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A NOTE ON NEIGHBOURHOOD CONTRACTION IN GRAPHS

M. Bhuvaneshwari*, Selvam Avadayappan & M. Pavithra Devi*****

Research Department of Mathematics, Virudhunagar Hindu Nadars' Senthikumara Nadar College, Virudhunagar, Tamilnadu

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Abstract:

Let G be a graph and let v be any vertex of G. The open neighbourhood of a vertex $v \in$ V is defined as $N(v)$ = $\{u \in V / uv \in E\}$, that is, the set of vertices adjacent to v. Then the neighbourhood contracted graph G_v of G, with respect to the vertex v is the graph with the vertex set $V - N(v)$, where two vertices u, w are adjacent in G_v if either $w = v$ and u is adjacent to any vertex of $N(v)$ in G or $u, w \notin N[v]$ and u, w are adjacent in G. In this paper, we prove some results on neighbourhood contraction of some special families of graphs. Also we construct n non isomorphic graphs with isomorphic neighbourhood contracted graph.

Key Words: Neighbourhood Contraction, Splitting Graph & Cosplitting Graph.

1. Introduction:

The graphs considered in this paper are finite, undirected and connected. Unless or otherwise stated, we consider only simple graphs. For notations and basic definitions, we refer [2]. Let $G(V, E)$ be a graph. Order of G is denoted by $n(G)$. A full vertex v is a vertex which is adjacent to every other vertices in G. That is, v is a full vertex, if $d(v) = n(G) - 1$. Distance between any two vertices u and v in a graph G is the length of a shortest path between them. It is denoted by $d(u, v)$. The neighbourhood of a vertex v $\in$ V is defined as $N(v) = \{u \in V / uv \in E\}$, that is, the set of vertices adjacent to v is called the neighbourhood of a vertex v. Or, more precisely, $N(v) = \{x \in V / d(x, v) = 1\}$ in G. In general, $N_k(v)$ is defined as $N_k(v) = \{u \in V(G) / d(u, v) = k\}$, $k = 1,2,3\ldots$. For a subset V_1 of V, the induced subgraph induced by V_1 is denoted by $G[V_1]$. A *clique* C is a subset of vertices such that $G[C]$ is complete. The *clique number* $\omega(G)$ of a graph G is the number of vertices in a maximum clique in G. The *corona* $G_1 \circ G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 which has p_1 vertices and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2. The concept of neighbourhood contraction was introduced by S.S. Kamath and Prameela Kolake[3]. The *Neighbourhood contracted graph* G_v of G, with respect to the vertex v is the graph with the vertex set $V - N(v)$, where two vertices $u, w \in V - N(v)$ are adjacent in G_v such that one of the following conditions holds:

1. $w = v$ and u is adjacent to any vertex of $N(v)$ in G.
2. $u, w \notin N[v]$ and u, w are adjacent in G.

For example, the graph G_{v_1} of G is shown in Figure 1.

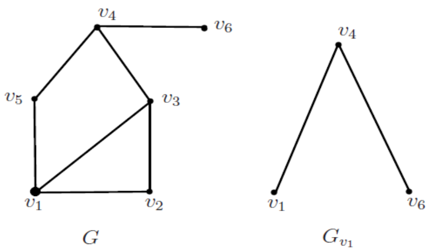

Figure 1

The concept of splitting graph was introduced by Sampath Kumar and Walikar[4]. The graph $S(G)$ obtained from G by adding a new vertex w for every vertex $v \in V$ and joining w to all vertices adjacent to v in G is called the *splitting graph* of G. For example, a graph G and its splitting graph $S(G)$ are shown in Figure 2.

