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PRIME CORDIAL GRAPHS

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Abstract:

Let $G=(V,E)$ be a graph. A prime cordial labeling of G with vertex set V is a bijection f from V to $\{1,2,...,|V|\}$ such that if each edge uv is assigned the label 1 when gcd $(f(u),f(v))=1$ and 0 otherwise, then the difference between the number of edges labeled with 1 and the number of edges labeled with 0 is at most 1. A graph which admits prime cordial labeling is called a prime cordial graph. In this paper, we prove that some corona graphs are prime cordial.

Key Words : Labeling, Cordial Labeling & Prime Cordial Labeling

1. Introduction:

The graphs considered in this paper are finite, simple, undirected and connected. For the notations and terminology, we refer [3]. Let C_n and P_n denote the cycle and path on n vertices respectively. Let $K_{1,n}$ and $B_{n,n}$ denote the star and bistar on n vertices respectively. The corona $G_1 \circ G_2$ of two graphs G_1 and G_2 is the graph G obtained by taking one copy of G_1 which has p_1 vertices and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. For example, the graph $C_6 \circ K_2$ is shown in Figure 1.

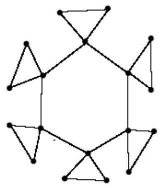

Figure 1

An edge $uv \in E(G)$ is subdivided if the edge uv is deleted and a new vertex x (called a subdivision vertex) is added together with the new edges ux and vx. A *subdivision graph* $S(G)$ of a graph G is obtained from G by subdividing all edges of G exactly once. For example, the subdivision graph $S(K_{1,6})$ is shown in Figure 2.

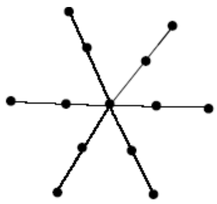

Figure 2

In 1960's Rosa [7] introduced the concept of graph labeling. A *graph labeling* is an assignment of numbers to the vertices or edges or both satisfying some constraints. A *vertex labeling* of a graph G is an assignment f of labels to the vertices of G that induces for each edge uv a label depending on the vertex label $f(u)$ and $f(v)$. The two best known labelings are graceful labeling and harmonious labeling [5]. Cordial labeling is a variation of both graceful and harmonious labelings [4]. Let $G=(V,E)$ be a graph. A mapping $f:V(G)\rightarrow\{0,1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the *label* of the vertex v of G under f. For an edge $e=uv$, the *induced edge labeling* $f^*:E(G)\rightarrow\{0,1\}$ is given by $f^*(e)=|f(u)-f(v)|$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0)$ and $e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*. The concept of cordial labeling was introduced by cahit [4]. A binary vertex labeling of a graph G is called a *cordial labeling* if $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$. A graph G is *cordial* if it admits cordial labeling. The concept of prime cordial labeling was introduced by Sundaram, Ponraj, and Somasundaram [8].

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A *prime cordial labeling* of a graph G with vertex set V is a bijection f from V to $\{1,2,...,|V|\}$ such that if each edge uv is assigned the label 1 when gcd $(f(u), f(v)) = 1$ and 0 otherwise, then the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1. A graph which admits prime cordial labeling is called a *prime cordial graph*. Many results on prime cordial labelings have been established in [1, 2, 6, 9, 10]. For example, a prime cordial labeling of a graph is shown in Figure 3.

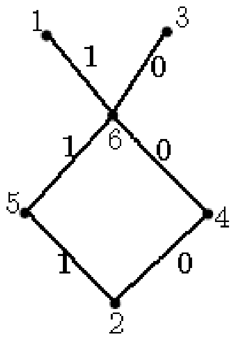


Figure 3

In this paper, we prove that the corona graphs $P_n \circ K_1$, $C_n \circ K_1$, $K_{1,n} \circ K_1$, $S(K_{1,n}) \circ K_1$, $B_{n,n} \circ K_1$ and some other graphs admit prime cordial labeling.

2. Main Results:

Theorem 2.1: *The graph* $P_n \circ K_1$ *is prime cordial.*

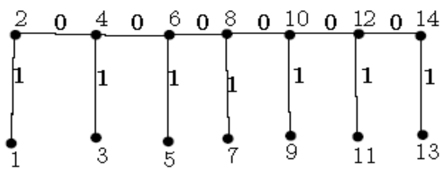
Proof: Let $V(P_n \circ K_1)$ = $\{u_i, v_i ;\ 1 \le i \le n\}$ be the vertex set and Let $E(P_n \circ K_1)$ = $\{u_i v_i ;\ 1 \le i \le n\} \bigcup \{u_i u_{i+1} ;\ 1 \le i \le n-1\}$ be the edge set of $P_n \circ K_1$.

