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ANALYTICAL EXPRESSIONS OF HEAT TRANSFER TO STEADY MHD FLOW

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Abstract:

In this paper we investigate the combined effect of a transverse magnetic field and radiative heat transfer to steady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non-uniform walls temperature. The approximate analytical expressions for the dimensionless axial velocity and dimensionless temperature of MHD fluid flow problem are derived. The shear stress function and rate of heat transfer are solved analytically and graphically.

Key Words: Oscillatory Flow, Porous Medium, Magnetic Field, Heat Transfer & Shear Stress

1. Introduction:

The flow of fluids through porous medium is governing by Darcy's law, which was formulated by the Henry Darcy and has applications in geophysics, engineering, thermal insulation, heat storage. Magnetohydrodynamics (MHD) is the study of flow of an electrically conducting fluid and it can be seen in many engineering problems such as MHD power generators, plasma studies, nuclear reactors, geothermal energy extraction and the boundary layer control in the field of aerodynamics [1]. Moreau [2] contains a survey of MHD studies in the technological fields. In the presence of magnetic field, the flow of Newtonian fluid has applications in many areas including the handling of biological fluids and flow of nuclear fuel slurries, liquid metals and alloys, plasma, mercury amalgams, and blood ([3], [4] and [5]). Another important field of application is electromagnetic propulsion. Basically, an electro-magnetic propulsion system consists of a power source (such as nuclear reactor), plasma and tube through which the plasma is accelerated by electro-magnetic forces. The study of such systems, which is closely associated with magneto-chemistry, requires a complete understanding of the equation of state and transport properties such as diffusion, the shear stress-shear rate relationship, thermal conductivity, electrical conductivity and radiation. Some of these properties will undoubtedly be influenced by the presence of an external magnetic field that sets plasma in hydrodynamic motion ([6] and [7]). The flow of fluid through porous media has become importance due to recovery of crude oil from the pores of reservoir rock. There is a lot of application in studying about the magnetohydrodynamic (MHD) flow and heat transfer in porous media because of the effect of magnetic fields on the performance of many systems [8]. Due to the increase in applications of free convective and heat transfer flows through porous medium under the influence of magnetic field many researchers have studied about the magnetohydrodynamic free convective heat transfer flow in a porous medium. For example Raptis et al. [9] we can study about the unsteady free convective motion through a porous medium bounded by an infinite vertical plate. Ajibade and Jha [10] investigated the effects of suction and injection on hydrodynamics of oscillatory fluid through parallel plates. Ram and Mishra [11] analyzed unsteady flow through MHD porous media. In Makinde et.al [12] we study the effect of thermal radiation on MHD oscillatory flow in a channel filled with saturated porous medium and non-uniform wall temperatures. Kumar et al.(2010)has considered the problem of unsteady MHD periodic flow of viscous fluid through a planar channel in porous medium using perturbation techniques. In the present paper, we are going to discuss about the combined effect of transverse magnetic field and radiative heat transfer on steady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non-uniform walls temperature.

2. Mathematical Formulation of the Problem:

Assume that the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Fig. 1.

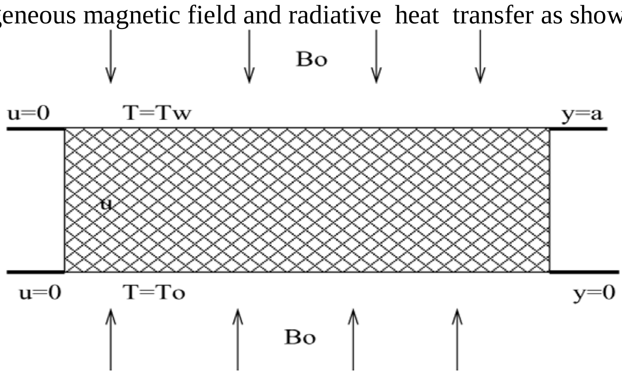

Figure 1: Geometry of the Problem

It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system (x, y) where x lies along the centre of the channel, y is the distance measured in the normal section. Using Boussinesq in compressible fluid model, the equations governing the motion are given as:

