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A STUDY ON NON SPLIT DOMINATION NUMBER

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Introduction:

The word "mathematics" comes from the Greek word "Mathema" which means science, knowledge or learning; Mthematiko's means fond of learning. Today, the term refers to specific body of knowledge - the deductive study of quantity, structure, space and change. Mathematics links the abstract world of mental concepts to real world of physical things. Graph theory is one of the most developing branches of mathematics with many modern applications. Its basic ideas were introduced in the 18^{th} century, by the great Swiss mathematician Leonard Euler. Certain problems in physics, chemistry, communications science, computer technology, genetics, psychology, sociology and linguistics can be formulated as problems in graph theory. Also it plays an important role in several areas of computer science such as switching theory and logic design, Artificial Intelligence, formal languages, operating system and information such as group theory, metric theory, probability and topology have interactions with graph theory. The concept of non split domination in graphs helps to find a subset, which covers the whole graph in unit distance. A communication network can be represented by a connected graph G, where the vertices of G represent processors and edges represent bi-directional communication channels. A dominating set in a graph can be interpreted as a set of processors from which information can be passed on to all the other processors. Hence determination of non split domination parameter of a graph is an important problem. The global non split domination number, cycle non split domination number and the path non split domination number was introduced by kulli and Nandargi. The block non split domination number was introduced by kulli and Janakiram. The total dominating set concept was introduced by Janakiram, soner and chaluvar.

Preliminaries:

Definition 1: A Graph G consists of a pair (V(G), E(G)) where V(G) is a non empty finite set whose elements are called points or vertices and E (G) is a set of unordered pairs of distinct elements of V (G). The elements of E(G) are called lines or edge of the graph G.

Example 1:

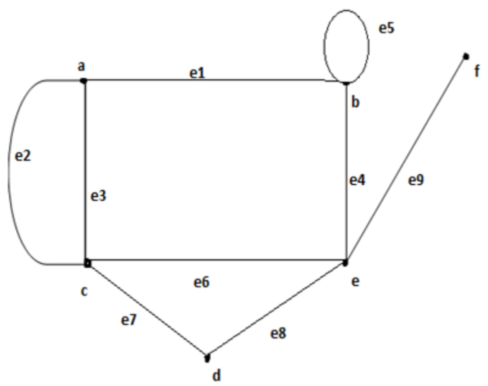


Figure 1

G = < V,E > where
V = { a, b, c, d, e, f }
E = {e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9}
and the ends of the edges are given by

$e_1 \leftrightarrow (a,b)$, $e_2 \leftrightarrow (a,c)$, $e_3 \leftrightarrow (a,c)$,
$e_4 \leftrightarrow (b,e)$, $e_5 \leftrightarrow (b,b)$, $e_6 \leftrightarrow (c,e)$,
$e_7 \leftrightarrow (c,d)$, $e_8 \leftrightarrow (d,e)$, $e_9 \leftrightarrow (e,f)$,

Definition 2: When a vertex v_i is an end vertex of some edges e_i , e_j and v_i are said to be incident with each other

Example 2: In figure 1 edge e_2, e_3, e_6, and e_7 are incident with vertex C

Definition 3: Two non parallel edges are said to adjacent if they are incident on a common vertex .

Example 3: In figure 1 e_7 and e_8 are adjacent

Definition 4: Two vertices are said to be adjacent if they are the end vertices of the same edge

Example 4: In figure 1 c and e are adjacent but c and b are not adjacent.

Definition 5: The number of edges incident on a vertex v_i with self loops counted twice is called the degree d (v_i) of vertex v_i

Example 5: In Figure 1

d (a) =3, d (b) =4, d (c) =3,
d (e) =3, d (f) =1,

Definition 6: A walk is defined as a finite alternating sequence of vertices and edges beginning and ending with vertices such that each edge is incident with vertices preceding and following it. No edge appears more than once in a walk.

Example 6: In figure 1, v_1 e_3 v_2 e_1 v_2 e_2 v_4 e_7 v_5 is a walk.

Definition 7: Vertices with which a walk begins and ends are called terminal vertices.

Definition 8: A walk in which the terminal vertices are distinct is called an open walk.

Definition 9: A walk which begins and ends with same vertex is called a closed walk.

Definition 10: A Closed walk in which no vertex incept the end vertices is repeated is called cycle.

Example 10: In figure 1, v_1 e_3 v_2 e_2 v_4 e_6 v_3 e_5 v_1 is a cycle.