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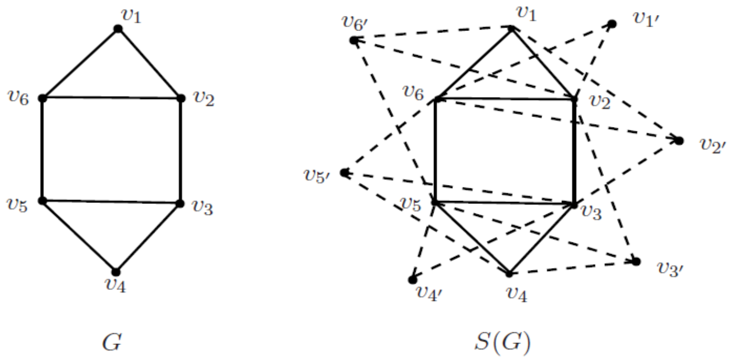

Figure 2

When we contract a vertex v in a splitting graph $S(G)$, the resultant graph is denoted by $[S(G)]_v$. Note that $S(G_v)$ denotes the splitting graph of the contracted graph G_v of G. For example, the graph G_{v_1}, $S(G_{v_1})$ and $[S(G)]_{v_1}$ for the graph in Figure 2 are shown in Figure 3.

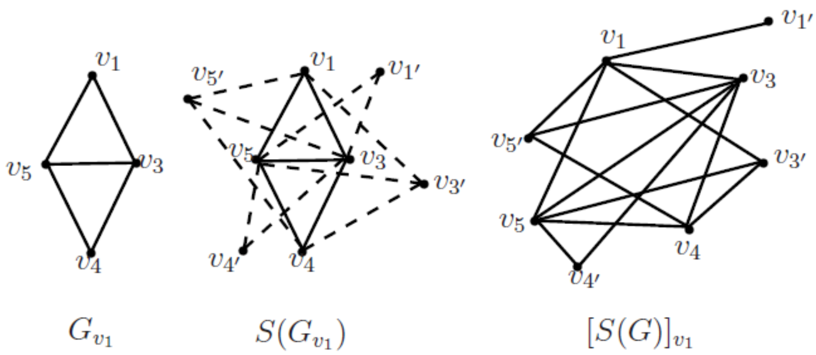

Figure 3

The concept of cosplitting graph $CS(G)$ was introduced by Selvam Avadayappan and M. Bhuvaneshwari[1]. The *cosplitting graph* $CS(G)$ obtained from G, by adding a new vertex w for each vertex $v \in V$ and joining w to those vertices of G which are not adjacent to v in G. For example, a graph G and its cosplitting graph $CS(G)$ are shown in Figure 4.

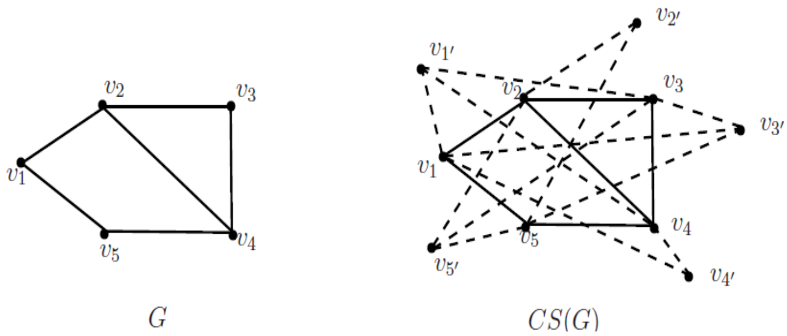

Figure 4

In a similar way, $[CS(G)]_v$ denotes the neighbourhood contracted graph with respect to v in a cosplitting graph $CS(G)$. Note that $CS(G_v)$ denotes the cosplitting graph of the contracted graph G_v of G. For example, the graph G_{v_1}, $CS(G_{v_1})$ and $[CS(G)]_{v_1}$ for the Figure 4 are shown in Figure 5.

