ISSN: 2456 – 3080
International Journal of Applied and Advanced Scientific Research
Impact Factor 5.255, Special Issue, February - 2017
International Conference on Advances in Theoretical and Applied Mathematics – ICATAM 2017
On 14th February 2017 Organized By
Madurai Sivakasi Nadars Pioneer Meenakshi Women's College, Poovanthi, Tamilnadu

RESULTS ON DISTANCE-2 DOMINATION SUBDIVISION NUMBER OF CARTESIAN PRODUCT GRAPH

G. Hemalatha* & P. Jeyanthi**

* Department of Mathematics, Shri Andal Alagar College of Engineering, Mamandur, Kancheepuram, Tamilnadu

** Research Centre, Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur, Tamilnadu

Cite This Article: G. Hemalatha & P. Jeyanthi, "Results on Distance-2 Domination Subdivision Number of Cartesian Product Graph", International Journal of Applied and Advanced Scientific Research, Special Issue, February, Page Number 87-93, 2017.

Abstract:

Let G be a simple graph on the vertex set $V(G)$. In a graph G, A set D $\subseteq V(G)$ is a dominating set of G if every vertex in $V(G) -$ D is adjacent to some vertex in D. The bondage number of a graph G[$Bd\,\gamma(G)$]is the cardinality of a smallest set of edges whose removal results in a graph with domination number larger than that of G. A set D $\subseteq V(G)$ is called a distance k dominating set of G if every vertex in $V(G) -$ D is with in distance k of at least one vertex in D, that is, for every vertex $V(G) -$ D, there exists a vertex $u \in$ D such that $d(u, v) \leq k$. In this paper we determine the domination number of Cartesian product graph in distance two dominating set and also find the subdivision number for Cartesian product graph.

Key Words: Dominating Set, Distance Two Domination, Cartesian Product Graph & Subdivision Number

1. Introduction:

The domination in graphs is one of the major areas in graph theory which attracts many researchers because it has the potential to solve many real life problems involving design and analysis of communication network, social network as well as defence surveillance. Many variants of domination models are available in the existing literature. The literature on domination and related parameters has been surveyed and beautifully presented in the two books by Haynes, Hedetniemi and Slater [2, 3]. The behaviour of several graph parameters in product graphs has become an interesting topic of research [4, 7]. The distance domination in the context of spanning tree is discussed in Griggs and Hutchinson [1] while bounds on the distance two-domination number and the classes of graphs attaining these bounds are reported in Sridharan et al. [6]. For more bibliographic references on distance k-domination readers are advised to refer the survey by Henning [4]. Let G be a simple graph on the vertex set $V(G)$. In a graph G, A set D $\subseteq V(G)$ is a dominating set of G if every vertex in $V(G) -$ D is adjacent to some vertex in D. The domination number of a graph G [γ(G)] is the minimum size of a dominating set of vertices in G. The domination subdivision number of a graph G [$Sd\,\gamma(G)$] is the minimum number of edges that must be subdivided in order to increase the domination number of a graph. The Cartesian product of G and H [G × H] is the graph with vertex set V(G) × V(H) specified by putting (u, v) adjacent to (u′, v′) if and only if (i) u = u′ and vv' belongs to E(H), or (ii) v = v′ and uu′ belongs to E(G). The neighbourhood of vertex u is denoted by $N(u) = \{v \in V(G)/uv \in E(G)\}$ and the close neighbourhood of vertex u is denoted by $N[u] = N(u) \cup \{u\}$. Let D $\subseteq V(G)$, the neighbourhood and closed neighbourhood of D are defined as $N(D) = \bigcup_{u \in D} N(u)$ and $N[D] = \bigcup_{u \in D} N[u]$. The distance $d(u, v)$ between two vertices u and v is the length of the shortest path between u and $v$in G.A set D $\subseteq V(G)$ is called a distance k dominating set of G if every vertex in $V(G) -$ D is withindistance $k$of at least one vertex in D, that is, for every vertex $V(G) -$ D, there exists a vertex $u \in$ D such that $d(u, v) \leq k$.Thus a set D $\subseteq V(G)$ is called a distance-2 dominating set if every vertex in $V(G) -$ D iswithin distance-2 of at least one vertex in D. That is, $N_2(u) = \{u \in V(G) /$ *du,v≤2*. The closed 2- neighbourhood set *N2u* of u is defined as *N2u=N2u∪u*. If D ⊆*VG*and*u∈VG*, then *du,D=mindu,v ;v∈D*. The minimum cardinality of a distance-2 dominating set in G is called the distance-2 domination number and is denoted$\gamma_{\leq 2}(G)$. A distance-2 dominating set of cardinality $\gamma_{\leq 2}(G)$ is called a $\gamma_{\leq 2}$-set.

Theorem 1.1[5]: (Ore's Theorem): In any graph $G = (V, E)$ having no isolated vertices, the complement $V \backslash S$ of any minimal dominating set S is a dominating set.

Theorem 1.2 [5]: A dominating set S is a minimal dominating set if and only if for each $v \in S$ one of the following two conditions hold:

- ✓ v is not adjacent to any vertex in S, or
- ✓ there is a vertex$u \notin S$ such that $N(u) \cap S = \{v\}$.

Example [5]: $\gamma_2(P_n) = \left\lceil \frac{n}{5} \right\rceil$ where P_n is the path on n vertices.

2. Distance 2- Domination Number:

In this paper, we establish the value of the distance-2 domination number of Cartesian product graph.

Observation 2.1: For any $P_1 \times P_n$, we have$\gamma_{\leq 2}(P_1 \times P_n) = \left\lceil \frac{n}{5} \right\rceil$.

Proposition 2.2: For $P_2 \times P_7$, we have $\gamma_{\leq 2}(P_2 \times P_7) = 2$.