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$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+v\frac{\partial^2 u}{\partial y^2}-\frac{v}{K}u-\frac{\sigma_e B_o^2}{\rho}u+g\beta(T-T_o) \quad (1)$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q}{\partial y} \quad (2)$$

with the corresponding boundary conditions are as follows

$$u=0, T=T_w, on\ y=1 \quad (3)$$

$$u=0, T=T_o, on\ y=0 \quad (4)$$

where $u(y)$ is the axial velocity, t the time, T the fluid temperature, P the pressure, g the gravitational force, q the radiative heat flux, β the coefficient of volume expansion due to temperature, c_p the specific heat at constant pressure, k the thermal conductivity, K the porous medium permeability coefficient, $B_o=(\mu_e H_o)$ the electromagnetic induction, μ_e the magnetic permeability, H_o the intensity of magnetic field, σ_e the conductivity of the fluid, ρ the fluid density and v is the kinematic viscosity coefficient. It is assumed that both walls temperature T_o, T_w are high enough to induce radiative heat transfer. We assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y}=4\alpha^2(T_o-T_w) \quad (5)$$

where α is the mean radiation absorption coefficient.
We introduce the following dimensionless variables and parameters:

$$Re=\frac{Ua}{v}, \bar{x}=\frac{x}{a}, \bar{y}=\frac{y}{a}, \bar{u}=\frac{u}{U}, \theta=\frac{T-T_0}{T_w-T_o}, H^2=\frac{a^2\sigma_e B_o^2}{\rho v}, \bar{t}=\frac{tU}{a}; \bar{P}=\frac{aP}{\rho v U},$$
$$Da=\frac{K}{a^2}, Gr=\frac{g\beta(T_w-T_o)a^2}{vU}, Pe=\frac{Ua\rho c_p}{k}, N^2=\frac{4\alpha^2 a^2}{k} \quad (6)$$

Where U is the flow mean velocity. The dimensionless governing equations together with the appropriate boundary conditions, (neglecting the bars for clarity) can be written as

$$Re\frac{\partial u}{\partial t}=-\frac{\partial P}{\partial x}+\frac{\partial^2 u}{\partial y^2}-(s^2+H^2)u+Gr\theta \quad (7)$$

$$Pe\frac{\partial \theta}{\partial t}=\frac{\partial^2 \theta}{\partial y^2}+N^2\theta \quad (8)$$

The steady flow eqns. of (7) and (8) are as follows:

$$\lambda+\frac{d^2u}{dy^2}-(s^2+H^2)u+Gr\theta=0 \quad (9)$$

$$\frac{d^2\theta}{dy^2}+N^2\theta=0 \quad (10)$$

The corresponding boundary conditions are as follows:

$$u=0, \theta=1, on\ y=1 \quad (11)$$

$$u=0, \theta=0, on\ y=0 \quad (12)$$

Where Gr means Grashoff number, H indicates Hartmann number, N denotes Radiation parameter, Pe represents Péclet number, Re refers Reynolds number, Da means Darcy number and $s=(1/Da)$ denotes porous medium shape factor parameter.

3. Solution of the Non-Linear Boundary Value Problem:

In recent days, a basic tool for solving nonlinear problem is Homotopy analysis method (HAM) which was generated by Liao [13], is employed to solve the non-linear differential equation. The Homotopy analysis method is based on a basic concept in topology, i.e. Homotopy by Hilton [14] which is widely applied in numerical techniques as in [15–18]. Homotopy analysis method is independent of the small/large parameters not like the perturbation techniques [19]. There is a simple way to adjust and control the convergence region and rate of approximation series in Homotopy analysis method. The Homotopy analysis method has applied in many nonlinear problems such as heat transfer [20], viscous flows [21], nonlinear oscillations [22], Thomas-Fermi's atom model [23], non-linear water waves [24], etc. Such varied successful applications of the Homotopy analysis method conform

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its validity for nonlinear problems in science and engineering. The auxiliary parameter h is used to adjust and control the convergence of the series solution. In [25], mathematical expression is solved using Homotopy analysis method.