Definition 11: An open walk in which no vertex appears more than once is called a path or simple path or an elementary path.

Example 11: In figure 1, v_1 e_3 v_2 e_2 v_4 e_7 v_5 is a path.

Definition 12: A Graph G is said to be connected if there exist at least one path between every pair of vertices in G

Example 12:

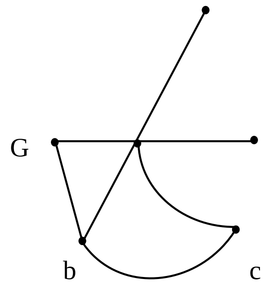


Figure 2: G is a connected graph

Definition 13: A set D of vertices in a graph G = <V,E> is said to be dominating set of G if every vertex not in D is adjacent to at least one vertex in D.

Example 13:

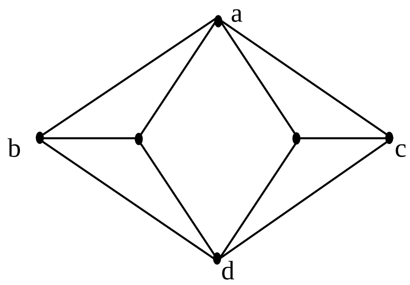


Figure 3: {b,c} is a dominating set

Definition 14: The minimum number of cardinality of the dominating set of G is called the domination number of G and is denoted by γ (G).

Example 14:

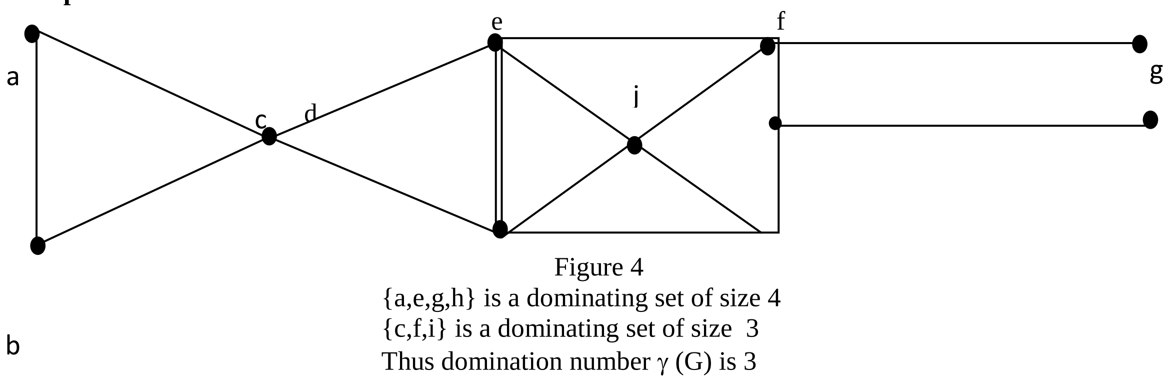


Figure 4

{a,e,g,h} is a dominating set of size 4
{c,f,i} is a dominating set of size 3
Thus domination number γ (G) is 3

Definition 15: A set S of vertices in a graph G is called an independent set if no two vertexes in S are adjacent.

Example 15:

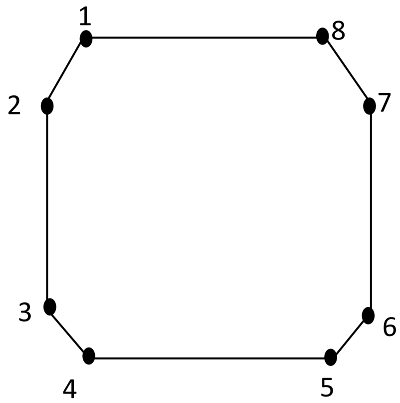


Figure 5: {2,5,8} are independent set

Definition 16: The independence number βo (G) is the maximum cardinality of an independent set in G.

Example 16: In figure 5

{2,5,8} is an independent set of size 3
{1,3,5,7} is an independent set of size 4
Thus βo (G) = 4

Definition 17: A cut point of a graph is a point whose removal increase no of components.

Example 17:

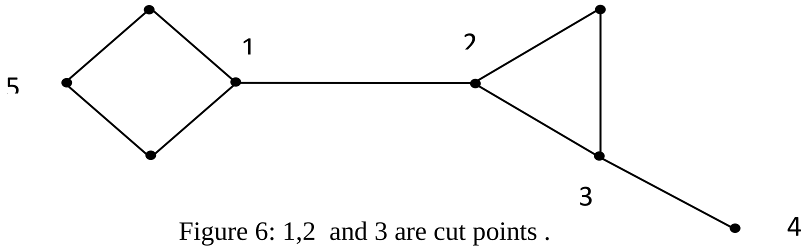


Figure 6: 1,2 and 3 are cut points .