Proof:

Let G be a Cartesian product graph of P_2 and P_7. To distance-2 domination, we need two vertices. Now the vertex (u_2, v_2) dominates $\{(u_2, v_1), (u_2, v_3), (u_2, v_4), (u_1, v_1), (u_1, v_2), (u_1, v_3)\}$ vertices at a distance 1 or 2, while the vertex (u_1, v_6) dominates $\{(u_2, v_5), (u_2, v_6), (u_2, v_7), (u_1, v_4), (u_1, v_5), (u_1, v_7)\}$ vertices at a distance 1 or 2. Therefore, $\gamma_{\leq 2}(P_2 \times P_7) = 2$.

Theorem 2.3: For any $n \geq 7$, we have

$$\gamma_{\trianglelefteq 2}(P_2 \times P_n) = \begin{cases} \frac{2n}{7} & for \quad n \equiv 0 \ (mod\ 7) \\ 2\left\lfloor \frac{n}{7} \right\rfloor + 1 & for \quad n \equiv 1,2,3,4 \ (mod\ 7) \\ 2\left\lfloor \frac{n}{7} \right\rfloor + 2 & for \quad n \equiv 5,6 \ (mod\ 7) \end{cases}$$

Proof:

Let $P_2 \times P_n$ be the Cartesian product graph and D_n the distance-2 dominating set of $P_2 \times P_n$. Now to describe our distance-2 dominating set $D = \left\{(u_2, v_{8i+2}), (u_1, v_{8j+6}) \ / \ i = 0, 1, 2, \dots \dots, \left\lfloor \frac{n}{10} \right\rfloor \ ; \ j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{14} \right\rfloor\right\}$for $n \equiv 0 \ (mod\ 7)$, we consider block $B \simeq P_2 \times P_7$. If $n \equiv 0 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\frac{n}{7}$number of blocks B. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{8i+2}), (u_1, v_{8j+6})$; $i = 0, 1, 2, \dots \dots, \left\lfloor \frac{n}{10} \right\rfloor$; $j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{14} \right\rfloor$. Moreover, since $|D| = \frac{2n}{7}$. Now, we consider the distance-2 dominating set $D_0 = D$. Hence $\gamma_{\trianglelefteq 2}(P_2 \times P_n) = \frac{2n}{7}$.

If $n \equiv 1 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{7} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_1$and $D \cap B' = \emptyset$. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{8i+2}), (u_1, v_{8j+6})$; $i = 0, 1, 2, \dots \dots, \left\lfloor \frac{n}{10} \right\rfloor$; $j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{14} \right\rfloor$ except the vertices of B'. Moreover, since $|D| = 2\left\lfloor \frac{n}{7} \right\rfloor$, to distance-2 dominate the vertices (u_1, v_n) and (u_2, v_n) of B', we need one vertex among $\{(u_2, v_n), (u_1, v_n)\}$. Now, we consider the distance-2 dominating set $D_1 = D \cup \{(u_2, v_n)\}$. Hence $\gamma_{\trianglelefteq 2}(P_2 \times P_n) = 2\left\lfloor \frac{n}{7} \right\rfloor + 1$.

If $n \equiv 2 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{7} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_2$and $D \cap B' = \emptyset$. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{8i+2}), (u_1, v_{8j+6})$; $i = 0, 1, 2, \dots \dots, \left\lfloor \frac{n}{10} \right\rfloor$; $j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{14} \right\rfloor$ except the vertices of B'. Moreover, since $|D| = 2\left\lfloor \frac{n}{7} \right\rfloor$, to distance-2 dominate the vertices $(u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)$ of B', we need one vertex among $\{(u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)\}$. Now, we consider the distance-2 dominating set $D_2 = D \cup \{(u_2, v_n)\}$. Hence $\gamma_{\trianglelefteq 2}(P_2 \times P_n) = 2\left\lfloor \frac{n}{7} \right\rfloor + 1$.

If $n \equiv 3 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{7} \right\rfloor$number of blocks B, plus a block $B' \simeq P_2 \times P_3$and $D \cap B' = \emptyset$. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{8i+2}), (u_1, v_{8j+6})$; $i = 0, 1, 2, \dots \dots, \left\lfloor \frac{n}{10} \right\rfloor$; $j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{14} \right\rfloor$ except the vertices of B'. Moreover, since $|D| = 2\left\lfloor \frac{n}{7} \right\rfloor$, to distance-2 dominate the vertices $(u_1, v_{n-2}), (u_2, v_{n-2}), (u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)$ of B', we need one vertex among $\{(u_1, v_{n-2}), (u_2, v_{n-2}), (u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)\}$. Now, we consider the distance-2 dominating set $D_3 = D \cup \{(u_2, v_n)\}$. Hence $\gamma_{\trianglelefteq 2}(P_2 \times P_n) = 2\left\lfloor \frac{n}{7} \right\rfloor + 1$.

If $n \equiv 4 \ (mod\ 5)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{7} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_4$and $D \cap B' = \emptyset$. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{8i+2}), (u_1, v_{8j+6})$; $i = 0, 1, 2, \dots \dots, \left\lfloor \frac{n}{10} \right\rfloor$; $j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{14} \right\rfloor$ except the vertices of B'. Moreover, since $|D| = 2\left\lfloor \frac{n}{7} \right\rfloor$, to distance-2 dominate the vertices $(u_1, v_{n-3}), (u_2, v_{n-3}), (u_1, v_{n-2}), (u_2, v_{n-2}), (u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)$ of B', we need one vertex among $\{(u_1, v_{n-1}), (u_2, v_{n-1})\}$. Now, we consider the distance-2 dominating set $D_4 = D \cup \{(u_2, v_{n-1})\}$. Hence $\gamma_{\trianglelefteq 2}(P_2 \times P_n) = 2\left\lfloor \frac{n}{7} \right\rfloor + 1$.