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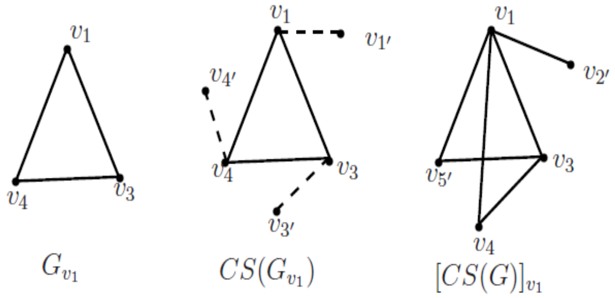

Figure 5

In this paper, we discuss about the relation between neighbourhood contraction in $S(G)$ and $CS(G)$ with the splitting and cosplitting graphs of the contracted graphs respectively. Also, we construct n non isomorphic graphs with isomorphic neighbourhood contracted graphs.

2. Main Results:

Proposition 2.1: *For any connected graph G,* $[S(G)]_{v_1} \ncong [S(G)]_{v_1'}$.

Proof: Let G be a graph and $S(G)$ be its splitting graph. Let $V(G)$ = $\{v_1, v_2, ..., v_n\}$ and $V(S(G))$ = $\{v_1, v_2, ..., v_n, v_1', v_2', ..., v_n'\}$ be the vertex set of G and $S(G)$ respectively. Let v_i' be the corresponding newly added vertex to v_i for all $1 \le i \le n$ in $S(G)$. $N(v_1')$ = $\{v_i / d(v_{1'}, v_i) = 1\}$. But $N(v_1)$ = $\{v_i, v_i' \ / d(v_1, v_i) = 1\}$. Then, $N(v_1)$ = $N(v_1') \cup \{v_i' / d(v_1, v_i') = 1\}$. Therefore, $N(v_1') \subsetneq N(v_1)$. Hence, $[S(G)]_{v_1'}$ has more vertices compared to $[S(G)]_{v_1}$. Therefore, $[S(G)]_{v_1} \ncong [S(G)]_{v_1'}$.

Figure 6 shows the illustration of the above proposition.

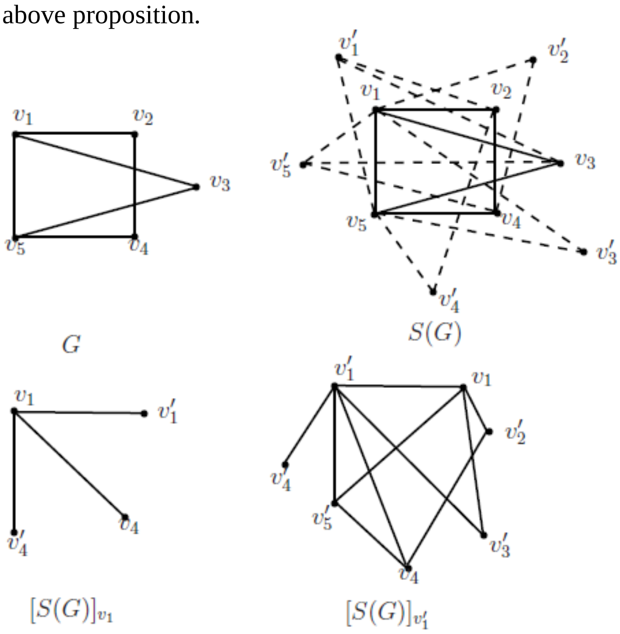

Figure 6

Theorem 2.2: *Let* G *be a graph and* $CS(G)$ *be its cosplitting graph. If* v *is a full vertex of G, then* $G \subseteq [CS(G)]_{v'}$.

Proof: Let G be a graph and v be a full vertex in G. Let $CS(G)$ be its cosplitting graph and v' be the newly added vertex in $CS(G)$ corresponding to v in G. Since v is a full vertex in G, by definition v' is a pendant vertex in $CS(G)$. Also, $N_{CS(G)}(v)$ = $N_G(v) \cup \{v'\}$. Now in $CS(G)$, $N_2(v')$ = $N_{CS(G)}(v) \setminus \{v'\}$ = $N_G(v) \setminus v'$. Hence $[CS(G)]_{v'} \subseteq (CS(G)) \setminus \{v'\}$. Since $G \subseteq CS(G)$, we get $G \subseteq [CS(G)]_{v'}$.

