x),\ (\mu(x\alpha y) \geq \mu(y))\ \ \forall x, y \in S.$ and $\alpha \in \Gamma$. if μ is both a fuzzy left and a fuzzy right ideal of S, then μ is called a fuzzy ideal of S.

Definition 2.10: [22] A fuzzy Γ-subsemigroup μ of a po-Γ- semigroup S is called a fuzzy bi-ideal of S, if
$i)\ x \leq y \Rightarrow \mu(x) \geq \mu(y)$
$ii)\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\}\ \forall x, y, z \in S.$ and $\alpha, \beta \in \Gamma$.

Definition 2.11: [17] A fuzzy Γ-subsemigroup μ of a po-Γ- semigroup S is called a fuzzy interior ideal of S, if
$i)\ x \leq y \Rightarrow \mu(x) \geq \mu(y)$
$ii)\mu(x\alpha y\beta z) \geq \mu(y)\ \ \ \forall x, y, z \in S.$ and $\alpha, \beta \in \Gamma$.

Definition 2.12: [3] An interval-valued fuzzy subset $\bar{\mu}$ of X is called a fuzzy left (right) ideal of X, if $\bar{\mu}(xy) \geq \bar{\mu}(y),\ (\bar{\mu}(xy) \geq \bar{\mu}(x)), \forall x, y \in X$. if $\bar{\mu}$ is both an interval-valued fuzzy left and right ideal of X, then $\bar{\mu}$ is called an interval-valued fuzzy ideal of X.

Definition 2.13: [7] Let S be a Γ- semigroup. Let us define a relation ρ on $S \times \Gamma$ as follows $(x, \alpha)\rho(y, \beta)$ if and only if $x\alpha s = y\beta s\ \forall\ s \in S$ and $\gamma x\alpha = \gamma y\beta\ \ \forall\ \gamma \in \Gamma$. The ρ is an equivalence relation. Let $[x, \alpha]$ denote the equivalence class containing (x, α). Let $L = \{[x, \alpha]: x \in S, \alpha \in \Gamma\}$. Then L is a semigroup with respect to the multiplication defined by $[x, \alpha][y, \beta] = [x\alpha y, \beta]$. This semigroup L is called the left operator semigroup of the Γ-semigroup S. Dually the right operator semigroup R of Γ-semigroup S is defined where the multiplication is defined by $[\alpha, a][\beta, b] = [\alpha a\beta, b]$.

Let $((S, \Gamma), \leq)$ be a po-Γ- semigroup. We define a relation $\leq$ on L by $[a, \alpha] \leq [b, \beta]$ if and only if $a\alpha s \leq b\beta s$ for all $s \in S$ and $\gamma a\alpha \leq \gamma b\beta$ for all $\gamma \in \Gamma$. With respect to this relation L becomes a po-semigroup. In a similar way R can be made into a po-semigroup.

If there exist an element $[e, \delta] \in L$ ($[\gamma, f] \in R$) such that $e\delta s = s$(resp. $s\gamma f = s$) for all $s \in S$ then $[e, \delta]$ (resp. $[\gamma, f]$) is called the left (resp. right) unity of S.

Now let S be a Γ- semigroup with unities, L and R are po-semigroups. We define a relation $\leq$ in S by $a \leq b$ if and only if $[a, \alpha] \leq [b, \alpha]$ in L and $[\alpha, a] \leq [\alpha, b]$ in R for all $\alpha \in \Gamma$. We also define a relation $\leq$ in Γ by $\alpha \leq \beta$ if and only if $[a, \alpha] \leq [a, \beta]$ in L and $[\alpha, a] \leq [\beta, a]$ in R for all $a \in S$. With respect these relations S becomes a po-Γ- semigroup.

Definition 2.14: [9] Let X be a non empty set. A cubic set $\mathcal{A}$ is a structure of the form $\mathcal{A}$=$\{\langle x, \bar{\mu}(x), \lambda(x)\rangle : x \in X\}$ and denoted by $\mathcal{A} = \langle \bar{\mu}_A, \lambda\rangle, \bar{\mu}_A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set (briefly. IVF) in X and $\lambda: X \to [0,1]$ is a fuzzy set in X.

Definition 2.15: [11] For any non-empty subset G of a set X, the characteristic cubic set of G is defined to be a structure$\chi_G(x) =< x, \bar{\mu}_{\chi_G}(x), \gamma_{\chi_G}(x): x \in X >$ which is briefly denoted by $\chi_G(x) =< \bar{\mu}_{\chi_G}(x), \gamma_{\chi_G}(x) >$. Where $\bar{\mu}_{\chi_G}(x) = \begin{cases}[1,1] & if\ x \in G\\ [0,0] & otherwise\end{cases}$ and $\gamma_{\chi_G}(x) = \begin{cases}0 & if\ x \in G\\ 1 & otherwise\end{cases}$

Definition 2.16: [2] A mapping f from a Γ- semigroup P to another Γ- semigroup Q is called a homomorphism, if $f(x\gamma y) = f(x)\gamma f(y)\ \ \ \forall\ x, y \in S\ and\ \forall\ \gamma \in \Gamma$.

Definition 2.17: [9] Let X and Y be given classical sets. A mapping f: X→Y induces two mappings
$C_f: C(X) \to C(Y), \mathcal{A}_1 \to C_f(\mathcal{A}_1)$ and $C_f^{-1}: C(Y) \to C(X), \mathcal{A}_2 \to C_f^{-1}(\mathcal{A}_2)$.
Where the mapping C_f is called cubic transformation and C_f^{-1} is called inverse cubic transformation.

Definition 2.18: [9] Let f be a mapping from a set X to Y and $\mathcal{A} = < \bar{\mu}, \omega >$ be a cubic set of X then the image of X $C_f(\mathcal{A}) = < C_f(\bar{\mu}), C_f(\omega) >$ is a cubic set of Q is defined by

$$C_f(\mathcal{A})(y) = \begin{cases} C_f(\bar{\mu})(y) = \begin{cases} \sup\limits_{f(x)=y} \bar{\mu}(x) & if\ f^{-1}(y) \neq \emptyset\ for\ all\ y \in Y \\ [0,0] & otherwise \end{cases} \\ C_f(\omega)(y) = \begin{cases} \inf\limits_{f(x)=y} \omega(x) & if\ f^{-1}(y) \neq \emptyset\ for\ all\ y \in Y \\ 1 & otherwise \end{cases} \end{cases}$$

And

Let f be a mapping from a set X to Y and $\mathcal{A} = < \bar{\mu}, \omega >$ be a cubic set of Y then the pre image of Y $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic set of X is defined by

$$C_f^{-1}(\mathcal{A})(x) = \begin{cases} C_f^{-1}(\bar{\mu}(x)) = \bar{\mu}(f(x))\ \ for\ all\ x \in X \\ C_f^{-1}(\omega(x)) =\ \omega(f(x))\ \ for\ all\ x \in X \end{cases}$$

3. Main Results:

In this section we introduced the notion of cubic ideals of po-Γ- semigroups and discuss some of its properties with examples.

Definition 3.1: A cubic subset $\mathcal{A} = < \bar{\mu}, \omega >$ of a po-Γ- semigroup S is called a cubic right (left) ideal of S, if
$i)\ x \leq y \Rightarrow \bar{\mu}(x) \geq \bar{\mu}(y)\ and\ \omega(x) \leq \omega(y)$
$ii)\bar{\mu}(x\alpha y) \geq \bar{\mu}(x)\ and\ \omega(x\alpha y) \leq \omega(x)\ \ (\bar{\mu}(x\alpha y) \geq \bar{\mu}(y)\ and\ \omega(x\alpha y) \leq \omega(y))\ \ \forall x, y \in S.$ and $\alpha \in \Gamma$.
if $\mathcal{A} = < \bar{\mu}, \omega >$ is both a cubic left and a cubic right ideal of S, then $\mathcal{A} = < \bar{\mu}, \omega >$ is called a cubic ideal of S.