In this paper, the non-linear boundary value problem which is expressed in the eqns. (9) - (12) can be solved directly. The approximate analytical expressions for the dimensionless axial velocity $u(y)$ and dimensional temperature $\theta(y)$ are as follows:

$$u(y) = C_1 \exp(\sqrt{s^2+H^2}\,y) + C_2 \exp(-\sqrt{s^2+H^2}\,y) + \frac{\lambda}{s^2+H^2} + \frac{Gr\sin(Ny)}{(N^2+s^2+H^2)\sin(N)} \tag{13}$$

Where

$$C_1 = \left(\frac{1}{(s^2+H^2)(\exp(-\sqrt{s^2+H^2}) - \exp(\sqrt{s^2+H^2}))}\right)\left(\begin{array}{l} -\lambda(-1+\exp(-\sqrt{s^2+H^2}) \\ +\dfrac{Gr(s^2+H^2)}{N^2+s^2+H^2}\end{array}\right) \tag{14}$$

$$C_2 = \left(\frac{1}{\exp(-\sqrt{s^2+H^2}) - \exp(\sqrt{s^2+H^2})}\right)\left(\begin{array}{l} \dfrac{\lambda}{s^2+H^2}(-1+\exp(\sqrt{s^2+H^2}) \\ -\dfrac{Gr}{N^2+s^2+H^2}\end{array}\right) \tag{15}$$

$$\theta(y) = \frac{\sin(Ny)}{\sin(N)} \tag{16}$$

The analytical expression for the shear stress at the upper wall of the channel is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=1} = -\sqrt{s^2+H^2}C_1 \exp(\sqrt{s^2+H^2}) + \sqrt{s^2+H^2}C_2 \exp(-\sqrt{s^2+H^2}) - \frac{NGr\cos(N)}{(N^2+s^2+H^2)\sin(N)} \tag{17}$$

where C_1 and C_2 are the constants which are defined by the eqns. (14) and (15) respectively.

The rate of heat transfer across the channel's wall is given as

$$Nu = -\frac{N\cos(N)}{\sin(N)} \tag{18}$$

4. Result and Discussion:

Figure 1 shows geometry of the MHD flow problem. Figure 2 represent the dimensionless temperature $\theta(y)$ versus the transverse distance y. From Fig. 2, it is noted that when the radiation parameter N increases the corresponding dimensionless temperature profile also increases in some fixed values of the other dimensionless parameters $Pc, s, H, Gr, \mathrm{Re}, \lambda$.

Figure 3 represent the axial velocity $u(y)$ versus the transverse distance y. From Fig.3(a) it is clear that when the radiation parameter N increases the corresponding axial velocity profile also increases in some fixed values of the other parameters. From Fig.3(b) it is inferred that when the Hartmann number H increases the corresponding axial velocity profile decreases in some fixed values of the other parameters. From Fig.3(c) it is depict that when the porous medium shape factor s increases the corresponding axial velocity profile decreases in some fixed values of the other parameters. From Fig.3(d) it is clear that when the Grashoff number Gr increases the corresponding axial velocity profile increases in some fixed values of the other parameters.

Figure 4represent the variation of wall shear stress with Hartmann number H. FromFig.4(a), it is noted that when the radiation parameter N increases the corresponding shear stress at the upper wall of the channel also increases in some fixed value of the other dimensionless parameters. From Fig.4(b) it is depict that when the porous medium shape factor s increases the corresponding shear stress at the upper wall of the channel decreases in some fixed values of the other parameters. From Fig.4(c) it is inferred that when the Grashoff number Gr also increases the corresponding shear stress increases in some fixed values of the other dimensionless parameters.

Figure 5 represent the rate of heat transverse versus the radiation parameter N and also it is observe that the value of the heat transverse increases along the radiation parameter.