Figure 7 shows the illustration of this theorem.

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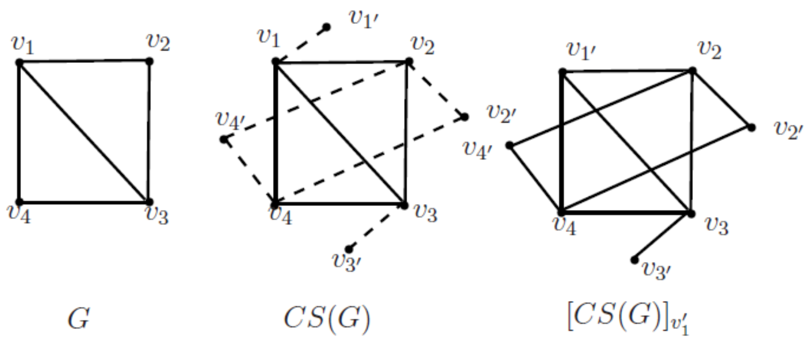

Figure 7

Theorem 2.3: *For any graph G, there exist* n *graphs,* $n \geq 2$, $G_1, G_2, \ldots, G_n$ *such that* G, $G_i \ncong G_j$ *for* $i \neq j$ *and* $(G_i)_v \cong (G_j)_v \cong G \vee K_1$ *and* $< N(v) >$ *is independent in each* G_i.

Proof: Let G be any graph. Construct a graph $H_i = G \circ K_i^{\,c}$, $1 \leq i \leq n.$ It is obvious that $H_i \ncong H_j$, $i \neq j, 1 \leq i, j \leq n$. Now construct G_i from H_i such that $V(G_i) = V(H_i) \cup \{v\}$. $E(G_i) = E(H_i) \cup \{uv / d(u) = 1 \text{ in } H_i\}$. Since $H_i \ncong H_j$, it can be easily verified that $G_i \ncong G_j$, forall $i \neq j$. It is easy to note that $< N(v) >$ is independent in each G_i. And in G_i, the neighbour of v are all pendant vertices in H_i. Since $H_i \cong G \circ K_i^{\,c}$, no vertex of G can be a pendant vertex in H_i. So, the vertices of G remain unaltered in $(G_i)_v$. Now in $(G_i)_v$, all pendant vertices of H_i are deleted and neighbours of all pendant vertices of H_i are made adjacent to v in $(G_i)_v$. Every vertex of G is adjacent to a pendant vertex in H_i. Therfore, $(G_i)_v \cong G \vee K_1$, forall $1 \leq i \leq n$.

Figure 8 illustrates the above theorem, for the case n =2.

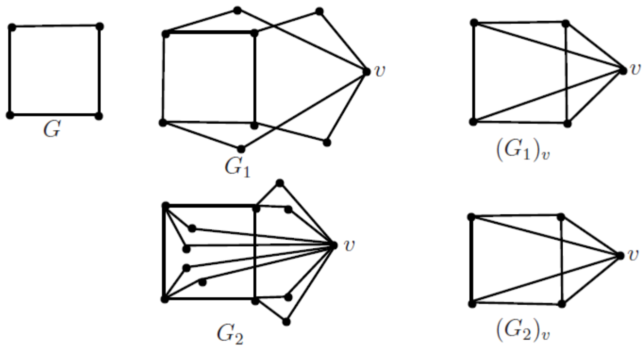

Figure 8

Note that the above constructed graph is not unique with this property. Next theorem shows another construction for n non isomorphic graphs G_i such that $(G_i)_v \cong G \vee K_1$, forall $1 \leq i \leq n$ and $< N(v) >$ is complete.