Definition 18: A connected non- trivial graph having no cut point is a block.

Example 18:

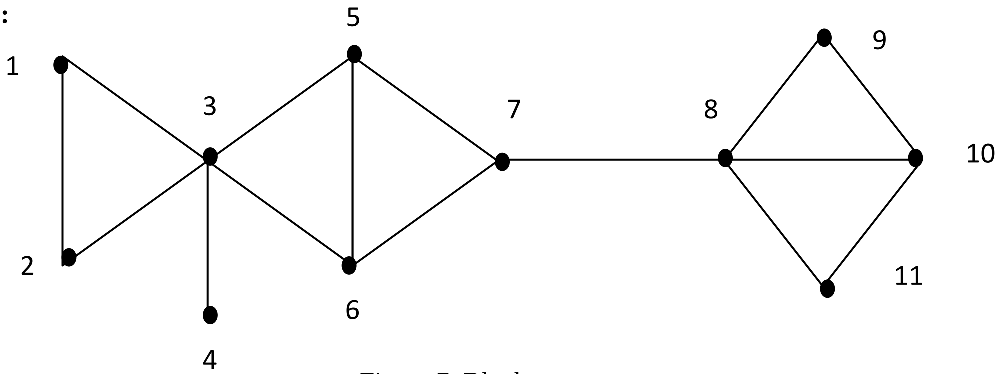
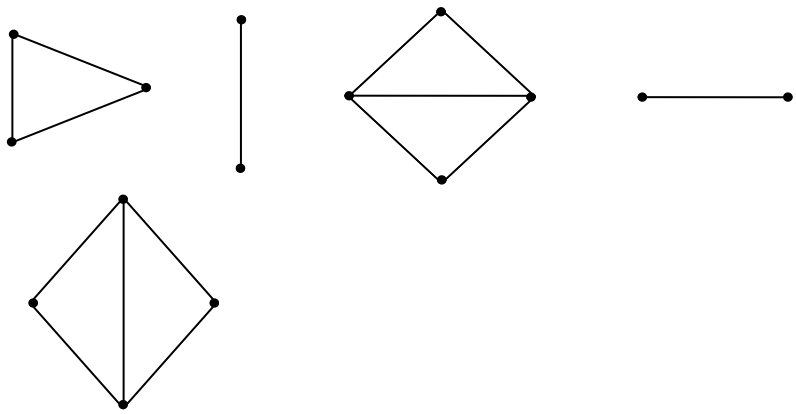


Figure 7: Blocks are



Some Types of Basic Non Split Domination Numbers:

The global non split domination number, cycle non split domination number and the path non split domination number was introducted by kulli and Nandargi. The block non split domination number was introduced by kulli and Janakiram. The total dominating set concept was introduced by Janakiram, soner and chaluvar. In this session, the necessary and sufficient condition for the domination number and the block non split domination number. The relation between cototal domination number and Global domination number and the relation between Global non split domination number and the cotototal domination number of a connected graph have been discussed.

Block Non Split Domination Number:

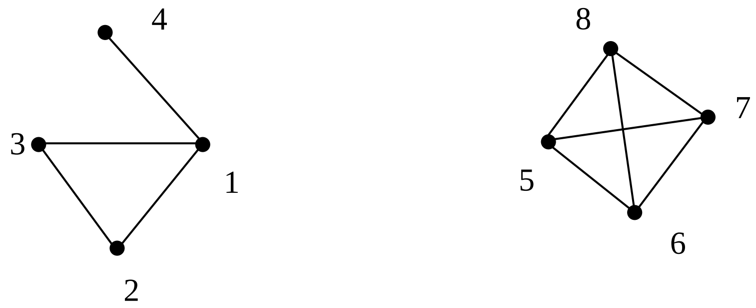
Definition 19: A Dominating set D of a connected graph G = < V, E> is a block non split domination set if the induced sub graph < V- D> is a block in G.