Example 3.2: Let S be the set of all non-positive integers without zero and Γ be the set of all non-positive even

integers without zero. Then S is a Γ- semigroup where $x\alpha y$ denote the usual multiplication of integers x, α, y for all $x, y \in S$ and $\alpha \in \Gamma$. Then by routine calculation it is observed that S is a po-Γ- semigroup.
Define an interval-valued fuzzy set $\bar{\mu}$:S→D[0,1] by

$$\bar{\mu}(x) = \begin{cases} [0.1,0.2] & if\ x = -1 \\ [0.3,0.4] & if x = -2 \\ [0.5,0.6] & if\ x < -2 \end{cases}$$

Define a fuzzy set ω:S→[0,1] by

$$\omega(x) = \begin{cases} 0.8 & if\ x = -1 \\ 0.4 & if x = -2 \\ 0.2 & if\ x < -2 \end{cases}$$

Hence $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic ideal of po-Γ-semigroup S.

Definition 3.3: A cubic subset $\mathcal{A} = <\bar{\mu}, \omega>$ of a po-Γ-semigroup S is called a cubic Γ- subsemigroup of S, if
$i)\ \bar{\mu}(x\gamma y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$
$ii)\ \omega(x\gamma y) \leq \max\{\omega(x), \omega(y)\}\ \ \forall\ x, y \in S\ and\ \forall\ \gamma \in \Gamma.$

Definition 3.4: A cubic Γ-subsemigroup $\mathcal{A} = <\bar{\mu}, \omega>$ of a po-Γ-semigroup S is called a cubic bi-ideal of S, if
$i)\ x \leq y \Rightarrow \bar{\mu}(x) \geq \bar{\mu}(y)\ and\ \omega(x) \leq \omega(y)$
$ii) \bar{\mu}(x\alpha y\beta z) \geq \min\{\bar{\mu}(x), \bar{\mu}(z)\}\ and\ \omega(x\alpha y\beta z) \leq \max\{\omega(x), \omega(z)\}\ \forall x, y, z \in S$ and $\alpha, \beta \in \Gamma$.

Definition 3.5: A cubic Γ-subsemigroup $\mathcal{A} = <\bar{\mu}, \omega>$ of a po-Γ-semigroup S is called a cubic interior ideal of S, if
$i)\ x \leq y \Rightarrow \bar{\mu}(x) \geq \bar{\mu}(y)\ and\ \omega(x) \leq \omega(y)$
$ii) \bar{\mu}(x\alpha y\beta z) \geq \bar{\mu}(y)\ and\ \omega(x\alpha y\beta z) \leq \omega(y)\ \ \ \forall x, y, z \in S$ and $\alpha, \beta \in \Gamma$.

Definition 3.6: Let f be a mapping from a po-Γ-semigroup S to a po-Γ-semigroup $S^{\prime}$ and $\mathcal{A} = <\bar{\mu}, \omega>$ be a cubic set of $S^{\prime}$ then the pre image of $S^{\prime}$ $C_f^{-1}(\mathcal{A}) = <C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega)>$ is a cubic set of S is defined by

$$C_f^{-1}(\mathcal{A})(x) = \begin{cases} C_f^{-1}(\bar{\mu}(x)) = \bar{\mu}(f(x)) & for\ all\ x \in S \\ C_f^{-1}(\omega(x)) = \omega(f(x)) & for\ all\ x \in S \end{cases}$$

Definition 3.7: Let S be a po-Γ-semigroup, L be the left operator and R be the right operator, then
For a cubic set (R) $\mathcal{A}_1 = <\bar{\mu}_1, \omega_1>$ we define a cubic set (S) $\mathcal{A}_1^* = <\bar{\mu}_1, \omega_1>^* = <(\bar{\mu}_1)^*, (\omega_1)^*>$ by $(\bar{\mu}_1)^*(x) = \inf_{\gamma \in \Gamma} \bar{\mu}_1([\gamma, x])$ and $(\omega_1)^*(x) = \sup_{\gamma \in \Gamma} \omega_1([\gamma, x])$ where $x \in S$.
For a cubic set (S) $\mathcal{A}_2 = <\bar{\mu}_2, \omega_2>$ we define a cubic set (R) $\mathcal{A}_2^{*\prime} = <\bar{\mu}_2, \omega_2>^{*\prime} = <(\bar{\mu}_2)^{*\prime}, (\omega_2)^{*\prime}>$ by $(\bar{\mu}_2)^{*\prime}([\alpha, x]) = \inf_{s \in S} \bar{\mu}_2(s\alpha x)$ and $(\omega_2)^{*\prime}([\alpha, x]) = \sup_{s \in S} \omega_2(s\alpha x)$ where $[\alpha, x] \in R$.
For a cubic set (L) $\mathcal{A}_3 = <\bar{\mu}_3, \omega_3>$ we define a cubic set (S) $\mathcal{A}_3^+ = <\bar{\mu}_3, \omega_3>^+ = <(\bar{\mu}_3)^+, (\omega_3)^+>$ by $(\bar{\mu}_3)^+(x) = \inf_{\gamma \in \Gamma} \bar{\mu}_3([x, \gamma])$ and $(\omega_3)^+(x) = \sup_{\gamma \in \Gamma} \omega_3([x, \gamma])$ where $x \in S$.
For a cubic set (S) $\mathcal{A}_4 = <\bar{\mu}_4, \omega_4>$ we define a cubic set (L) $\mathcal{A}_4^{+\prime} = <\bar{\mu}_4, \omega_4>^{+\prime} = <(\bar{\mu}_4)^{+\prime}, (\omega_4)^{+\prime}>$ by $(\bar{\mu}_4)^{+\prime}([x, \alpha]) = \inf_{s \in S} \bar{\mu}_4(x\alpha s)$ and $(\omega_4)^{+\prime}([x, \alpha]) = \sup_{s \in S} \omega_4(x\alpha s)$ where $[x, \alpha] \in L$.

Theorem 3.8: Every cubic ideal of a po-Γ-semigroup is cubic bi-ideal of a po- Γ-semigroup.
Proof: Let S be a po- Γ-semigroup and $\mathcal{A} = <\bar{\mu}, \omega>$be a cubic ideal of S.
For any $x, y \in S$ with x≤ $y, \bar{\mu}(x) \geq \bar{\mu}(y)$ and $\omega(x) \leq \omega(y)$
case(i) Suppose $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic left ideal of a po-Γ-semigroup S
$\bar{\mu}(x\alpha y) \geq \bar{\mu}(y), \omega(x\alpha y) \leq \omega(y)\ for\ all\ x, y \in S\ and\ \alpha \in \Gamma.$
For any $x, y, z \in S$ and $\alpha, \beta \in \Gamma$, we have
$\bar{\mu}(x\alpha y\beta z) = \bar{\mu}(x\alpha(y\beta z)) \geq \bar{\mu}(y\beta z) \geq \bar{\mu}(z)$ and
$\omega(x\alpha y\beta z) = \omega(x\alpha(y\beta z)) \leq \omega(y\beta z) \leq \omega(z)$
case(ii) suppose $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic right ideal of a po-Γ-semigroup S
$\bar{\mu}(x\alpha y) \geq \bar{\mu}(x), \omega(x\alpha y) \leq \omega(x)\ for\ all\ x, y \in S\ and\ \alpha \in \Gamma.$
For any $x, y, z \in S$ and $\alpha, \beta \in \Gamma$, we have
$\bar{\mu}(x\alpha y\beta z) = \bar{\mu}((x\alpha y)\beta z)) \geq \bar{\mu}(x\alpha y) \geq \bar{\mu}(x)$ and
$\omega(x\alpha y\beta z) = \omega((x\alpha y)\beta z) \leq \omega(x\alpha y) \leq \omega(x)$
From the above two cases, we have
$\bar{\mu}(x\alpha y) \geq \bar{\mu}(x) \wedge \bar{\mu}(y) = \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and
$\omega(x\alpha y) \leq \omega(x) \vee \omega(y) = \max\{\omega(x), \omega(y)\}$
Again, $\bar{\mu}(x\alpha y\beta z) \geq \bar{\mu}(x) \wedge \bar{\mu}(z) = \min\{\bar{\mu}(x), \bar{\mu}(z)\}$ and
$\omega(x\alpha y\beta z) \leq \omega(x) \vee \omega(z) = \max\{\omega(x), \omega(z)\}$
Hence $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic bi-ideal of S.