Theorem 2.4: *For any given graph G and any positive integer* n, *there exists* n *graphs* $n \geq 2$, $G_1, G_2, \ldots, G_n$ *such that* $G_i \ncong G_j$, *for all* $i \neq j$ *and* $1 \leq i, j \leq n$ *and* $(G_i)_v \cong (G_j)_v \cong G \vee K_1$, *forall* $1 \leq i \leq n$ *and* $\omega(G_i) \leq \omega(G_j)$, *for all* $i \leq j, 1 \leq i, j \leq n$.

Proof: Let G be any graph. Construct a graph $H_i = G \vee K_i, 1 \leq i \leq n$ with $\{u_1, u_2, \ldots, u_i\}$ be the vertices of K_i. It is obvious that $H_i \ncong H_j$, for every $i \neq j$, $1 \leq i, j \leq n$. Now construct G_i from H_i such that $V(G_i) = V(H_i) \cup \{v\}$. $E(G_i) = E(H_i) \cup \{u_j v / 1 \leq j \leq i\}$ where $1 \leq i \leq n$. Since $H_i \ncong H_j$, it can be easily verified that $G_i \ncong G_j$, for all $i \neq j.$ And in G_i, $< N(v) > \cong K_i$. Since $H_i = G \vee K_i$, no vertex of G can be a pendant vertex in H_i. So, vertices of G remain unaltered in $(G_i)_v$. In fact, $(G_i)_v = G \vee K_1$. Now note that $\omega(G_i) = \omega(G) + i$. Hence we get $\omega(G_i) \leq \omega(G_j)$, for all $i \leq j, 1 \leq i, j \leq n$.

Figure 9 shows the illustration of this theorem.

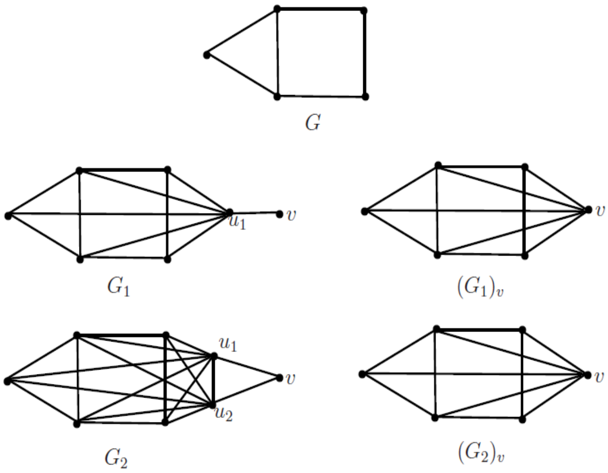

Figure 9

Also, the theorem can be extended to a method of constructing n non-isomorphic graphs which on contraction yield G itself.

Theorem 2.5: *For any given graph G, and any positive integer* n, *there exist n graphs* $n \geq 2$, $G_1, G_2, \ldots, G_n$ *such that* $G_i \ncong G_j$, *forall* $i \neq j$ *and* $1 \leq i, j \leq n$ *and* $(G_i)_v \cong (G_j)_v \cong G$.

Proof: Let G be a connected graph. Then there exists a vertex u in G such that $uv \in E(G)$. Now, construct a multigraph H_i from G by replacing uv by i multiedges, $1 \leq i \leq n$. Now, construct a simple graph G_i from H_i, such that $V(G_i) = V(H_i) \cup \{w_1, w_2, \ldots, w_{d(v)+(i-1)}\}$ and $E(G_i) = E(H_i) \cup \{vw_i, w_i u / u \in N(v)\} \setminus \{vu / u \in N(v)\}$. Also, $N_{G_i}(v) = \{w_j / 1 \leq j \leq d(v) + (i-1)\}$ Hence in G_i, $N_2(v) = N_G(v)$. Therefore, $(G_i)_v \cong G$, for all $1 \leq i \leq n$. Hence the proof. Figure 10 shows the illustration of this theorem.