If $n \equiv 5 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{7} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_5$and $D \cap B' = \emptyset$. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{8i+2}), (u_1, v_{8j+6})$; $i = 0, 1, 2, \dots \dots, \left\lfloor \frac{n}{10} \right\rfloor$; $j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{14} \right\rfloor$ except the vertices of B'. Moreover, since $|D| = 2\left\lfloor \frac{n}{7} \right\rfloor$, to distance-2 dominate B', we need two vertices. Now, we consider the distance-2 dominating set $D_5 = D \cup \{(u_2, v_{n-2}), (u_1, v_n)\}$. Hence$\gamma_{\trianglelefteq 2}(P_2 \times P_n) = 2\left\lfloor \frac{n}{7} \right\rfloor + 2$.

If $n \equiv 6 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{7} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_6$and $D \cap B' = \emptyset$. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{8i+2}), (u_1, v_{8j+6})$; $i = 0, 1, 2, \dots \dots, \left\lfloor \frac{n}{10} \right\rfloor$; $j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{14} \right\rfloor$ except the vertices of B'. Moreover, since $|D| = 2\left\lfloor \frac{n}{7} \right\rfloor$, to distance-2 dominate B', we need two vertices. Now, we consider the distance-2 dominating set $D_5 = D \cup \{(u_2, v_{n-3}), (u_1, v_n)\}$. Hence $\gamma_{\trianglelefteq 2}(P_2 \times P_n) = 2\left\lfloor \frac{n}{7} \right\rfloor + 2$. This completes the proof.

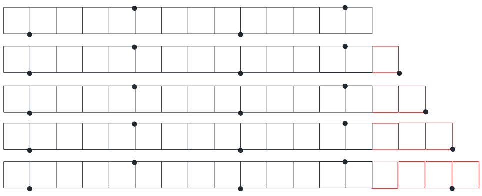

Figure 1: $P_2 \times P_n$ *with distance-2 domination number n=0,6?*

Theorem 2.4: For any $n \geq 4$, we have

$$\gamma_{\leq 2}(P_3 \times P_n) = \begin{cases} \frac{n}{3} & for\ \ if\ n \equiv 0\ (mod\ 3) \\ \left\lfloor \frac{n}{3} \right\rfloor + 1 & for\ if\ n \equiv 1, 2\ (mod\ 3) \end{cases}.$$

Proof:

Let $P_3 \times P_n$ be the Cartesian product graph and D_n the distance-2 dominating set of $P_3 \times P_n$. Now to describe our distance-2 dominating set $D = \left\{(u_2, v_{3i+2});\ i = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1\right\}$for $n \equiv 0\ (mod\ 3)$, we consider block $B \simeq P_3 \times P_3$ and $D \cap B = \{(u_2, v_2)\}$.

If $n \equiv 0\ (mod\ 3)$ then $P_3 \times P_n$ can be partitioned with $\frac{n}{3}$ number of blocks B.Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{3i+2});\ i = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1$. Moreover, since $|D| = \frac{n}{3}$. Now, we consider the distance-2 dominating set $D_0 = D$.Hence$\gamma_{\leq 2}(P_3 \times P_n) = \frac{n}{3}$.

If $n \equiv 1\ (mod\ 3)$ then $P_3 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{3} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_1$ and $D \cap B' = \emptyset$. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{3i+2});\ i = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1$ except the vertices ofB'.Moreover, since $|D| = \left\lfloor \frac{n}{3} \right\rfloor$, to distance-2 dominate the vertices (u_1, v_n) and (u_2, v_n) of B',we need one vertex among $\{(u_2, v_n), (u_1, v_n)\}$. Now, we consider the distance-2 dominating set $D_1 = D \cup \{(u_2, v_n)\}$. Hence $\gamma_{\leq 2}(P_3 \times P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$.

If $n \equiv 2\ (mod\ )$ then $P_3 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{3} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_2$and $D \cap B' = \emptyset$. Now, all the vertices of B can be distance-2 dominated by the vertices $(u_2, v_{3i+2});\ i = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1$ except the vertices of B'. Moreover, since $|D| = \left\lfloor \frac{n}{3} \right\rfloor$, to distance-2 dominate the vertices $(u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)$ of B', we need one vertex among $\{(u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)\}$.Now, we consider the distance-2 dominating set $D_1 = D \cup \{(u_2, v_n)\}$. Hence $\gamma_{\leq 2}(P_3 \times P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$.

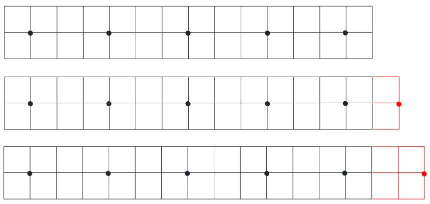

Figure 2: $P_3 \times P_n$ *with distance-2 domination number*

Observation 2.5: For any $n \geq 3$, we have $P_k \times P_n \simeq P_n \times P_k$.

Theorem 2.6: Let $B'_R \simeq P_1 \times P_nandB'_C \simeq P_n \times P_1$. For any ≥ 3 , we have

$$\gamma_{\leq 2}(P_n \times P_n) = \begin{cases} \frac{n^2}{9} & for\ \ n \equiv 0\ (mod\ 3) \\ \left\lfloor \frac{n}{3} \right\rfloor^2 + 2\left\lceil \frac{n}{5} \right\rceil - \gamma_{\leq 2}(\mathcal{P}) for\ \ n \equiv 1\ (mod\ 3) \\ \left\lfloor \frac{n}{3} \right\rfloor^2 + 2\left\lfloor \frac{n}{4} \right\rfloor + 2 - \gamma_{\leq 2}(\mathcal{P}) for\ \ n \equiv 2\ (mod\ 3) \end{cases}$$

Where$\gamma_{\leq 2}(\mathcal{P}) = \left|B'_R \cap B'_C\right| = \begin{cases} 0 & for\ \ n \equiv 0,3,4 (mod\ 5) \\ 1 & for\ \ n \equiv 1,2 (mod\ 5) \end{cases}$