Theorem 3.9: Let $\mathcal{A} = <\bar{\mu}, \omega>$be a non-empty cubic subsets of a po- Γ-semigroup S and $\chi_{\mathcal{A}} = <\bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}}>$ be the characteristic cubic set of $\mathcal{A}$. Then $\mathcal{A}$ is a left ideal (resp. right ideal, ideal) of S if and only if the characteristic cubic set $\chi_{\mathcal{A}} = <\bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}}>$ of $\mathcal{A}$ in S is a cubic left ideal

(resp. right ideal, ideal) of S.
Proof: Let $\mathcal{A}$ be a non-empty cubic subsets of S and $\mathcal{A}$ be a left ideal of S.
Let $x, y \in S$ and $\alpha \in \Gamma$. Then $x\alpha y \in \mathcal{A}$.
If $y \in \mathcal{A}$, it follows that $\bar{\mu}_{\chi_{\mathcal{A}}}(x\alpha y) = \bar{1} \geq \bar{\mu}_{\chi_{\mathcal{A}}}(y)$ and $\omega_{\chi_{\mathcal{A}}}(x\alpha y) = 0 \leq \omega_{\chi_{\mathcal{A}}}(y)$
If $y \notin \mathcal{A}$, it follows that $\bar{\mu}_{\chi_{\mathcal{A}}}(x\alpha y) = \bar{0} \geq \bar{\mu}_{\chi_{\mathcal{A}}}(y)$ and $\omega_{\chi_{\mathcal{A}}}(x\alpha y) = 1 \leq \omega_{\chi_{\mathcal{A}}}(y)$
Again, let $x, y \in S$ be such that $x \leq y$, if $y \in \mathcal{A}$ then $\bar{\mu}_{\chi_{\mathcal{A}}}(y) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(y) = 0$
Since $x \leq y \in \mathcal{A}$, we have $x \in \mathcal{A}$, then $\bar{\mu}_{\chi_{\mathcal{A}}}(x) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(x) = 0$
Hence $\bar{\mu}_{\chi_{\mathcal{A}}}(x) \geq \bar{\mu}_{\chi_{\mathcal{A}}}(y)$ and $\omega_{\chi_{\mathcal{A}}}(x) \leq \omega_{\chi_{\mathcal{A}}}(y)$.
If $y \notin \mathcal{A}$, then $\bar{\mu}_{\chi_{\mathcal{A}}}(y) = \bar{0}$ and $\omega_{\chi_{\mathcal{A}}}(y) = 1$
Hence we have $\bar{\mu}_{\chi_{\mathcal{A}}}(x) \geq \bar{0} = \bar{\mu}_{\chi_{\mathcal{A}}}(y)$ and $\omega_{\chi_{\mathcal{A}}}(x) \leq 1 = \omega_{\chi_{\mathcal{A}}}(y)$
Therefore $\chi_{\mathcal{A}} = < \bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}} >$ is a cubic left ideal of S.
Conversly, let $\chi_{\mathcal{A}} = < \bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}} >$ be the cubic left ideal of S, let $x, y \in S$ and $y \in \mathcal{A}$.
Then $\bar{\mu}_{\chi_{\mathcal{A}}}(y) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(y) = 0$. Since $\chi_{\mathcal{A}}$ is a cubic left ideal of S, then
$\bar{\mu}_{\chi_{\mathcal{A}}}(x\alpha y) \geq \bar{\mu}_{\chi_{\mathcal{A}}}(y)$ and $\omega_{\chi_{\mathcal{A}}}(x\alpha y) \leq \omega_{\chi_{\mathcal{A}}}(y)$.
So, $\bar{\mu}_{\chi_{\mathcal{A}}}(x\alpha y) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(x\alpha y) = 0$, then $x\alpha y \in \mathcal{A}$.
Now, if $x \leq y$ and $y \in \mathcal{A}$, then $\bar{\mu}_{\chi_{\mathcal{A}}}(y) = \bar{1}$, $\omega_{\chi_{\mathcal{A}}}(y) = 0$ and
$\bar{\mu}_{\chi_{\mathcal{A}}}(x) \geq \bar{\mu}_{\chi_{\mathcal{A}}}(y)$, $\omega_{\chi_{\mathcal{A}}}(x) \leq \omega_{\chi_{\mathcal{A}}}(y)$
So, $\bar{\mu}_{\chi_{\mathcal{A}}}(x) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(x) = 0$, then $x \in \mathcal{A}$.
Thus $x \leq y$ and $y \in \mathcal{A}$, then $x \in \mathcal{A}$.
Therefore $\mathcal{A} = < \bar{\mu}, \omega >$ is left ideal of S.
Theorem 3.10: Let $\mathcal{A} = < \bar{\mu}, \omega >$be a non-empty cubic subsets of a po- Γ-semigroup S and $\chi_{\mathcal{A}} = < \bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}} >$ be the characteristic cubic set of $\mathcal{A}$. Then $\mathcal{A}$ is an interior ideal of S if and only if the characteristic cubic set $\chi_{\mathcal{A}} = < \bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}} >$ of $\mathcal{A}$ in S is a cubic interior ideal of S.
Theorem 3.11: Let $\mathcal{A} = < \bar{\mu}, \omega >$ be a non-empty cubic subsets of a po- Γ-semigroup S and $\chi_{\mathcal{A}} = < \bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}} >$ be the characteristic cubic set of $\mathcal{A}$. Then $\mathcal{A}$ is a bi-ideal of S if and only if the characteristic cubic set $\chi_{\mathcal{A}} = < \bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}} >$ of $\mathcal{A}$ in S is a cubic bi- ideal of S.
Proof: Let $\mathcal{A}$ be a non-empty subset of a po-Γ-semigroup S and $\mathcal{A}$ be a bi-ideal of S.
Let $x, y, z \in S$. Then $xyz \in \mathcal{A}$, if $x, z \in \mathcal{A}$ and $x\alpha y\beta z \in \mathcal{A}$, if $x, y, z \in \mathcal{A}$ and $\alpha, \beta \in \Gamma$.
It follows that $\bar{\mu}_{\chi_{\mathcal{A}}}(x\alpha y\beta z) = \bar{1} = \min\{\bar{\mu}_{\chi_{\mathcal{A}}}(x), \bar{\mu}_{\chi_{\mathcal{A}}}(z)\}$ and $\omega_{\chi_{\mathcal{A}}}(x\alpha y\beta z) = 0 = \max\{\omega_{\chi_{\mathcal{A}}}(x), \omega_{\chi_{\mathcal{A}}}(z)\}$
Let $x \notin \mathcal{A}$ or $z \notin \mathcal{A}$. Then,
case(i) If $x\alpha y\beta z \notin \mathcal{A}$, then $\bar{\mu}_{\chi_{\mathcal{A}}}(x\alpha y\beta z) \geq \bar{0} = \min\{\bar{\mu}_{\chi_{\mathcal{A}}}(x), \bar{\mu}_{\chi_{\mathcal{A}}}(z)\}$ and
$\omega_{\chi_{\mathcal{A}}}(x\alpha y\beta z) \leq 1 = \max\{\omega_{\chi_{\mathcal{A}}}(x), \omega_{\chi_{\mathcal{A}}}(z)\}$
case(ii) If $x\alpha y\beta z \in \mathcal{A}$, then $\bar{\mu}_{\chi_{\mathcal{A}}}(x\alpha y\beta z) \geq \bar{1} = \min\{\bar{\mu}_{\chi_{\mathcal{A}}}(x), \bar{\mu}_{\chi_{\mathcal{A}}}(z)\}$ and
$\omega_{\chi_{\mathcal{A}}}(x\alpha y\beta z) \leq 0 = \max\{\omega_{\chi_{\mathcal{A}}}(x), \omega_{\chi_{\mathcal{A}}}(z)\}$
Again, let $x, y \in S$ be such that $x \leq y$.
If $y \in \mathcal{A}$, then $\bar{\mu}_{\chi_{\mathcal{A}}}(y) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(y) = 0$.
Since $x \leq y \in \mathcal{A}$, we have $x \in \mathcal{A}$, then $\bar{\mu}_{\chi_{\mathcal{A}}}(x) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(x) = 0$
Thus $\bar{\mu}_{\chi_{\mathcal{A}}}(x) \geq \bar{\mu}_{\chi_{\mathcal{A}}}(y)$ *and* $\omega_{\chi_{\mathcal{A}}}(x) \leq \omega_{\chi_{\mathcal{A}}}(y)\}$
Hence $\chi_{\mathcal{A}} = < \bar{\mu}_{\chi_{\mathcal{A}}}, \omega_{\chi_{\mathcal{A}}} >$ is a cubic bi-ideal of S.
Conversly, let $\chi_{\mathcal{A}}$ be the cubic bi-ideal of S.
Let $x, y, z \in S$ be such that $x, z \in \mathcal{A}$ then $\bar{\mu}_{\chi_{\mathcal{A}}}(x) = \bar{1} = \bar{\mu}_{\chi_{\mathcal{A}}}(z)$ and $\omega_{\chi_{\mathcal{A}}}(x) = 0 = \omega_{\chi_{\mathcal{A}}}(z)$.
Also, $\bar{\mu}_{\chi_{\mathcal{A}}}(x\alpha y\beta z) \geq \min\{\bar{\mu}_{\chi_{\mathcal{A}}}(x), \bar{\mu}_{\chi_{\mathcal{A}}}(z)\} = \bar{1}$ and
$\omega_{\chi_{\mathcal{A}}}(x\alpha y\beta z) \leq \max\{\omega_{\chi_{\mathcal{A}}}(x), \omega_{\chi_{\mathcal{A}}}(z)\} = 0$.
Hence $x\alpha y\beta z \in \mathcal{A}$.
Again, let $x \in \mathcal{A}$ and $x \leq y$. Then we have $y \in \mathcal{A}$.
This implies that $\bar{\mu}_{\chi_{\mathcal{A}}}(y) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(y) = 0$.
Since $\chi_{\mathcal{A}}$ is a cubic bi-ideal of S, then $\bar{\mu}_{\chi_{\mathcal{A}}}(x) = \bar{1} \geq \bar{\mu}_{\chi_{\mathcal{A}}}(y)$ and $\omega_{\chi_{\mathcal{A}}}(x) = 0 \leq \omega_{\chi_{\mathcal{A}}}(y)$
Since $y \in$ S, we have $\bar{\mu}_{\chi_{\mathcal{A}}}(y) \geq \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(y) \leq 0$.
Hence $\bar{\mu}_{\chi_{\mathcal{A}}}(y) = \bar{1}$ and $\omega_{\chi_{\mathcal{A}}}(y) = 0$. So, $y \in \mathcal{A}$.
Consequently, $\mathcal{A}$ is a bi- ideal of S.
Theorem 3.12: Let $\mathcal{A} = < \bar{\mu}, \omega >$ be a cubic left (right) ideal of a po- Γ-semigroup S. Then so is $\mathcal{A}^{\alpha} = < (\bar{\mu})^{\alpha}, (\omega)^{\alpha} >$ for every real number $\alpha \geq 0$, where $\mathcal{A}^{\alpha}$ defined by $\mathcal{A}^{\alpha}(x) = (\mathcal{A}(\mathrm{x}))^{\alpha}$ for all $x \in S$.
Proof: Let $\mathcal{A} = < \bar{\mu}, \omega >$ be a cubic left ideal of a po-Γ-semigroup S. For any $x, y \in S$ and $\gamma \in \Gamma$.
We have $\bar{\mu}(x\alpha y) \geq \bar{\mu}(y)$ *and* $\omega(x\alpha y) \leq \omega(y)$