Definition 20: The block non split domination number γ_{bns} (G) of G is the minimum cardinality of a block non split dominating set of G

Example 19:

Figure 8: Dominating set = {1,5}

< V-D >



< V-D > is a block

γ_{bns} **(G) = 3**

In Figure 8

γ Set = {1,5}

γ_{ns} Set = {3,4,6} and

γ_{bns} set ={2,3,4,6,7,8}

γ (G) = 2 γ_{ns} (G) = 3 γ_{bns} (G) = 6

Therefore γ (G) < γ ns (G) < γ_{bns} (G)

Theorem 1:

A connected graph G has a block non split dominating set if and only if there exist a block in G containing only cut vertices of G.

Proof:

Let D be a block non split dominating set of a connected graph G. Then < V-D > is a block in G. Since D is dominating set, every vertex
< V-D > is a cut vertex of G.

Conversely,

Suppose a connected graph G has block < s > containing only cut vertices.

Then < V-S > is a block non split dominating set

Note: If a connected graph G has a block non split dominating set, then G has atleast two cut vertices.

- ✓ A tree T has a γ_{bns} set if and only if diam (T) $\geq$ 3.
- ✓ If a connected graph G has a block containing all cut vertices of G, then G has almost one block non split dominating set.

- ✓ If D is a block non split dominating set of a connected graph G, then every vertex in D is adjacent to atmost one vertex in V-D.

Theorem 2:

For any connected graph G, $p/2 \leq \gamma_{bns}$ (G) $\leq$ P-2. Furthermore, the lower bound is attained if and only if there exist a block in G Containing P/2 cut vertices, and the upper bound is attained if and only if there exist two adjacent cut vertices u and v such that $<\{u,v\}>$ is a block in G and every other block is either K_2 or has atleast one cut vertex. (There exist exactly two adjacent cut vertices u and v such that $<\{u,v\}>$ is a block in G).

Proof:

Let D be a γ_{bhs} set of a connected graph G. By theorem 2.2.3, each vertex in D is adjacent to atmost one vertex in V-D

Hence $|D| \geq |V - D|$ thus γ_{bns} (G) $\geq$ p/2

Let us consider the upper bound since every block has atleast two vertices, γ_{bns} (G) $\leq$ p-2.

Thus $p/2 \leq \gamma_{bns}$ (G) $\leq$ p-2

Suppose lower bound it attained , then there exist a block in G containing p/2 cut vertices.

Conversely, there exists a block <B> in G containing p/2 cut vertices. Then each vertex in B is a cut vertex. Hence by theorem 1, γ_{bns} = p/2.

Suppose the upper bound is attained. On the contrary, if every block in G contains at least one non cut vertex.

By theorem 1 G has no block non split dominating set, which is a contradiction.

suppose there exists a block containing atleast 3 vertices such that each vertex is a cut vertex, then γ_{bns} (G) $\leq$ p-3, which is a contradiction.

Hence the given condition holds

Conversely, if G is a graph which has two adjacent cut vertices u and v such that $<\{u,v\}>$ is a block in G and every other block in G- {u,v} has atleast one non cut

vertex or if G is a tree with atleast two(adjacent) cut vertices, then in either case, it is easy to check that γ_{bns} (G) = p-2.

Note: Kulli and Janakiram gave another lower bound on γ_{bns} (G) in terms of the maximum degree Δ (G)

Theorem 3:

For any connected graph G, γ_{bns} (G) $\geq \Delta$ (G)

Proof:

Let D be a γ_{bns} set of a connected graph G .

For each vertex vεD , Let Dv = {u / uεD and u is adjacent to v}

Since v is cut vertex Dv $\neq \phi$ and for any cut vertex w ε V-D, u is not adjacent to w. Thus any dominating set must contain either v or w.

Hence V (G)$\geq$|V-D| or p-γ (G) $\leq \gamma_{bns}$ (G).

Since γ(G) $\leq$ p- Δ (G), Δ (G) $\leq$, γ_{bns}(G)

Hence proved

Note: If a γ_{bns} set D of a connected graph G is independent then γ(G) + γ_{bns} (G) = P

Theorem 4:

Let D be a γ_{bns} set of a connected graph G if V (G) = γ_{bns} (G) then D is independent.

Proof:

Suppose γ (G) = γ_{bns} (G)

Then γ_{bns} (G) = p/2 .Hence it follows that for each vertex vεD, there exists exactly one vertex vεD such that v is adjacent to u. Since v is a cut vertex, u is not adjacent to any vertex in D – {u}

Hence D is independent.

Theorem 5:

Let D be a γ_{bns} set of a connected graph G having no cut vertices. Then D is independent if and only if γ_{ns} (G).

Proof:

Suppose D is a γ_{bns} – set of a connected graph G having no cut vertices and is independent.