Proof:

Let D_n be a distance-2 dominating set of$P_n \times P_n$. First five rows of $P_n \times P_n$ is considered as block$B \simeq P_3 \times P_n\ for\ n \equiv 0\ (mod\ 3)$ .If $n \equiv 0\ (mod\ 3)$ then $P_n \times P_n$ can be partitioned with $\frac{n}{3}$ number ofblocks B .By Theorem 2.4,$|D_0| = \frac{n}{3}\left(\frac{n}{3}\right)$.Hence$\gamma_{\leq 2}(P_n \times P_n) = \frac{n^2}{9}$.If $n \equiv 1\ (mod\ 3)$ then $P_n \times P_n$ can be partitioned with $\left\lfloor \frac{n}{3} \right\rfloor$ number of blocks B, plus a block $B'_R \simeq P_1 \times P_n$, plus a block $B'_C \simeq P_n \times P_1$ .By Theorem 2.4, we have $|D_1| = |D_0| + |B'_R| + |B'_C| - \left|B'_R \cap B'_C\right| = \left\lfloor \frac{n}{3} \right\rfloor \left\lfloor \frac{n}{3} \right\rfloor + \left\lceil \frac{n}{5} \right\rceil + \left\lceil \frac{n}{5} \right\rceil - \left|B'_R \cap B'_C\right|$. Hence $\gamma_{\leq 2}(P_n \times P_n) = \left\lfloor \frac{n}{3} \right\rfloor^2 + 2\left\lceil \frac{n}{5} \right\rceil - 1$. If $n \equiv 2\ (mod\ 3)$ then $P_n \times P_n$ can be partitioned with

ISSN: 2456 – 3080
International Journal of Applied and Advanced Scientific Research
Impact Factor 5.255, Special Issue, February - 2017
International Conference on Advances in Theoretical and Applied Mathematics – ICATAM 2017
On 14th February 2017 Organized By
Madurai Sivakasi Nadars Pioneer Meenakshi Women's College, Poovanthi, Tamilnadu

$\left\lfloor\frac{n}{3}\right\rfloor$ number of blocks B, plus a block $B'_R \simeq P_2 \times P_n$, plus a block $B'_C \simeq P_n \times P_2$. By observation 2.5 and Theorem 2.4, we have $|D_2| = |D_0| + |B'_R| + |B'_C| - \left|B'_R \cap B'_C\right| = \left\lfloor\frac{n}{3}\right\rfloor\left(\left\lfloor\frac{n}{3}\right\rfloor\right) + \left\lfloor\frac{n}{4}\right\rfloor + 1 + \left\lfloor\frac{n}{4}\right\rfloor + 1 - \left|B'_R \cap B'_C\right|$. Hence $\gamma_{\leq 2}(P_n \times P_n) = \left\lfloor\frac{n}{3}\right\rfloor^2 + 2\left\lfloor\frac{n}{4}\right\rfloor + 2 - \left|B'_R \cap B'_C\right|$.

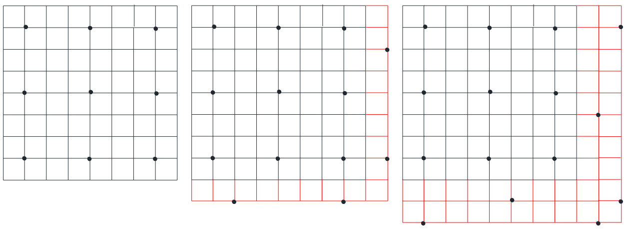

Figure 3: $P_n \times P_n$ with distance-2 domination number

3. Subdivision Number for Cartesian Product Graph:

Theorem 3.1: For $P_1 \times P_n$, we have $Sd\gamma_{\leq 2}(P_1 \times P_n) = 1$.

Proof:

Let D be a distance-2 dominating set of $P_1 \times P_n$. Since $|D| = \left\lceil\frac{n}{5}\right\rceil$. Let $(P_1 \times P_n)'$ be obtain from $P_1 \times P_n$ by subdividing an edge $(u_1, v_1)(u_1, v_2)$ and adding new vertex called x. Since the vertices $(u_1, v_1), (u_1, v_2)$ are not belongs to D. To distance-2 dominate the vertex (u_1, v_1), we need one vertex among $\{x, (u_1, v_1), (u_1, v_2)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_1, v_1)\}$. Thus, we have $|D'| = |D| + 1$. So we obtain that the distance-2 domination number of $(P_1 \times P_n)'$ is greater than the distance-2 domination number of $P_1 \times P_n$. This completes the proof.

Theorem3.2: For $P_2 \times P_n$, we have $Sd\gamma_{\leq 2}(P_2 \times P_n) = 1$.

Proof:

Let $D = \left\{(u_2, v_{8i+2}), (u_1, v_{8j+6}) \ ; \ i = 0, 1, 2, \dots\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor \ ; j = 0, 1, 2, \dots\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}$ be the distance-2 dominating set for $n \equiv 0 \ (mod\ 7)$, we consider block $B \simeq P_2 \times P_7$. We prove this theorem in each following cases.

If $n \equiv 0 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\frac{n}{7}$ number of blocks B. Let $(P_2 \times P_n)'$ be obtained from $P_2 \times P_n$ by subdividing an edge $(u_2, v_3)(u_2, v_4)$ and adding new vertex called x. The vertex x can be dominated from the vertex (u_2, v_2) at a distance 2. The vertices of D distance-2 dominate all of the vertices of $(P_2 \times P_n)'$ exactly once, except the vertex (u_2, v_4). To distance-2 dominate this vertex we need one vertex among $\{x, (u_1, v_3), (u_1, v_4), (u_1, v_5), (u_2, v_3), (u_2, v_5), (u_2, v_6)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_1, v_4)\}$. By Theorem2.3, we have $\gamma_{\leq 2}(P_2 \times P_n)' = \gamma_{\leq 2}(P_2 \times P_n) + 1$. Hence we obtain that the distance-2 domination number of $(P_2 \times P_n)'$ is greater than the distance-2 domination number of $P_2 \times P_n$.