Now $(\bar{\mu})^{\alpha}(x\alpha y) = \left(\bar{\mu}(x\alpha y)\right)^{\alpha} \geq \left(\bar{\mu}(y)\right)^{\alpha} = (\bar{\mu})^{\alpha}(y)$ and

$(\omega)^{\alpha}(x\alpha y) = \left(\omega(x\alpha y)\right)^{\alpha} \leq \left(\omega(y)\right)^{\alpha} = (\omega)^{\alpha}(y)$.

Next, let $x, y \in S$, be such that $x \leq y$. Since $\mathcal{A}$ is a cubic left ideal of S.

So, $\bar{\mu}(x) \geq \bar{\mu}(y)$ *and* $\omega(x) \leq \omega(y)$ for all $x, y \in S$.

Then $\left(\bar{\mu}(x)\right)^{\alpha} \geq \left(\bar{\mu}(y)\right)^{\alpha}$ and $\left(\omega(x)\right)^{\alpha} \leq \left(\omega(y)\right)^{\alpha}$ for all $\alpha \geq 0$.

therefore $(\bar{\mu})^{\alpha}(x) \geq (\bar{\mu})^{\alpha}(y)$ and $(\omega)^{\alpha}(x) \leq (\omega)^{\alpha}(y)$

Hence $\mathcal{A}^{\alpha} =< (\bar{\mu})^{\alpha}, (\omega)^{\alpha} >$ is a cubic left (right) ideal of S.

Theorem 3.13: Let $\mathcal{A} = < \bar{\mu}, \omega >$ be a cubic interior ideal of a po- Γ-semigroup S. Then so is $\mathcal{A}^{\alpha} =< (\bar{\mu})^{\alpha}, (\omega)^{\alpha} >$ for every real number $\alpha \geq 0$, where $\mathcal{A}^{\alpha}$ defined by $\mathcal{A}^{\alpha}(x) = \left(\mathcal{A}(x)\right)^{\alpha}$ for all $x \in S$.

Proof: Let $\mathcal{A} = < \bar{\mu}, \omega >$ be a cubic interior ideal of a po-Γ-semigroup S. For any $x, y, z \in S$ and $\gamma, \beta \in \Gamma$.

We have $\bar{\mu}(x\gamma y\beta z) \geq \bar{\mu}(y)$ *and* $\omega(x\gamma y\beta z) \leq \omega(y)$

Now $(\bar{\mu})^{\alpha}(x\gamma y\beta z) = \left(\bar{\mu}(x\gamma y\beta z)\right)^{\alpha} \geq \left(\bar{\mu}(y)\right)^{\alpha} = (\bar{\mu})^{\alpha}(y)$ and

$(\omega)^{\alpha}(x\gamma y\beta z) = \left(\omega(x\gamma y\beta z)\right)^{\alpha} \leq \left(\omega(y)\right)^{\alpha} = (\omega)^{\alpha}(y)$.