By Note, each vertex in D is an end vertex.

Hence D is a γ_{bns} set of G and γ_{ns} (G) = γ_{bns} (G)

Conversely,

Suppose γ_{ns} (G) = γ_{bns} (G) and D is a γ_{bns} –set of G. on the contrary, Suppose D is not independent. Then there exists a components G, in <D> which is non- trivial.

Let u and w be two vertices in G, which are adjacent to a common cut vertex vεV-D.

This implies that D- {u} or D – {w} is a non split dominating set of G, contradiction.

Hence D is independent.

Cycle Non Split Domination Number:

Definition 21: A dominating set D of a connected graph G is a cycle dominating set if the induced sub graph <V-D> is a cycle in G

Definition 22: The cycle non split domination γ_{cns} (G) of G is the minimum cardinality of a cycle nonsplit dominating set

Example 20:

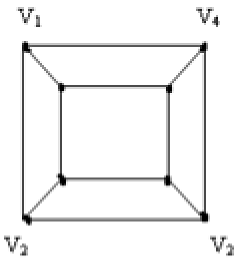


Figure 9: D={ V_1, V_2, V_3, V_4}

<V-D>

The induced subgraph <V-D> is connected and a cycle.

Hence γ_{cns} (G) = 4 and γ (G) =2

Example 21:

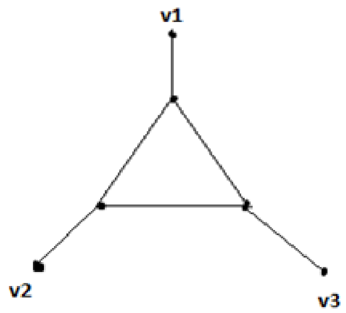
Figure 10: D = {V_1 , V_2}

<V-D>

The induced sub graph <V-D> is connected and a cycle.

The Graph G has γ_{cns} (G)=2, γ(G) =2

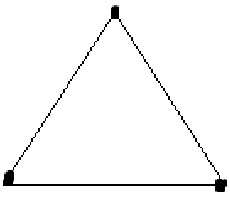
Example 22:



C_3 o K_1

Figure 11: D = {a,b,c}

<V-D>



The induced subgraph < V-D> is connected and cycle.

γ_{cns} (c) =3 and γ (C_3 o K_1) =3

Note: For any connected graph G with p≥4 vertices γ (G) + γ_{cns} (G) ≤ p and this bound is sharp.

Path Non Split Domination Number:

Definition 23: A dominating set D of a connected graph G is a path non split dominating set if the induced sub graph <V-D> is a path in G

Definition 24: The path non split domination number γ_{pns} (G) of G is the minimum cardinality of a path non split dominating set.

Example 23:

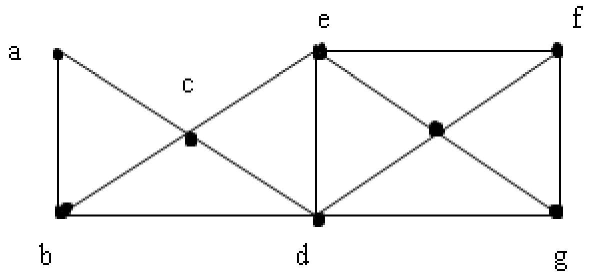
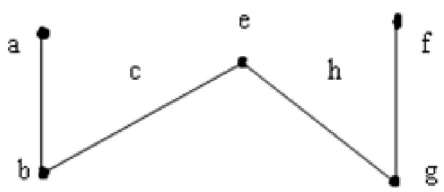


Figure 12: Dominating set = {d}

<V-D>



<V-D> is a path

Hence γ_{pns} (G) = 1.

Note:

- ✓ For any nontrivial connected graph G, γ(G) ≤ γ_{pns} (G) and the equality holds if G is P_4 or C_4
- ✓ For any nontrivial connected graph G, γ(G) + γ_{pns} (G) ≤ p and this bound is sharp

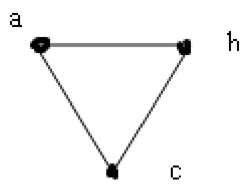
Proposition 1:

γ_{pns} (K_p) = p-2, p≥3

Proof:

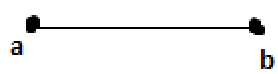
Let p=3

K_3 is



Let Dominating set D = {c}

<V-D>



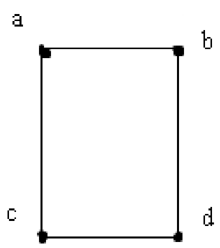
Which is a path γ_{pns} (K_3) = 1= 3-2

Hence γ_{pns} (K_p) = p-2, p≥3.