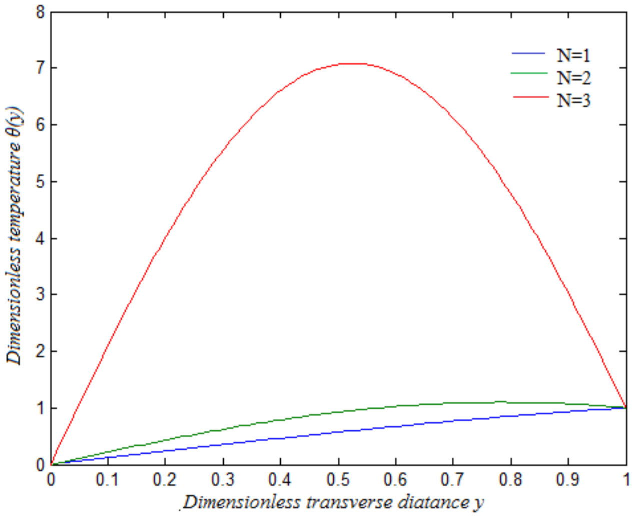
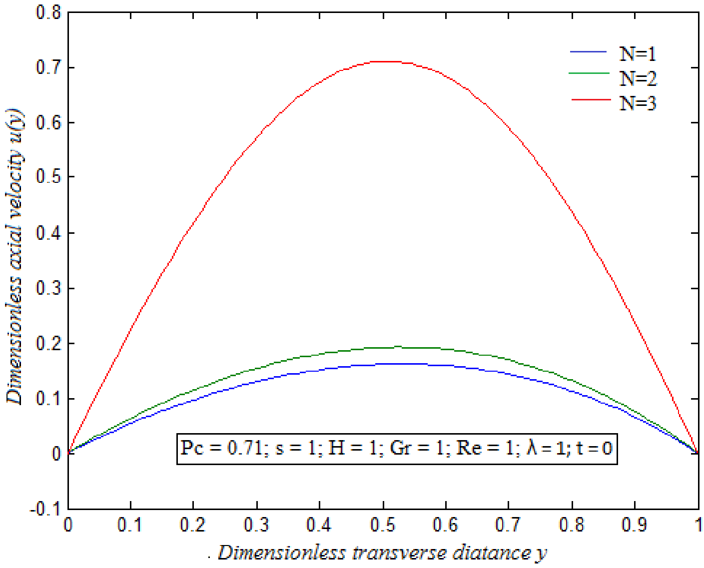
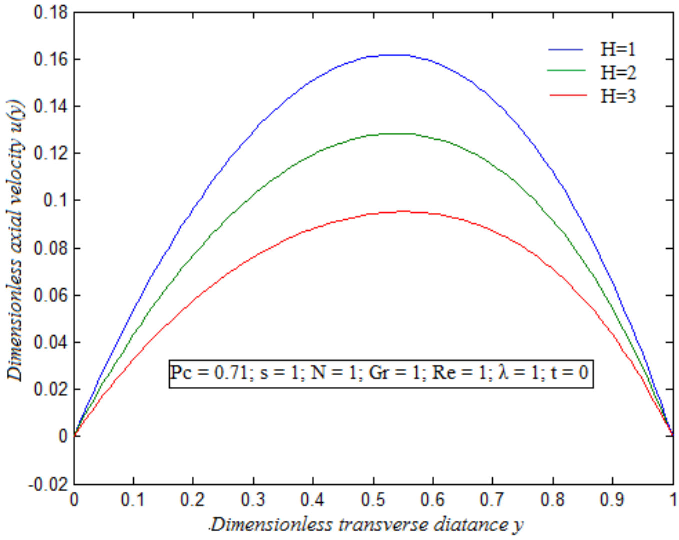


Figure 2: Dimensionless temperature $\theta(y)$ versus dimensionless transverse distance y .The curves are plotted using the eqn.(16) for various values of the dimensionless radiation parameter N .

(a)



(b)



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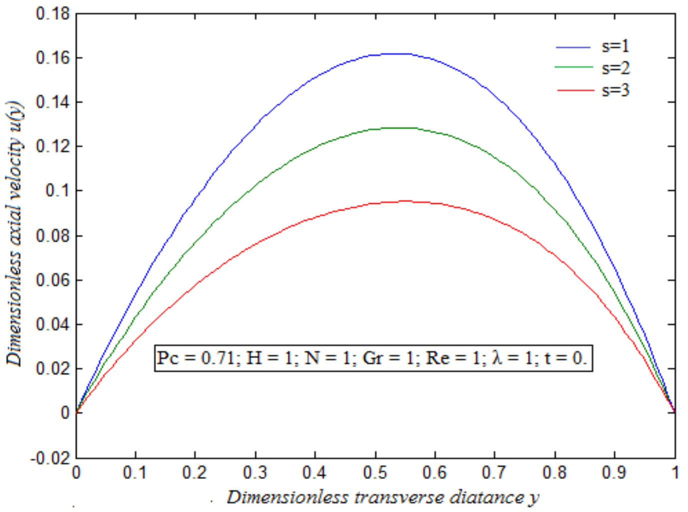
c)