If $n \equiv 1 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor\frac{n}{7}\right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_1$. The vertices of D distance-2 dominate all of the vertices of $(P_2 \times P_n)$ exactly once, except the vertices $(u_1, v_n), (u_2, v_n)$. To distance-2 dominate these vertices we need one vertex (u_2, v_n). By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n) = 2\left\lfloor\frac{n}{7}\right\rfloor + 1$. Let $(P_2 \times P_n)'$ be obtained from $P_2 \times P_n$ by subdividing an edge $(u_2, v_3)(u_2, v_4)$ and adding new vertex called x. The vertex x can be dominated from the vertex in D at a distance 2. But the vertex (u_2, v_4) is not dominated by any vertex in D. We can rewrite $D = \left\{(u_2, v_{8i+3}), (u_1, v_{8j+7}) \ ; \ i = 0, 1, 2, \dots\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor \ ; j = 0, 1, 2, \dots\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}$ and distance-2 dominating set of B' is $\{(u_1, v_1)\}$. Thus, the vertices x and (u_2, v_4) distance-2 dominated by the vertex (u_2, v_3) in D. But the vertex (u_2, v_5) is not dominated by any vertex in D. To distance-2 dominate this vertex, we need one vertex among $\{x, (u_2, v_4), (u_2, v_6), (u_2, v_7), (u_1, v_4), (u_1, v_5), (u_1, v_6)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_1, v_1), (u_1, v_5)\}$. By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n)' > \gamma_{\leq 2}(P_2 \times P_n)$.

If $n \equiv 2 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor\frac{n}{7}\right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_2$. The vertices of D distance-2 dominate all of the vertices of $(P_2 \times P_n)$ exactly once, except the vertices $(u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)$. To distance-2 dominate these vertices we need one vertex (u_2, v_n). By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n) = 2\left\lfloor\frac{n}{7}\right\rfloor + 1$. Let $(P_2 \times P_n)'$ be obtained from $P_2 \times P_n$ by subdividing an edge $(u_2, v_4)(u_2, v_5)$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. We can rewrite $D = \left\{(u_2, v_{8i+4}), (u_1, v_{8j+8}) \ ; \ i = 0, 1, 2, \dots\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor \ ; j = 0, 1, 2, \dots\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}$ and distance-2 dominating set of B' is $\{(u_1, v_1)\}$. Thus, the vertex x distance-2 dominated by the vertex (u_2, v_4) in D. But the vertex (u_2, v_6) is not dominated by any vertex in D. To distance-2 dominate this vertex, we need one vertex among $\{x, (u_2, v_5), (u_2, v_7), (u_2, v_8), (u_1, v_5), (u_1, v_6), (u_1, v_7)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_1, v_1), (u_1, v_6)\}$. By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n)' > \gamma_{\leq 2}(P_2 \times P_n)$.

If $n \equiv 3 \ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor\frac{n}{7}\right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_3$. The vertices of D distance-2 dominate all of the vertices of $(P_2 \times P_n)$ exactly once, except the vertices

$$(u_1, v_{n-3}), (u_2, v_{n-3}), (u_1, v_{n-2}), (u_2, v_{n-2}), (u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n).$$

ISSN: 2456 – 3080
International Journal of Applied and Advanced Scientific Research
Impact Factor 5.255, Special Issue, February - 2017
International Conference on Advances in Theoretical and Applied Mathematics – ICATAM 2017
On 14th February 2017 Organized By
Madurai Sivakasi Nadars Pioneer Meenakshi Women's College, Poovanthi, Tamilnadu

To distance-2 dominate these vertices we need one vertex(u_2, v_n). By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n) = 2\left\lfloor\frac{n}{7}\right\rfloor + 1$. Let $(P_2 \times P_n)'$ be obtained from $P_2 \times P_n$ by subdividing an edge $(u_2, v_3)(u_2, v_4)$ and adding new vertex called x. The vertex x can be dominated from the vertex in D at a distance 2. But the vertex(u_2, v_4) is not dominated by any vertex in D. We can rewrite $D = \left\{(u_2, v_{8i+5}), (u_1, v_{8j+9})\ ;\ i = 0,1,2,\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor ; j = 0,1,2,\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}$and distance-2 dominating set of $B'$is $\{(u_1, v_1)\}$. Thus, the vertex x and (u_2, v_4) distance-2 dominated by the vertex(u_2, v_5)in D. But the vertex(u_2, v_3) is not dominated by any vertex in D. To distance-2 dominate this vertex, we need one vertex among

$$\{x, (u_2, v_4), (u_2, v_2), (u_2, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4)\}.$$

Now, we consider the distance-2 dominating set $D' = D \cup \{(u_1, v_1), (u_1, v_3)\}$. By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n)' > \gamma_{\leq 2}(P_2 \times P_n)$.

If $n \equiv 4\ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor\frac{n}{7}\right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_4$. The vertices of D distance-2 dominate all of the vertices of $(P_2 \times P_n)$ exactly once, except the vertices

$$(u_1, v_{n-3}), (u_2, v_{n-3}), (u_1, v_{n-2}), (u_2, v_{n-2}), (u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n).$$

To distance-2 dominate these vertices we need one vertex(u_2, v_{n-1}). By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n) = 2\left\lfloor\frac{n}{7}\right\rfloor + 1$. Let $(P_2 \times P_n)'$ be obtained from $P_2 \times P_n$ by subdividing an edge $(u_2, v_4)(u_2, v_5)$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. We can rewrite $D = \left\{(u_2, v_{8i+6}), (u_1, v_{8j+10})\ ;\ i = 0,1,2,\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor ; j = 0,1,2,\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}$and distance-2 dominating set of $B'$is $\{(u_1, v_2)\}$. Thus, the vertices x distance-2 dominated by the vertex(u_2, v_6)in D. But the vertex(u_2, v_4) is not dominated by any vertex in D. To distance-2 dominate this vertex, we need one vertex among$\{x, (u_2, v_5), (u_2, v_3), (u_2, v_2), (u_1, v_3), (u_1, v_4), (u_1, v_5)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_1, v_2), (u_1, v_4)\}$. By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n)' > \gamma_{\leq 2}(P_2 \times P_n)$.