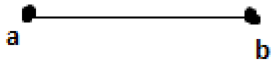
Proposition 2:

γ_{pns} (C_p) = p-2, p≥3

Proof:



Let p = 4
C_4 is
Dominating set D= { c,d}
<V – D>



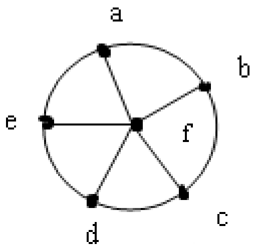
<V-D> is a path
γ_{pns} $(C_4) = 2 = 4\text{-}2$
Hence γ_{pns} (Cp) = p-2, p≥3.

Proposition 3:

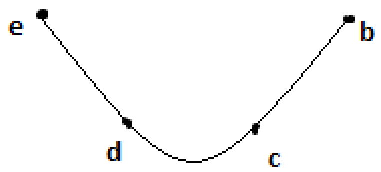
γ_{pns} $(W_p) = 2$, p≥4

Proof:

Let p=6
W_6 is



Dominating set D= { a,f}
<V-D>



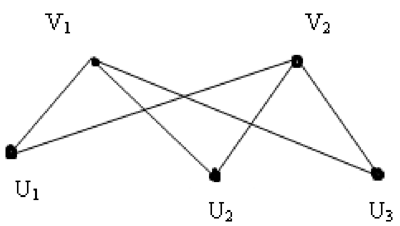
<V-D> is a path
γ_{pns} $(w_6) = 2$
Hence γ_{pns} (Wp) = 2, P≥4

Proposition 4:

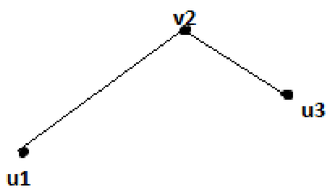
γ_{pns} $(K_{m,n})$ = m+n-3, m≥ 2, n≥3.

Proof:

m=2, n=3
$K_{2,3}$



Dominating set D= $\{V_1, U_2\}$
<V-D> is



<V-D> is a Path
γ_{pns} $(K_{2,3}) = 2 = 2+3-3$
Hence γ_{pns} $(K_{m,n})$ = m+n-3, m≥2, n≥3

Note:

γ_{pns} (P_P) = p-2, p≥3

Some Types of Advanced Non Split Domination Number:

Cototal Domination Number:

Definition 25: A dominating set D of G is a cototal dominating set if the induced subgraph <V-D> has no isolated vertices.

Definition 26: The cototal domination number γ_{cot} (G) of G is the minimum cardinality of cototal domating set.

Example 24:

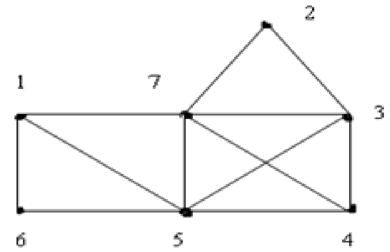
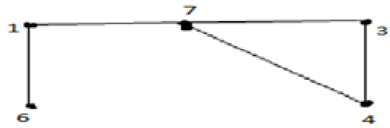


Figure 13: Dominating set ={ 2,5}

< V-D >



<V-D> has no isolated vertices

Hence γ_{cot} (G) = 2

Theorem 6:

For any graph G p-(2/3) q≤ γ_{cot} (G)

Proof:

Let D be a γ_{cot} set of G

Then $q \geq |V\text{-}D| + \frac{|V\text{-}D|}{2}$

Thus p– (2/3)q ≤ γ_{cot} (G)

Theorem 7:

For any graph G, 2(p-q) - p_0 <V-D> has no isolated vertices, where p_0 is the number of isolated vertices in G

Proof:

Let D be a γ_{cot} set of G .Then

$$2q = \sum_{i=1}^{|D|} \deg v_i + \sum_{j=1}^{|D|} \deg v_j$$

$2q \geq |D| - p_0 + 2\,|V\text{-}D|$

Thus 2 (p-q)- $p_0 \leq \gamma_{cot}$ (G)

Note:

- ✓ Let G be a graph such that each component of G is not a stat .Then γ_{cot} (G) ≤ P-δ (G)
- ✓ A cototal dominating set D of G is minimal if and only if for vertex vεD one of the following conditions holds;
  - There exist a vertex vεV-D such that N(u) n D = {v};
  - v is an isolated vertex in <D>;
  - v is an isolated vertex in < (V-D) U { V } >

Theorem 8:

Let G be a graph with at least three vertices and D be a γ set of G such that every vertex in D is adjacent to at most one vertex in V-D. If γ(G) ≤ δ (G) then γ(G) = γ_{cot} (G) = p/2

Proof:

Clearly G has no isolated vertices, and every vertex in D is adjacent to exactly one vertex in V-D. Suppose there exists a vertex in V-D which is adjacent to all the vertices in D. Then G= K_2, a contradiction.