(d)

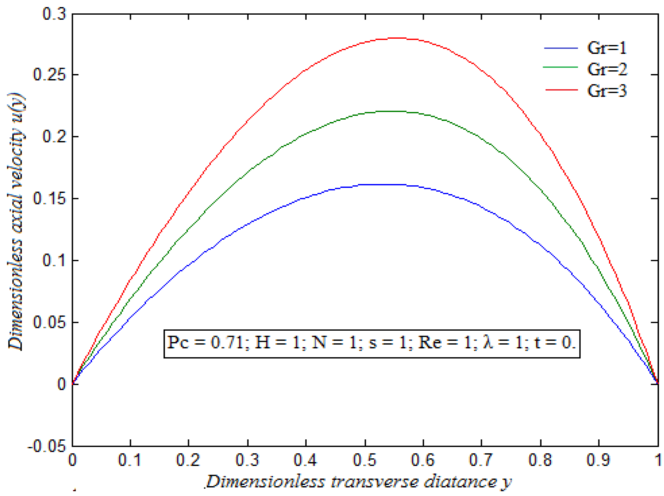
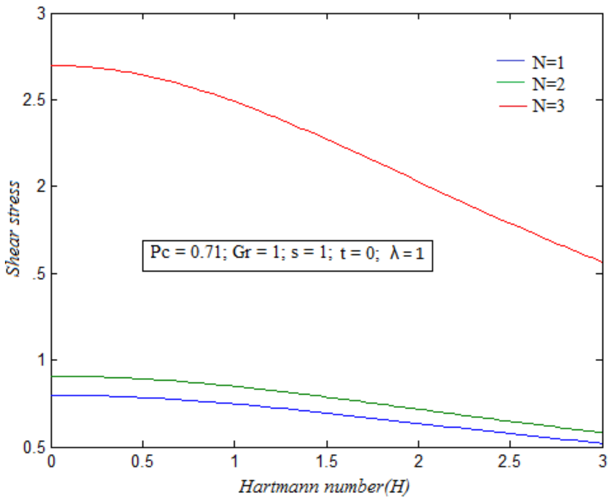


Figure 3: Dimensionless axial velocity *u(y)* versus dimensionless transverse distance y. The curves are plotted using the eqn. (13) for various values of the dimensionless parameter H, N, s, Gr and in some fixed values of the other dimensionless parameters, when

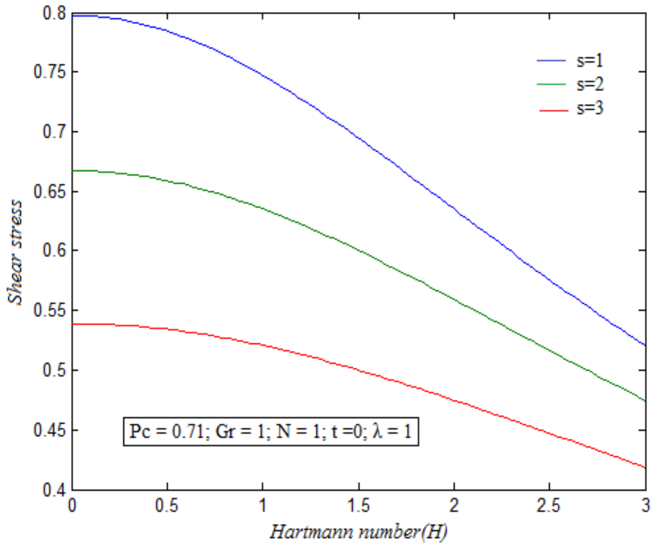
$(a) Pc = 0.71, H = 1, s = 1, Gr = 1, \lambda = 1, \mathrm{Re} = 1;$ $(b)\ Pc = 0.71, s = 1, Gr = 1, \mathrm{Re} = 1, \lambda = 1;$

$(c) Pc = 0.71, H = 1, Gr = 1, \mathrm{Re} = 1, \lambda = 1;$ $(d) Pc = 0.71, s = 1, \mathrm{Re} = 1, H = 1, \lambda = 1$