If $n \equiv 5\ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor\frac{n}{7}\right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_5$. Now, we consider the distance-2 dominating set

$$D = \left\{(u_2, v_{n-2}), (u_2, v_{8i+2}), (u_1, v_{8j+6})\ ;\ i = 0,1,2,\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor ; j = 0,1,2,\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}.$$

The vertices of D distance-2 dominate all of the vertices of $(P_2 \times P_n)$ exactly once, except the vertex(u_1, v_n). To distance-2 dominate this vertex we need one vertex(u_1, v_n). By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n) = 2\left\lfloor\frac{n}{7}\right\rfloor + 2$. Let $(P_2 \times P_n)'$ be obtained from $P_2 \times P_n$ by subdividing an edge $(u_2, v_4)(u_2, v_5)$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. We can rewrite $D = \left\{(u_1, v_3), (u_2, v_{8i+7}), (u_1, v_{8j+11})\ ;\ i = 0,1,2,\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor ; j = 0,1,2,\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}$and distance-2 dominating set of $B'$is $\{(u_2, v_1)\}$. Here also the vertex$x$ is not dominated by any vertex in D. To distance-2 dominate this vertex, we need one vertex among $\{(u_2, v_3), (u_2, v_4), (u_2, v_5), (u_2, v_6), (u_1, v_4), (u_1, v_5)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_2, v_1), (u_1, v_5)\}$. By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n)' > \gamma_{\leq 2}(P_2 \times P_n)$.

If $n \equiv 6\ (mod\ 7)$ then $P_2 \times P_n$ can be partitioned with $\left\lfloor\frac{n}{7}\right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_6$. Now, we consider the distance-2 dominating set

$$D = \left\{(u_2, v_{n-3}), (u_2, v_{8i+2}), (u_1, v_{8j+6})\ ;\ i = 0,1,2,\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor ; j = 0,1,2,\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}.$$

The vertices of D distance-2 dominate all of the vertices of $(P_2 \times P_n)$ exactly once, except the vertex (u_1, v_n). To distance-2 dominate this vertex we need one vertex(u_1, v_n). By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n) = 2\left\lfloor\frac{n}{7}\right\rfloor + 2$. Let $(P_2 \times P_n)'$ be obtained from $P_2 \times P_n$ by subdividing an edge $(u_1, v_4)(u_1, v_5)$ and adding new vertex called x. The vertex x can be dominated from the vertex in D at a distance 2. But the vertex (u_1, v_4) is not dominated by any vertex in D. We can rewrite $D = \left\{(u_1, v_4), (u_2, v_{8i+8}), (u_1, v_{8j+12})\ ;\ i = 0,1,2,\dots\dots, \left\lfloor\frac{n}{10}\right\rfloor ; j = 0,1,2,\dots\dots, \left\lfloor\frac{n}{14}\right\rfloor\right\}$and distance-2 dominating set of $B'$is $\{(u_2, v_1)\}$.Here $(u_1, v_4) \in D$.but the vertex (u_1, v_6) is not dominated by any vertex in D. To distance-2 dominate this vertex, we need one vertex among$\{(u_2, v_5), (u_2, v_6), (u_2, v_7), (u_1, v_5), (u_1, v_7), (u_1, v_8)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_2, v_1), (u_2, v_6)\}$. By Theorem 2.3, we have $\gamma_{\leq 2}(P_2 \times P_n)' > \gamma_{\leq 2}(P_2 \times P_n)$. This completes the proof.

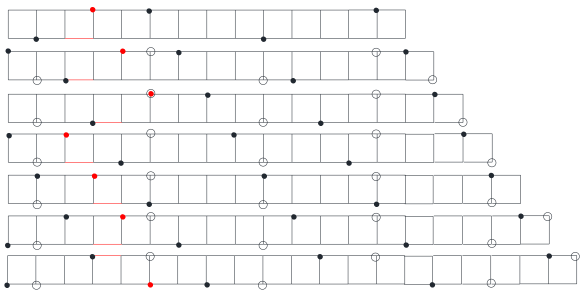

Figure 4: $P_2 \times P_n$ with distance-2 domination subdivision number $Sd\gamma_{\leq 2}(P_2 \times P_n) = 2$

ISSN: 2456 – 3080
International Journal of Applied and Advanced Scientific Research
Impact Factor 5.255, Special Issue, February - 2017
International Conference on Advances in Theoretical and Applied Mathematics – ICATAM 2017
On 14th February 2017 Organized By
Madurai Sivakasi Nadars Pioneer Meenakshi Women's College, Poovanthi, Tamilnadu

Theorem 3.3: For $P_3 \times P_n$, we have

$$Sd\gamma_{\leq 2}(P_3 \times P_n) = \begin{cases} 1 & for\ if\ n \equiv 0\ (mod\ 3) \\ 2 & for\ if\ n \equiv 1, 2, (mod\ 3) \end{cases}$$

Proof:

Let $D = \left\{(u_2, v_{3i+2});\ i = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1\right\}$ be the distance-2 dominating set for $n \equiv 0\ (mod\ 3)$.We consider block $B \simeq P_3 \times P_3$. We prove this theorem in each following cases.