This implies that <V-D> has no isolated vertices and hence D is a γ_{cot} set of G, since for each vertex vεV –D there exist a vertex uεD such that

N (U) n <V-D> = V,

|D| ≥ |V-D|,

Thus γ(G) = γ_{cot} (G) = p/2

Note: The result relates γ_t (G), γ_{cot} (G) and βo (G), If G is a graph without isolated vertices then γ_{cot} (G) ≤ γ_t (G) + βo (G)

Theorem 9:

Let G be a connected graph with βo (G)) ≤ p-2 then γ_{cot} (G) ≤ γ_{ns} (G)

Proof:

Let D be a γ_{ns} set of G

Then <V-D> is connected with at least two vertices and hence D is a cototal dominating set of G

Thus γ_{cot} (G) ≤ γ_{ns} (G)

Theorem 10:

If γ_{cot} (G) ≤ δ (G) Then γ_{cot} (G) +γ_g (G), ≤ p+1, furthermore, If G= K_p then the bound is attained.

Proof

Let D be a γ_{cot} –set of G, Then V-D is a dominating set of G. Suppose each vertex in D is not adjacent to some vertex in V-D, Then V-D is a global dominating set of G and hence γ_{cot} (G) + γ_g (G) ≤ P+1. If there exist a vertex u ε D which is adjacent to all the vertices in V-D. By note u is an isolated vertex in, <D> or there exists a vertex v ε V-D such that N (V) n D = {u}.

This implies that <V-D> U {u} is a global dominating set of G.

Hence γ_{cot} (G) +γ_g (G) ≤ p+1

Global Non Split Domination Number:

Definition 27: A dominating set D of a connected graph G is a global nonslip dominating set if D is a nonslip dominating set of both G and G.

Definition 28: The global nonslip domination number γ_{gns} (G) of G is the minimum cardinality of global nonslip dominating set of G

Example 25:

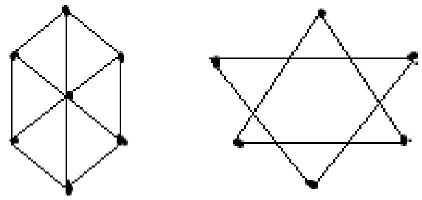
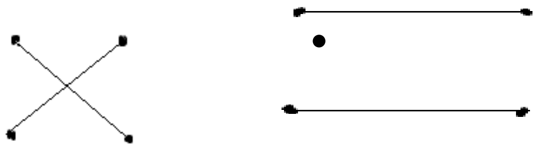


Figure 14: D = {a,d}

< V-D >



γ_{gns} (G) = 2

Note: A nonslip dominating set D of a connected graph G is a global non split dominating set if the following conditions are satisfied.

- ✓ Every condition in <V-D> is not adjacent to some vertex in D.
- ✓ <V-D> is K or there exists a set S≤V-D such that diam (<S>) ≥ 3 and for every vertex vε V-D, there exist a vertex uεs with u is not adjacent to V.

Theorem 11:

Let G be a connected graph with at least two adjacent non end vertices. A non split dominating set D of G is a global non split dominating set if the following conditions are satisfied.

- ✓ Every vertex in V-D is not adjacent to some vertex in D;
- ✓ D is not a vertex connectivity of $\overline{G}$

Proof:

Suppose D satisfies the given condition. Then by (i) D is a dominating set of $\overline{G}$

By (ii) <V-D> is connected in $\overline{G}$

This implies that D is a global non split dominating set of G

Conversely, Let D be a global non split dominating set G on the contrary,

Suppose one of the given condition say (i) is not satisfied. Then there exist a vεV –D adjacent to every vertex of D. This implies in $\overline{G}$. V is not adjacent to any vertex of D. Hence, D is not a dominating set of $\overline{G}$

Which is a contradiction.

Suppose (ii) doesn't hold. Then <V-D> is disconnected in $\overline{G}$, again a contradiction. Hence the given conditions are satisfied.