Next, let $x, y \in S$, be such that $x \leq y$. Since $\mathcal{A}$ is a cubic left ideal of S.

So, $\bar{\mu}(x) \geq \bar{\mu}(y)$ *and* $\omega(x) \leq \omega(y)$ for all $x, y \in S$.

Then $\left(\bar{\mu}(x)\right)^{\alpha} \geq \left(\bar{\mu}(y)\right)^{\alpha}$ and $\left(\omega(x)\right)^{\alpha} \leq \left(\omega(y)\right)^{\alpha}$ for all $\alpha \geq 0$.

therefore $(\bar{\mu})^{\alpha}(x) \geq (\bar{\mu})^{\alpha}(y)$ and $(\omega)^{\alpha}(x) \leq (\omega)^{\alpha}(y)$

Hence $\mathcal{A}^{\alpha} =< (\bar{\mu})^{\alpha}, (\omega)^{\alpha} >$ is a cubic interior ideal of S.

Proposition 3.14: Let S be a po-Γ-semigroup S and $\{\mathcal{A}_i\}_{i\in\wedge}$ be a non-empty family of cubic subsets of S. Then $\sqcap_{i\in\wedge}\mathcal{A}_i =< \cap_{i\in\wedge}\bar{\mu}_i, \cup_{i\in\wedge}\omega_i >$ is a cubic subset of S.

Proof: Let $x \in S$. Then the set $\{\bar{\mu}_i(x)\}_{i\in\wedge}$ is non-empty bounded below subset of R.

By the completeness axiom, there exist $inf\{\bar{\mu}_i(x)\}_{i\in\wedge}$ in R. Since $[0,0] \leq \bar{\mu}_i \leq [1,1]$ for each $i \in\wedge$.

We have $[0,0] \leq inf\{\bar{\mu}_i(x)\}_{i\in\wedge} \leq [1,1]$. Thus $[0,0] \leq (\cap_{i\in\wedge}\bar{\mu}_i)(x) \leq [1,1]$.

If $x, y \in S$, such that $x = y$, then $\{\bar{\mu}_i(x)\}_{i\in\wedge} = \{\bar{\mu}_i(y)\}_{i\in\wedge}$. Thus $inf\{\bar{\mu}_i(x)\}_{i\in\wedge} = inf\{\bar{\mu}_i(y)\}_{i\in\wedge}$.

So, $\cap_{i\in\wedge}\bar{\mu}_i\,(x) = \cap_{i\in\wedge}\bar{\mu}_i\,(y)$.

Thus $\cap_{i\in\wedge}\bar{\mu}_i$ is an interval-valued fuzzy subset of S.

And, the set $\{\omega_i(x)\}_{i\in\wedge}$ is non-empty bounded above subset of R. By the completeness axiom, there exist $sup\{\omega_i(x)\}_{i\in\wedge}$ in R. Since $0 \leq \omega_i \leq 1$ for each $i \in\wedge$, we have $0 \leq sup\{\omega_i(x)\}_{i\in\wedge} \leq 1$.

Thus $0 \leq \cup_{i\in\wedge}\omega_i\,(x) \leq 1$.

If $x, y \in S$ such that $x = y$ then $\{\omega_i(x)\}_{i\in\wedge} = \{\omega_i(y)\}_{i\in\wedge}$. Thus $sup\{\omega_i(x)\}_{i\in\wedge} = sup\{\omega_i(y)\}_{i\in\wedge}$.

This implies that $\cup_{i\in\wedge}\omega_i\,(x) = \cup_{i\in\wedge}\omega_i\,(y)$

Thus $\cup_{i\in\wedge}\omega_i$ is a fuzzy subset of S.

Hence $\sqcap_{i\in\wedge}\mathcal{A}_i =< \cap_{i\in\wedge}\bar{\mu}_i, \cup_{i\in\wedge}\omega_i >$ is a cubic subset of S.

Proposition 3.15: Let S be a po- Γ-semigroup and $\{\mathcal{A}_i\}_{i\in\wedge}$ be a family of cubic Γ-subsemigroup of S. Then $\sqcap_{i\in\wedge}\mathcal{A}_i =< \cap_{i\in\wedge}\bar{\mu}_i, \cup_{i\in\wedge}\omega_i >$ is a cubic Γ-subsemigroup of S.

Proof: By proposition (3.14) $\sqcap_{i\in\wedge}\mathcal{A}_i$ is a cubic subset of S.

Let $\{\mathcal{A}_i\}_{i\in\wedge}$ be the family of cubic Γ-subsemigroup of S, let $x, y, z \in S$ *and* $\alpha \in \Gamma$.

Let $\cap_{i\in\wedge}\bar{\mu}_i\,(x) = \inf\{\bar{\mu}_i(x)\}_{i\in\wedge}$ and $\cup_{i\in\wedge}\omega_i\,(x) = \sup\{\omega_i(x)\}_{i\in\wedge}$.

Then $\cap_{i\in\wedge}\bar{\mu}_i\,(x\alpha y) = \inf\{\bar{\mu}_i(x\alpha y)\}_{i\in\wedge} \geq \inf\{min\{\bar{\mu}_i(x), \bar{\mu}_i(y)\}_{i\in\wedge} = \min\{\inf\{\bar{\mu}_i(x)\}_{i\in\wedge}, \inf\{\bar{\mu}_i(y)\}_{i\in\wedge}\}$

$= \min\{\cap_{i\in\wedge}\bar{\mu}_i\,(x), \cap_{i\in\wedge}\bar{\mu}_i\,(y)$

and $\cup_{i\in\wedge}\omega_i\,(x\alpha y) = \sup\{\omega_i(x\alpha y)\}_{i\in\wedge} \geq \sup\{\max\{\omega_i(x), \omega_i(y)\}_{i\in\wedge} = \max\{\sup\{\omega_i(x)\}_{i\in\wedge}, \sup\{\omega_i(y)\}_{i\in\wedge}\}$

$= \max\{\cup_{i\in\wedge}\omega_i(x), \cup_{i\in\wedge}\omega_i(y)\}$

Hence $\sqcap_{i\in\wedge}\mathcal{A}_i =< \cap_{i\in\wedge}\bar{\mu}_i, \cup_{i\in\wedge}\omega_i >$ is a cubic Γ-subsemigroup of S.

Theorem 3.16: Let S be a po-Γ-semigroup and $\{\mathcal{A}_i\}_{i\in\wedge}$ be the family of cubic left (right) ideal of S. Then $\sqcap_{i\in\wedge}\mathcal{A}_i =< \cap_{i\in\wedge}\bar{\mu}_i, \cup_{i\in\wedge}\omega_i >$ is a cubic left (right) ideal of S.

Proof: By proposition (3.14) $\sqcap_{i\in\wedge}\mathcal{A}_i$ is a cubic subset of S.

Now, let $x, y \in S$ be such that $x \leq y$.

Since $\{\mathcal{A}_i\}_{i\in\wedge}$ is the family of cubic left ideal of S then $\bar{\mu}_i(x) \geq \bar{\mu}_i(y)$ and $\omega_i(x) \leq \omega_i(y)$ for all $i \in\wedge$.

Thus $inf\bar{\mu}_i(y) \leq \bar{\mu}_i(y) \leq \bar{\mu}_i(x)$ and $\sup\omega_i(y) \geq \omega_i(y) \geq \omega_i(x)$ for all $i \in\wedge$.

$\inf\{\bar{\mu}_i(y)\}_{i\in\wedge}$ is an lower bound of $\bar{\mu}_i(x)$ and $\sup\{\omega_i(y)\}_{i\in\wedge}$ is an upper bound of $\omega_i(x)$.

Hence $inf\bar{\mu}_i(x) \geq inf\bar{\mu}_i(y)$ and $\sup\omega_i(x) \leq \sup\omega_i(y)$ for all $i \in\wedge$.