(a)



(b)



(c)

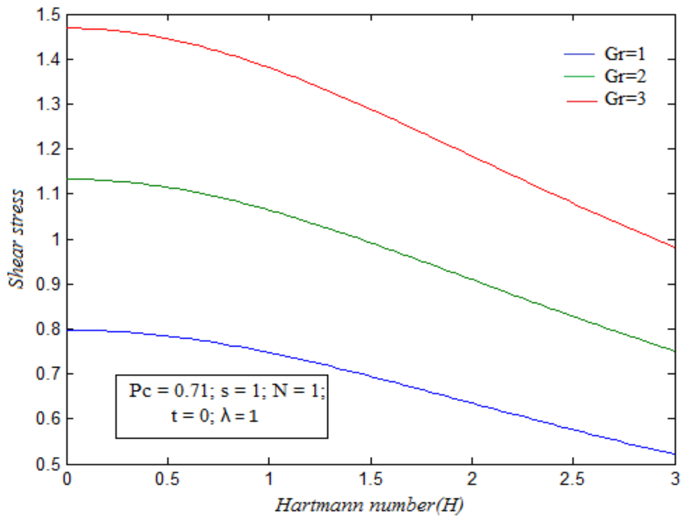


Figure 4: Shear stress at the wall of the channel versus Hartmann number H. The curves are plotted using the eqn.(17) for various values of the dimensionless parameter N, s, Gr and in some fixed values of the other dimensionless parameters, when $(a) Pc = 0.71, s = 1, Gr = 1, \lambda = 1;\ (b) Pc = 0.71, Gr = 1, N = 1, \lambda = 1;\ (c) Pc = 0.71, s = 1, \lambda = 1, N = 1.$

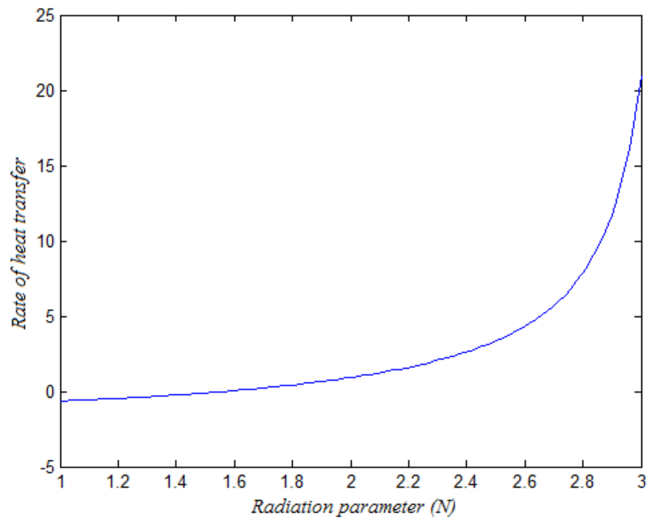


Figure 5: The rate of heat transverse versus dimensionless radiation parameter N using the eqn.(18)

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5. Conclusion

This paper investigates the combined effect of transverse magnetic field and radiativeof heat transfer across the channel with porous medium. We depict the analytical solution for the velocity and temperature profiles are obtained and it is used to find the shear stress and rate of heat transfer at the channel wall. We conclude from the plotted velocity profile as fluid velocity increases while increasing Grashoff number Gr, and Radiation number N and it is decreases with increasing Hartmann number H and also Porous medium shape factor s .

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Appendix-A: Nomenclature

Symbol	Meaning
B_o	Electromagnetic induction
c_p	Specific heat at constant pressure
Da	Darcy number
Gr	Grashoff number
H	Hartmann number
k	Thermal conductivity
K	The porous medium permeability
N	Radiation parameter
Pe	Peclet number
q	Radiative heat flux
Re	Flow Renolds number
s	Porous medium shape factor
t	Time variable
T	Fluid temperature
T_o	Temperature at $y = 0$
T_w	Temperature at $y = a$
u	The axial velocity
x	Axial distance
y	Transverse distance
β	Co-efficient of volume expansion due to temperature
λ	A constant
μ_e	Magnetic permeability
v	Kinetic viscosity coefficient
ρ	Fluid density
σ_e	Conductivity of the fluid
θ	Dimensionless temperature