Theorem 12:

For any connected graph G, [(p (9-p)-6)/6] , ≤ γ_{gns} (G)Furthermore, the bound is attained if and only if there exist a γ_{gns} (G) set D satisfying the following conditions:

- ✓ D has exactly two vertices.
- ✓ Every vertex in V-D is adjacent to exactly one vertex in D and<V-D>is self complementary.

Proof:

Let D be a γ_{gns} G set of G and q_1 denote the number of edges in G U $\overline{G}$ incident to the vertices of V-D only. Similarly q_2 denotes the number of edges in G U $\overline{G}$ incident to the vertices of D only.

Then [P (P-1)]/2 ≥ q_1+q_2 ≥ 4|V-D| - 2+| D-1|

Thus [P (q-p)-6)] / 6 ≤ γ_{gns} (G)

To prove: <V-D> is a self complementary graph

Suppose bound is attained. Then 6 ≤ p ≤ 8 and q_1 = 4 |V –D| -2 and q_2 = |D-1|

If P=δ then either D contains three vertices or V-D has six vertices.

In both cases either q_2 > |D-1| or

q_1= 4 |V-D|-2 which is a contradiction.

Hence p=6 or 7 and D has exactly two vertices.

Since D is a global dominating set, each vertex in V-D is adjacent to exactly one vertex in D.

As 4 ≤ | V-D | ≤ 5 and <V-D> has same number of edges in G and $\overline{G}$,<V-D> is a self complementary graph.

Note:

- ✓ For a connected graph G, γ_{gns} (G) ≤ p-4 if and only if G contains p_4 such that for every vertex v in p_4 there exist two vertices u and w is not in P_4 such that v is adjacent to u but not w
- ✓ Let G be a connected graph with diam (G)>5 and δ (G) >3.Then γ_{gns} (G) ≤ p –diam(G)-1

Theorem 13:

Relationship between global non split domination number and the cototal domination number of a connected graph. For any connected graph G, γ_{cot} (G) ≤ γ_{gns} (G)

Proof:

Let D be a global non split dominating set of G.

Case 1: Suppose γ_{gns} (G) = p-1

Since G has atleast two non adjacent end vertices u and v, v-{u,v} is a cototal dominating set of G

Case 2: Suppose γ_{gns} (G) = p-2

Since <V-D> has no isolated vertices, D is a cototal dominating set of G.

Hence in both cases γ_{cot} (G) ≤ γ_{gns} (G)

Theorem 14:

Relation Between γ_{gns} (G) and γ_{bns} (G), If g (G) is the length of a shortest cycle in a connected graph G such that g (G) >5 and γ_{bns} (G) <p-2, then γ_{gns} (G) ≤ γ_{bns} (G)

Proof:

Let D be a γ_{bns} set of G. Then each cycle in <V-D> is of length atleast five as g (G)≥5

⇒ <V-D> is a cycle of length at least five.

Also each vertex in V-D is not adjacent to some vertex in D, since <V-D> is a block in G

By theorem 2.5.4, D is a global non split dominating set of G.

This γ_{gns} (G) ≤ γ_{bns} (G)

Note: If g (G) ≥ 5 and δ (G) ≥ 3 then γ_{gns} (G) ≤ p-g (G)

Other Parameters:

A total dominating set D of a connected graph G is a total split dominating set if the induced sub graph <V-D> is disconnected. The total split domination number γ_{ts} (G) of G is the minimum cardinality of a total split dominating set of G. This concept was introduced by Janakiram, soner and chaluvaraju.

An independent dominating set D of a graph G is a split independent dominating set if the induced sub graph <V-D> is disconnected. The split independent domination number i_s (G) of G is the minimum cardinality of a split independent dominating set. A connected dominating set D of a connected graph G is a split connected dominating set if the induced sub graph <V-D> is disconnected. The split connected domination number γ_{sc} (G) of a connected graph G is theminimum cardinality of a split connected dominating set. The total dominating set D of a connected graph G is total nonsplit dominating set if the induced subgraph <V-D> is connected. The total non split domination number γ_{tnd} (G) of G is the minimum cardinality of total non split dominating set.

Conclusion:

In previous sections, definitions and examples of non split domination number, block, block non split dominating sets, path non split domination number are studied. Cototal domination number and some theorems related to these non split domination numbers also studied. This concept of non split domination number has application in communication networks, centrality problems, land surveying, setting radio stations etc., hence the study of non split domination parameters is very much useful.

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