So, $\cap_{i\in\wedge}\bar{\mu}_i\,(x) \geq \cap_{i\in\wedge}\bar{\mu}_i\,(y)$ and $\cup_{i\in\wedge}\omega_i(x) \leq \cup_{i\in\wedge}\omega_i(y)$.

Finally, let $x, y \in S$ and $\alpha \in \Gamma$. Since $\{\mathcal{A}_i\}_{i\in\wedge}$ is the family of cubic left ideal of S.

We have $\bar{\mu}_i(x\alpha y) \geq \bar{\mu}_i(y)$ and $\omega_i(x\alpha y) \leq \omega_i(y)$ for all $i \in\wedge$.

Thus $(\cap_{i\in\wedge}\bar{\mu}_i)(x\alpha y) = (inf\bar{\mu}_i)(x\alpha y)_{i\in\wedge} = inf\bar{\mu}_i(x\alpha y)_{i\in\wedge} \geq inf\bar{\mu}_i(y)_{i\in\wedge} = \cap_{i\in\wedge}\bar{\mu}_i\,(y)$.

And $(\cup_{i\in\wedge}\omega_i)(x\alpha y) = (sup\omega_i)(x\alpha y)_{i\in\wedge} = sup\omega_i(x\alpha y)_{i\in\wedge} \leq sup\omega_i(y)_{i\in\wedge} = \cup_{i\in\wedge}\omega_i\,(y)$.

Hence $\sqcap_{i\in\wedge}\mathcal{A}_i =< \cap_{i\in\wedge}\bar{\mu}_i, \cup_{i\in\wedge}\omega_i >$ is a cubic left (right) ideal of S.

Theorem 3.17: Let S be a po-Γ-semigroup and $\{\mathcal{A}_i\}_{i\in\wedge}$ be the family of cubic bi-ideals of S. Then $\sqcap_{i\in\wedge}\mathcal{A}_i = < \cap_{i\in\wedge}\bar{\mu}_i , \cup_{i\in\wedge}\omega_i >$ is a cubic bi-ideal of S.

Proof: By proposition (3.15) we have $\sqcap_{i\in\wedge}\mathcal{A}_i$ is a cubic Γ-subsemigroup of S.
Now, let $x, y \in S$ be such that $x \le y$. Since $\sqcap_{i\in\wedge}\mathcal{A}_i$ is a cubic Γ-subsemigroup of S,
then $\bar{\mu}_i(x) \ge \bar{\mu}_i(y)$ and $\omega_i(x) \le \omega_i(y)$ for all $i \in\wedge$.
Thus $inf\bar{\mu}_i(y) \le \bar{\mu}_i(y) \le \bar{\mu}_i(x)$ and $\sup\omega_i(y) \ge \omega_i(y) \ge \omega_i(x)$ for all $i \in\wedge$.
$\inf\{\bar{\mu}_i(y)\}_{i\in\wedge}$ is an lower bound of $\bar{\mu}_i(x)$ and $\sup\{\omega_i(y)\}_{i\in\wedge}$ is an upper bound of $\omega_i(x)$.
Hence $inf\bar{\mu}_i(x) \ge inf\bar{\mu}_i(y)$ and $\sup\omega_i(x) \le \sup\omega_i(y)$ for all $i \in\wedge$.
So, $\cap_{i\in\wedge}\bar{\mu}_i(x) \ge \cap_{i\in\wedge}\bar{\mu}_i(y)$ and $\cup_{i\in\wedge}\omega_i(x) \le \cup_{i\in\wedge}\omega_i(y)$.
Finally, let $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Since $\{\mathcal{A}_i\}_{i\in\wedge}$ is the family of cubic bi-ideals of S, we have
$\bar{\mu}_i(x\alpha y\beta z) \ge \min\{\bar{\mu}_i(x), \bar{\mu}_i(z)\}$ and $\omega_i(x\alpha y\beta z) \le \max\{\omega_i(x), \omega_i(z)\}$ for all $i \in\wedge$.

$$(\cap_{i\in\wedge}\bar{\mu}_i)(x\alpha y\beta z) = (inf\bar{\mu}_i)(x\alpha y\beta z)_{i\in\wedge} = inf\bar{\mu}_i(x\alpha y\beta z)_{i\in\wedge} \ge \inf\{\min\{\bar{\mu}_i(x), \bar{\mu}_i(z)\}\}_{i\in\wedge}$$
$$= \min\{\inf\{\bar{\mu}_i(x)\}_{i\in\wedge}, \inf\{\bar{\mu}_i(z)\}_{i\in\wedge}\}$$
$$(\cap_{i\in\wedge}\bar{\mu}_i)(x\alpha y\beta z) = \min\{\cap_{i\in\wedge}\bar{\mu}_i(x), \cap_{i\in\wedge}\bar{\mu}_i(z)\} \text{ and}$$
$$(\cup_{i\in\wedge}\omega_i)(x\alpha y\beta z) = (sup\omega_i)(x\alpha y\beta z)_{i\in\wedge} = sup\omega_i(x\alpha y\beta z)_{i\in\wedge} \le \sup\{\max\{\omega_i(x), \omega_i(z)\}_{i\in\wedge}$$
$$= \max\{\sup\{\omega_i(x)\}_{i\in\wedge}, \sup\{\omega_i(z)\}_{i\in\wedge}\}$$
$$(\cup_{i\in\wedge}\omega_i)(x\alpha y\beta z) = \max\{\cup_{i\in\wedge}\omega_i(x), \cup_{i\in\wedge}\omega_i(z)\}$$

Hence $\sqcap_{i\in\wedge}\mathcal{A}_i = < \cap_{i\in\wedge}\bar{\mu}_i , \cup_{i\in\wedge}\omega_i >$ is a cubic bi-ideal of S.

Theorem 3.18: Let S be a po-Γ-semigroup and $\{\mathcal{A}_i\}_{i\in\wedge}$ be the family of cubic interior ideal of S. Then $\sqcap_{i\in\wedge}\mathcal{A}_i = < \cap_{i\in\wedge}\bar{\mu}_i , \cup_{i\in\wedge}\omega_i >$ is a cubic interior ideal of S.

Proof: By proposition (3.15) we have $\sqcap_{i\in\wedge}\mathcal{A}_i$ is a cubic Γ-subsemigroup of S.
Now, let $x, y \in S$ be such that $x \le y$. Since $\sqcap_{i\in\wedge}\mathcal{A}_i$ is a cubic Γ-subsemigroup of S,
then $\bar{\mu}_i(x) \ge \bar{\mu}_i(y)$ and $\omega_i(x) \le \omega_i(y)$ for all $i \in\wedge$.
Thus $inf\bar{\mu}_i(y) \le \bar{\mu}_i(y) \le \bar{\mu}_i(x)$ and $\sup\omega_i(y) \ge \omega_i(y) \ge \omega_i(x)$ for all $i \in\wedge$.
And, let $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Since $\{\mathcal{A}_i\}_{i\in\wedge}$ is the family of cubic interior ideals of S, we have
$\bar{\mu}_i(x\alpha y\beta z) \ge \bar{\mu}_i(y)$ and $\omega_i(x\alpha y\beta z) \le \omega_i(y)$ for all $i \in\wedge$.