If $n \equiv 0\ (mod\ 3)$ then $P_3 \times P_n$ can be partitioned with $\frac{n}{3}$ number of blocks B. Let $(P_3 \times P_n)'$ be obtained from $P_3 \times P_n$ by subdividing an edge $(u_1, v_3)(u_1, v_4)$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. To distance-2 dominate this vertex we need one vertex among $\{(u_1, v_2), (u_1, v_3), (u_1, v_4), (u_1, v_5), (u_2, v_3), (u_2, v_4)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_2, v_4)\}$. By Theorem 2.4, we have $\gamma_{\leq 2}(P_3 \times P_n)' = \gamma_{\leq 2}(P_3 \times P_n) + 1$. Hence we obtain that the distance-2 domination number of $(P_3 \times P_n)'$ is greater than the distance-2 domination number of $P_3 \times P_n$.

If $n \equiv 1\ (mod\ 3)$ then $P_3 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{3} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_3 \times P_1$. The vertices of D distance-2 dominate all of the vertices of $(P_3 \times P_n)$ exactly once, except the vertices $(u_1, v_n), (u_2, v_n)$. To distance-2 dominate these vertices we need one vertex(u_2, v_n). By Theorem 2.4, we have $\gamma_{\leq 2}(P_3 \times P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$. Let $(P_3 \times P_n)'$ be obtained from $P_3 \times P_n$ by subdividing an edge $(u_1, v_3)(u_1, v_4)$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. We can rewrite $D = \left\{(u_2, v_{3i+3});\ i = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1\right\}$and distance-2 dominating set of B' is $\{(u_2, v_1)\}$. Thus, the vertex$x$ distance-2 dominated by the vertex(u_2, v_3)in D. By Theorem 2.4, we have $\gamma_{\leq 2}(P_3 \times P_n)' = \gamma_{\leq 2}(P_3 \times P_n)$. Let $(P_3 \times P_n)''$ be obtained from $P_3 \times P_n$ by subdividing the edges $(u_1, v_3)(u_1, v_4)$, $(u_1, v_4)(u_1, v_5)$ and adding new vertices respectively called $x$and y. The vertices x can be dominated from the vertex(u_2, v_3). The vertices of D distance-2 dominate all of the vertices of $(P_3 \times P_n)''$ exactly once, except the vertex y . To distance-2 dominate this vertex we need one vertex among $\{x, (u_1, v_4), (u_1, v_5), (u_1, v_6), (u_2, v_4), (u_2, v_5)\}$. Now, we consider the distance-2 dominating set $D'' = D \cup \{(u_2, v_1), (u_2, v_5)\}$. Thus, we have $\gamma_{\leq 2}(P_3 \times P_n)'' = \gamma_{\leq 2}(P_3 \times P_n) + 1$. Hence we obtain that the distance-2 domination number of $(P_3 \times P_n)''$is greater than the distance-2 domination number of $P_3 \times P_n$.

If $n \equiv 2\ (mod\ 3)$ then $P_3 \times P_n$ can be partitioned with $\left\lfloor \frac{n}{3} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_3 \times P_2$. The vertices of D distance-2 dominate all of the vertices of $(P_3 \times P_n)$ exactly once, except the vertices $(u_1, v_{n-1}), (u_2, v_{n-1}), (u_1, v_n), (u_2, v_n)$. To distance-2 dominate these vertices we need one vertex(u_2, v_n). By Theorem 2.4, we have $\gamma_{\leq 2}(P_3 \times P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$. Let $(P_3 \times P_n)'$ be obtained from $P_3 \times P_n$ by subdividing an edge $(u_1, v_6)(u_1, v_7)$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. We can rewrite $D = \left\{(u_2, v_{3i+4});\ i = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1\right\}$and distance-2 dominating set of $B'$is $\{(u_2, v_1)\}$. Thus, the vertex x distance-2 dominated by the vertex(u_2, v_7)in D. By Theorem 2.4, we have $\gamma_{\leq 2}(P_3 \times P_n)' = \gamma_{\leq 2}(P_3 \times P_n)$. Let $(P_3 \times P_n)''$ be obtained from $P_3 \times P_n$ by subdividing the edges $(u_1, v_6)(u_1, v_7)$, $(u_1, v_5)(u_1, v_6)$ and adding new vertices respectively called $x$and y. The vertices x can be dominated from the vertex(u_2, v_7). The vertices of D distance-2 dominate all of the vertices of $(P_3 \times P_n)''$ exactly once, except the vertex y. To distance-2 dominate this vertex we need one vertex among $\{x, (u_1, v_4), (u_1, v_5), (u_1, v_6), (u_2, v_5), (u_2, v_6)\}$. Now, we consider the distance-2 dominating set $D'' = D \cup \{(u_2, v_1), (u_2, v_6)\}$. Thus, we have $\gamma_{\leq 2}(P_3 \times P_n)'' = \gamma_{\leq 2}(P_3 \times P_n) + 1$. Hence we obtain that the distance-2 domination number of $(P_3 \times P_n)''$is greater than the distance-2 domination number of $P_3 \times P_n$.

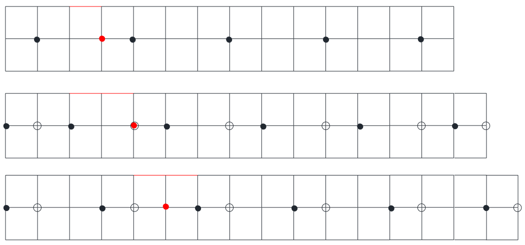

Figure 5: $P_3 \times P_n$with distance-2 domination subdivision number $Sd\gamma_{\leq 2}(P_3 \times P_n) = 2$

Theorem 3.4: For $P_n \times P_n$, we have

$$Sd\gamma_{\leq 2}(P_n \times P_n) = \begin{cases} 1 & for\ if\ n \equiv 0,2\ (mod\ 3) \\ 2 & for\ if\ n \equiv 1 (mod\ 3) \end{cases}..$$

Proof:

Let $D = \left\{(u_{3i+2}, v_{3i+2}), (u_{3i+2}, v_{3j+2})\ ;\ i \neq j, ;\ i, j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1\right\}$ be thedistance-2 dominating set for $n \equiv 0\ (mod\ 3)$, we consider block $B \simeq P_3 \times P_n$. We prove this theorem in each following cases.