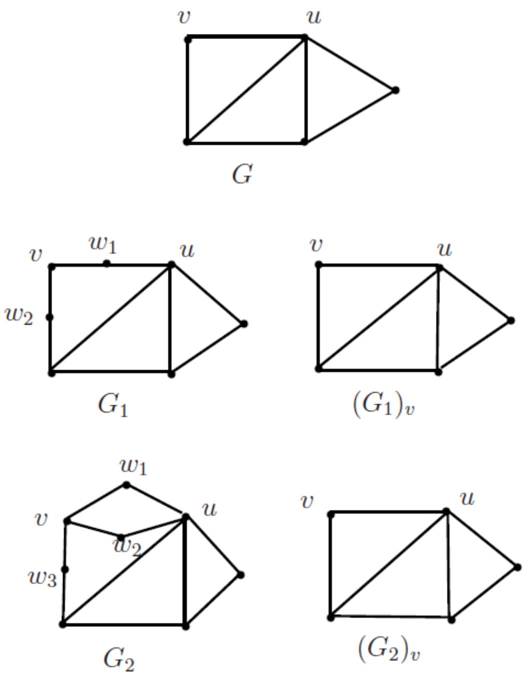

Figure 10

Theorem 2.6: *For any two given graphs* G *and* H, *there exist* n *non isomorphic graphs* $G_1, G_2, \ldots, G_n$ *such that* $G_i \ncong G_j$, *forall* $i \neq j$ *and* $1 \leq i, j \leq n$ *and* $(G_i)_v \cong G \vee K_1$, *and* $< N(v) >$ = iH, *for any* i, $1 \leq i \leq n$, *where* $n \in Z^+$.

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Proof: Let G and H be any graph. Construct a graph H_i = $G \circ (iH)$, $1 \le i \le n$. It is obvious that $H_i \ncong H_j$, for every $i \ne j$, $1 \le i, j \le n$. Now construct a graph G_i from H_i such that $V(G_i)$ = $V(H_i) \cup \{v\}$ and $E(G_i)$ = $E(H_i) \cup \{uv / u \in H\}$. Since $H_i \ncong H_j$, it can be easily verified that $G_i \ncong G_j$, forall $i \ne j$. It is easy to note that $< N(v) >$ = iH, $1 \le i \le n$. And no vertex of G is adjacent to v in G_i. So, the vertices of G remain unaltered in $(G_i)_v$. Now in $(G_i)_v$, the vertices of iH are deleted and the neighbours of iH are made adjacent to v in $(G_i)_v$. Every vertex of G is adjacent to a copy of H. Therefore, $(G_i)_v \cong G \vee K_1$.

Figure 11 illustrates the above theorem for the case n = 2.

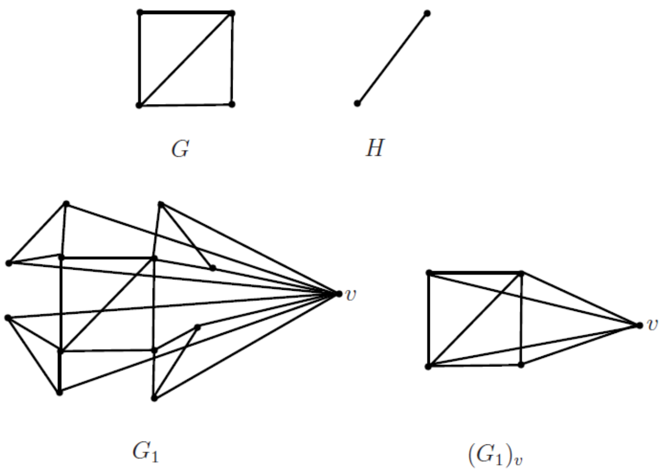

Figure 11

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