Thus
$$(\cap_{i\in\wedge}\bar{\mu}_i)(x\alpha y\beta z) = (inf\bar{\mu}_i)(x\alpha y\beta z)_{i\in\wedge}$$
$$= inf\bar{\mu}_i(x\alpha y\beta z)_{i\in\wedge}$$
$$\ge inf\bar{\mu}_i(y)_{i\in\wedge}$$
$$(\cap_{i\in\wedge}\bar{\mu}_i)(x\alpha y\beta z) = \cap_{i\in\wedge}\bar{\mu}_i(y) \text{ and}$$
$$(\cup_{i\in\wedge}\omega_i)(x\alpha y\beta z) = (sup\omega_i)(x\alpha y\beta z)_{i\in\wedge}$$
$$= sup\omega_i(x\alpha y\beta z)_{i\in\wedge}$$
$$\le sup\omega_i(y)_{i\in\wedge}$$
$$(\cup_{i\in\wedge}\omega_i)(x\alpha y\beta z) = \cup_{i\in\wedge}\omega_i(y)$$

Hence $\sqcap_{i\in\wedge}\mathcal{A}_i = < \cap_{i\in\wedge}\bar{\mu}_i , \cup_{i\in\wedge}\omega_i >$ is a cubic interior ideal of S.

Theorem 3.19: Let f: S→ $S^{\prime}$ be an epimorphism on po-Γ-semigroup S and $S^{\prime}$. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic Γ-sub semigroup of $S^{\prime}$ by the cubic property then $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic Γ-sub semigroup of S, provided $C_f^{-1}(\mathcal{A})$ is non-empty.

Proof: Let $x, y \in S$. Then $C_f(x), C_f(y) \in S^{\prime}$. For any $\alpha \in \Gamma$ we have
since $\bar{\mu}$ is an interval-valued fuzzy Γ-sub semigroup of $S^{\prime}$

$$C_f^{-1}(\bar{\mu}(x\alpha y)) = \bar{\mu}(f(x\alpha y))$$
$$= \bar{\mu}(f(x)\alpha f(y))$$
$$\ge \min\{\bar{\mu}(f(x)), \bar{\mu}(f(y))$$
$$C_f^{-1}(\bar{\mu}(x\alpha y)) = \min\{C_f^{-1}(\bar{\mu}(x)), C_f^{-1}(\bar{\mu}(y))\} \text{ and}$$

since ω is a fuzzy Γ-sub semigroup of $S^{\prime}$

$$C_f^{-1}(\omega(x\alpha y)) = \omega(f(x\alpha y))$$
$$= \omega(f(x)\alpha f(y))$$
$$\le \max\{\omega(f(x)), \omega(f(y))\}$$
$$C_f^{-1}(\omega(x\alpha y)) = \max\{C_f^{-1}(\omega(x)), C_f^{-1}(\omega(y))\}$$

Hence $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic Γ-sub semigroup of S.

Theorem 3.20: Let f: S→ $S^{\prime}$ be an epimorphism on po-Γ-semigroup S and $S^{\prime}$. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic left (right) ideal of $S^{\prime}$ by the cubic property then $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic left (right) ideal of S, provided $C_f^{-1}(\mathcal{A})$ is non-empty.

Proof: Let $x, y \in S$. Then $C_f(x), C_f(y) \in S^{\prime}$. For any $\alpha \in \Gamma$.
Since $\bar{\mu}$ is an interval-valued fuzzy left ideal of $S^{\prime}$

$$C_f^{-1}(\bar{\mu}(x\alpha y)) = \bar{\mu}(f(x\alpha y))$$
$$= \bar{\mu}(f(x)\alpha f(y))$$

$\geq \bar{\mu}(f(y))$
$C_f^{-1}(\bar{\mu}(x\alpha y)) = C_f^{-1}(\bar{\mu}(y))$ and
since ω is a fuzzy left ideal of S^{I}
$C_f^{-1}(\omega(x\alpha y)) = \omega(f(x\alpha y))$
$= \omega(f(x)\alpha f(y))$
$\leq \omega(f(y))$
$C_f^{-1}(\omega(x\alpha y)) = C_f^{-1}(\omega(y))$
Also, let $x, y \in S$ be such that $x \leq y$. Then $C_f(x), C_f(y) \in S^{\mathsf{I}}$ with $C_f(x) \leq C_f(y)$
since $\bar{\mu}$ is an interval-valued fuzzy left ideal of S^{I}
so $\bar{\mu}(f(x)) \geq \bar{\mu}(f(y))$ then $C_f^{-1}(\bar{\mu}(x)) \geq C_f^{-1}(\bar{\mu}(y))$ and
since ω is a fuzzy left ideal of S^{I}
so $\omega(f(x)) \leq \omega(f(y))$ then $C_f^{-1}(\omega(x)) \leq C_f^{-1}(\omega(y))$
Hence $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic left (right) ideal of S.

Theorem 3.21: Let f: S→ S^{I} be an epimorphism on po-Γ-semigroup S and S^{I}. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic bi-ideal of S^{I} by the cubic property then $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic bi-ideal of S, provided $C_f^{-1}(\mathcal{A})$ is non-empty.

Proof: By theorem (3.19) $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic Γ-sub semigroup of S.
Let $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic bi-ideal of S^{I} *and* $x, y, z \in S$. Then $C_f(x), C_f(y), C_f(z) \in S^{\mathsf{I}}$. For any $\alpha, \beta \in \Gamma$.
since $\bar{\mu}$ is an interval-valued fuzzy bi-ideal of S^{I}
$C_f^{-1}(\bar{\mu}(x\alpha y\beta z)) = \bar{\mu}(f(x\alpha y\beta z))$
$= \bar{\mu}(f(x)\alpha f(y)\beta f(z))$
$\geq \min\{\bar{\mu}(f(x)), \bar{\mu}(f(z))$
$C_f^{-1}(\bar{\mu}(x\alpha y\beta z)) = \min\{C_f^{-1}(\bar{\mu}(x)), C_f^{-1}(\bar{\mu}(z))\}$ and
since ω is a fuzzy bi- ideal of S^{I}
$C_f^{-1}(\omega(x\alpha y\beta z)) = \omega(f(x\alpha y\beta z))$
$= \omega(f(x)\alpha f(y)\beta f(z))$
$\leq \max\{\omega(f(x)), \omega(f(z))$
$C_f^{-1}(\omega(x\alpha y\beta z)) = \max\{C_f^{-1}(\omega(x)), C_f^{-1}(\omega(z))\}$
Also, let $x, y \in S$ be such that $x \leq y$. Then $C_f(x), C_f(y) \in S^{\mathsf{I}}$ with $C_f(x) \leq C_f(y)$
since $\bar{\mu}$ is an interval-valued fuzzy bi-ideal of S^{I}
so $\bar{\mu}(f(x)) \geq \bar{\mu}(f(y))$ then $C_f^{-1}(\bar{\mu}(x)) \geq C_f^{-1}(\bar{\mu}(y))$ and
since ω is a fuzzy bi-ideal of S^{I}
so $\omega(f(x)) \leq \omega(f(y))$ then $C_f^{-1}(\omega(x)) \leq C_f^{-1}(\omega(y))$
Hence $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic bi-ideal of S.

Theorem 3.22: Let f: S→ S^{I} be an epimorphism on po-Γ-semigroup S and S^{I}. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic interior ideal of S^{I} by the cubic property then $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic interior ideal of S, provided $C_f^{-1}(\mathcal{A})$ is non-empty.