If $n \equiv 0 \ (mod\ 3)$ then $P_n \times P_n$ can be partitioned with $\frac{n}{3}$ number of blocks B. Let $(P_n \times P_n)'$ be obtained from $P_n \times P_n$ by subdividing an edge $(u_1, v_3)(u_1, v_4)$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. To distance-2 dominate this vertex we need one vertex among $\{(u_1, v_2), (u_1, v_3), (u_1, v_4), (u_1, v_5), (u_2, v_3), (u_2, v_4)\}$. Now, we consider the distance-2 dominating set $D' = D \cup \{(u_2, v_4)\}$. By Theorem 2.6, we have $\gamma_{\leq 2}(P_n \times P_n)' = \gamma_{\leq 2}(P_n \times P_n) + 1$. Hence we obtain that the distance-2 domination number of $(P_n \times P_n)'$ is greater than the distance-2 domination number of $P_n \times P_n$.

If $n \equiv 1 \ (mod\ 3)$ then $P_n \times P_n$ can be partitioned with $\left\lfloor \frac{n}{3} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_1 \times P_n$. Let $(P_n \times Pn'$ be obtained from $Pn \times Pn$ by subdividing an edge $u1,v3u1,v4$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. We can rewrite $D = \left\{(u_{3i+3}, v_{3i+2}), (u_{3i+3}, v_{3j+2}) \ ; \ i \neq j, ; \ i, j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1\right\}$. Thus, the vertices x and (u_1, v_5) are distance-2 dominated by the vertices (u_1, v_3) and (u_3, v_5) respectively. By Theorem 2.6, we have $\gamma_{\leq 2}(P_n \times P_n)' = \gamma_{\leq 2}(P_n \times P_n)$. Let $(P_n \times P_n)''$ be obtained from $P_n \times P_n$ by subdividing the edges $(u_1, v_3)(u_1, v_4)$, $(u_1, v_5)(u_2, v_5)$ and adding new vertices respectively called x and y. The vertices of D distance-2 dominate all of the vertices of $(P_n \times P_n)''$ exactly once, except the vertex (u_1, v_5) . To distance-2 dominate this vertex we need one vertex among $\{x, y, (u_1, v_4), (u_1, v_6), (u_1, v_7), (u_2, v_4), (u_2, v_5), (u_2, v_6)\}$. By Observation2.1 and Theorem 2.6, we have $\gamma_{\leq 2}(P_n \times P_n)'' = \gamma_{\leq 2}(P_n \times P_n) + 1$. Hence we obtain that the distance-2 domination number of $(P_n \times P_n)''$is greater than the distance-2 domination number of $P_n \times P_n$.

If $n \equiv 2 \ (mod\ 3)$ then $P_n \times P_n$ can be partitioned with $\left\lfloor \frac{n}{3} \right\rfloor$ number of blocks B, plus a block $B' \simeq P_2 \times P_n$. Let $(P_n \times Pn'$ be obtained from $Pn \times Pn$ by subdividing an edge $u1,v3u1,v4$ and adding new vertex called x. All the vertices can be dominated from the vertex in D at a distance 1 or 2. But the vertex x is not dominated by any vertex in D. We can rewrite $D = \left\{(u_{3i+4}, v_{3i+2}), (u_{3i+4}, v_{3j+2}) \ ; \ i \neq j, ; \ i, j = 0, 1, 2, \dots \dots \dots, \left\lfloor \frac{n}{3} \right\rfloor - 1\right\}$. Thus, the vertex x is distance-2 dominated by the vertiex(u_1, v_3). But the vertex (u_1, v_4) is not dominated by any vertex in D. By Theorems2.3 and 2.6, we have $\gamma_{\leq 2}(P_n \times P_n)' > \gamma_{\leq 2}(P_n \times P_n)$. This completes the proof.

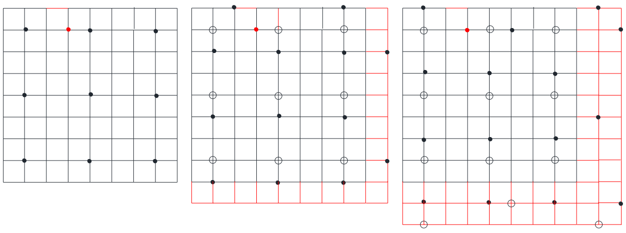

Figure 6: $P_n \times P_n$ with distance-2 domination subdivision number $Sd\gamma_{\leq 2}(P_n \times P_n) = 2$

References:

1. J. R. Griggs and J. P. Hutchinson, On the r-domination number of a graph, Discrete Mathematics, 101, (1992), 65-72.
2. T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, New York, 1997.
3. T. W. Haynes, S. T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics, Marcel Dekker, New York, 1998.
4. W. Imrich, S. Klavˇzar, D. F. Rall: Topics in Graph Theory. A. K. Peters Ltd.,Wellesley, MA, 2008.
5. Ore, Oystein, Theory of Graphs, No. 38, American Mathematical Soc., 1962.
6. N. Sridharan, V.S.A. Subramanian and M.D. Elias, Bounds on the Distance Two-Domination Number of a Graph, Graphs and Combinatorics, 18(3), (2002), 667-675.
7. G. Yero, J. A. Rodr´ıguez-Vel´azquez, Roman domination in Cartesian product graphs and Cartesian product graph, Discrete Math. 7(2013), 262-274.