Proof: By theorem (3.19) $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic Γ-sub semigroup of S.
Let $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic interior-ideal of S^{I} *and* $x, y, z \in S$. Then $C_f(x), C_f(y), C_f(z) \in S^{\mathsf{I}}$.
For any $\alpha, \beta \in \Gamma$.
since $\bar{\mu}$ is an interval-valued fuzzy interior ideal of S^{I}
$C_f^{-1}(\bar{\mu}(x\alpha y\beta z)) = \bar{\mu}(f(x\alpha y\beta z))$
$= \bar{\mu}(f(x)\alpha f(y)\beta f(z))$
$\geq \bar{\mu}(f(y))$
$C_f^{-1}(\bar{\mu}(x\alpha y\beta z)) = C_f^{-1}(\bar{\mu}(y))$ and
since ω is a fuzzy interior ideal of S^{I}
$C_f^{-1}(\omega(x\alpha y\beta z)) = \omega(f(x\alpha y\beta z))$
$= \omega(f(x)\alpha f(y)\beta f(z))$
$\leq \omega(f(y))$
$C_f^{-1}(\omega(x\alpha y\beta z)) = C_f^{-1}(\omega(y))$
Also, let $x, y \in S$ be such that $x \leq y$. Then $C_f(x), C_f(y) \in S^{\mathsf{I}}$ with $C_f(x) \leq C_f(y)$
since $\bar{\mu}$ is an interval-valued fuzzy interior ideal of S^{I}
so $\bar{\mu}(f(x)) \geq \bar{\mu}(f(y))$ then $C_f^{-1}(\bar{\mu}(x)) \geq C_f^{-1}(\bar{\mu}(y))$ and
since ω is a fuzzy interior ideal of S^{I}
so $\omega(f(x)) \leq \omega(f(y))$ then $C_f^{-1}(\omega(x)) \leq C_f^{-1}(\omega(y))$

Hence $\complement_f^{-1}(\mathcal{A}) = < \complement_f^{-1}(\bar{\mu}), \complement_f^{-1}(\omega) >$ is a cubic interior ideal of S.

Proposition 3.23: Let S be a po-Γ-semigroup with unities, L be its left operator po-semigroup and $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic ideal of L (resp. cubic left ideal of L). Then $\mathcal{A}^+ =< \bar{\mu}, \omega >^+ =< (\bar{\mu})^+, \omega^+ >$ is a cubic ideal of S (resp. cubic left ideal of S).

Proof: Let $x, y \in S$ and $\gamma \in \Gamma$. Then

$$(\bar{\mu})^+(x\gamma y) = \inf_{\alpha\in\Gamma} \bar{\mu}([x\gamma y, \alpha])$$
$$= \inf_{\alpha\in\Gamma} \bar{\mu}([x, \gamma][y, \alpha])$$
$$\geq \inf_{\alpha\in\Gamma} \bar{\mu}([y, \alpha])\big(\ \bar{\mu} \text{ is an } CLI(L)\big)$$
$(\bar{\mu})^+(x\gamma y) = (\bar{\mu})^+(y)$ and
$$(\omega)^+(x\gamma y) = \sup_{\alpha\in\Gamma} \omega([x\gamma y, \alpha])$$
$$= \sup_{\alpha\in\Gamma} \omega([x, \gamma][y, \alpha])$$
$$\leq \sup_{\alpha\in\Gamma} \omega([y, \alpha])\big(\ \omega \text{ is an } CLI(L)\big)$$
$(\omega)^+(x\gamma y) = (\omega)^+(y)$

Let $x, y \in S$ be such that $x \leq y$. Then $[x, \alpha] \leq [y, \alpha]$ in L for all $\alpha \in \Gamma$.
Since $\mathcal{A} = < \bar{\mu}, \omega >$ is a CI(L), then $\bar{\mu}[x, \alpha] \geq \bar{\mu}[y, \alpha]$ and $\omega[x, \alpha] \leq \omega[y, \alpha]$ for all $\alpha \in \Gamma$.

So, $(\bar{\mu})^+(x) = \inf_{\alpha\in\Gamma} \bar{\mu}([x, \alpha])$
$$\geq \inf_{\alpha\in\Gamma} \bar{\mu}([y, \alpha])$$
$(\bar{\mu})^+(x) \geq (\bar{\mu})^+(y)$ and
$$(\omega)^+(x) = \sup_{\alpha\in\Gamma} \omega([x, \alpha])$$
$$\leq \sup_{\alpha\in\Gamma} \omega([y, \alpha])$$
$(\omega)^+(x) \leq (\omega)^+(y)$

Hence $\mathcal{A}^+ =< (\bar{\mu})^+, \omega^+ >$ is a cubic ideal of S.

Corollary 3.24: Let S be a po-Γ-semigroup with unities, R be its right operator po-semigroup and $\mathcal{A} =< \bar{\mu}, \omega >$ is a cubic ideal of R (resp. cubic left ideal of R). Then $\mathcal{A}^* =< \bar{\mu}, \omega >^* =< (\bar{\mu})^*, \omega^* >$ is a cubic ideal of S (resp. cubic left ideal of S).

Proposition 3.25: Let S be a po-Γ-semigroup with unities, L be its left operator po-semigroup and $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic ideal of S (resp. cubic left ideal of S).Then $\mathcal{A}^{+'} =< \bar{\mu}, \omega >^{+'} =< (\bar{\mu})^{+'}, \omega^{+'} >$ is a cubic ideal of L (resp. cubic left ideal of L)

Proof: Let $[x, \alpha], [y, \beta] \in$ L. Then,

$$(\bar{\mu})^{+'}([x, \alpha][y, \beta]) = (\bar{\mu})^{+'}([x\alpha y, \beta])$$
$$= \inf_{s\in S} \bar{\mu}(x\alpha y\beta s)$$
$$= \inf_{s\in S} \bar{\mu}(x\alpha(y\beta s))$$
$$\geq \inf_{s\in S} \bar{\mu}(y\beta s)\ \big(\ \bar{\mu} \text{ is an } CLI(S)\big)$$
$(\bar{\mu})^{+'}([x, \alpha][y, \beta]) = (\bar{\mu})^{+'}([y, \beta])$ and
$$(\omega)^{+'}([x, \alpha][y, \beta]) = (\omega)^{+'}([x\alpha y, \beta])$$
$$= \sup_{s\in S} \omega(x\alpha y\beta s)$$
$$= \sup_{s\in S} \omega(x\alpha(y\beta s))$$
$$\leq \sup_{s\in S} \omega(y\beta s)\ \big(\ \omega \text{ is an } CLI(S)\big)$$
$(\omega)^{+'}([x, \alpha][y, \beta]) = (\omega)^{+'}([y, \beta])$

Now, let $[x, \alpha], [y, \alpha] \in$ L be such that $[x, \alpha] \leq [y, \alpha]$. Then $x\alpha s \leq y\alpha s$ for all $\alpha \in \Gamma$ and s ∈ S.
Since $\mathcal{A} = < \bar{\mu}, \omega >$ is a CI(S), then $\bar{\mu}(x\alpha s) \geq \bar{\mu}(y\alpha s)$ and $\omega(x\alpha s) \leq \omega(y\alpha s)$ for all $\alpha \in \Gamma$ and s ∈ S.

$$(\bar{\mu})^{+'}([x, \alpha]) = \inf_{s\in S} \bar{\mu}(x\alpha s)$$
$$\geq \inf_{s\in S} \bar{\mu}(y\alpha s)$$
$(\bar{\mu})^{+'}([x, \alpha]) \geq (\bar{\mu})^{+}([y, \alpha])$ and
$$(\omega)^{+'}([x, \alpha]) = \sup_{s\in S} \omega(x\alpha s)$$
$$\leq \sup_{s\in S} \omega(y\alpha s)$$
$(\omega)^{+'}([x, \alpha]) \leq (\omega)^{+'}([y, \alpha])$

Hence $\mathcal{A}^{+'} =< (\bar{\mu})^{+'}, \omega^{+'} >$ is a cubic ideal of L.

Corollary 3.26: Let S be a po-Γ-semigroup with unities, R be its right operator po-semigroup and $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic ideal of S (resp. cubic left ideal of S).Then $\mathcal{A}^{*'} =< \bar{\mu}, \omega >^{*'} =< (\bar{\mu})^{*'}, \omega^{*'} >$ is a cubic ideal of R (resp. cubic left ideal of R).

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