Define $f : V \to \{1,2,...,2n\}$ by $f(u_i) = 2i \ ;\ 1 \le i \le n$ and $f(v_i) = 2i-1 \ ;\ 1 \le i \le n$.

Then the induced edge labelings are $f^*(u_i u_{i+1}) = 0 \ ;\ 1 \le i \le n-1$ and $f^*(u_i v_i) = 1 \ ;\ 1 \le i \le n$.

Thus, $e_f(0) = n-1$, $e_f(1) = n$.

It follows that f is a prime cordial labeling. Thus the graph $P_n \circ K_1$ is prime cordial. The case when $n = 7$ is illustrated in Figure 4.



A Prime Cordial Labeling of $P_7 \circ K_1$

Figure 4

Theorem 2.2: *The graph* $C_n \circ K_1$ *admits prime cordial labeling.*

Proof: Let $V(C_n \circ K_1) = \{u_i, v_i ;\ 1 \le i \le n\}$ be the vertex set and let $E(C_n \circ K_1)$ = $\{u_n u_1, u_i v_i ;\ 1 \le i \le n\} \bigcup \{u_i u_{i+1} ;\ 1 \le i \le n-1\}$ be the edge set of $C_n \circ K_1$.

Define $f : V \to \{1,2,...,2n\}$ by $f(u_i) = 2i \ ;\ 1 \le i \le n$ and $f(v_i) = 2i-1 \ ;\ 1 \le i \le n$.

Then the induced edge labelings are $f^*(u_i u_{i+1}) = 0 \ ;\ 1 \le i \le n-1$, $f^*(u_n u_1) = 0$ and $f^*(u_i v_i) = 1 \ ;\ 1 \le i \le n$.

Thus, $e_f(0) = e_f(1) = n$.

It follows that f is a prime cordial labeling. Thus the graph $C_n \circ K_1$ is prime cordial. The case when $n = 6$ is illustrated in Figure 5.

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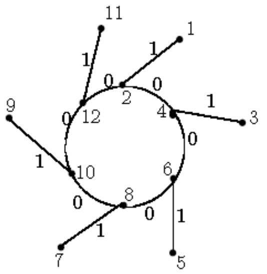


A Prime Cordial Labeling of $C_6 \circ K_1$

Figure 5

Theorem 2.3: *The graph* $K_{1,n} \circ K_1$ *is prime cordial.*

Proof: Let $V(K_{1,n} \circ K_1) = \{v, w, u_i, v_i \; ; 1 \leq i \leq n\}$ be the vertex set and let $E(K_{1,n} \circ K_1) = \{vv_i, v_iu_i, vw \; ; 1 \leq i \leq n\}$ be the edge set of $K_{1,n} \circ K_1$.

Define $f : V \to \{1,2,\ldots,2n+2\}$ by $f(v) = 2$, $f(w) = 1$, $f(u_i) = 2i+1 \; ; 1 \leq i \leq n$ and $f(v_i) = 2i+2 \; ; 1 \leq i \leq n$.

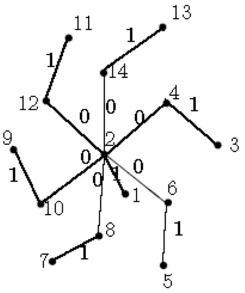
Then the induced edge labelings are $f^*(vv_i) = 0 \; ; 1 \leq i \leq n$

$f^*(v_iu_i) = 1 \; ; 1 \leq i \leq n$ and $f^*(vw) = 1$.

Thus, $e_f(0) = n$, $e_f(1) = n+1$.

It follows that f is a prime cordial labeling. Thus the graph $K_{1,n} \circ K_1$ is prime cordial.

The case when $n = 6$ is illustrated in Figure 6.



A Prime Cordial Labeling of $K_{1,6} \circ K_1$

Figure 6

Theorem 2.4: *The graph* $S(K_{1,n}) \circ K_1$ *is prime cordial.*

Proof: Let $V(S(K_{1,n}) \circ K_1) = \{v, w, u_i, v_i, x_i, y_i \; ; 1 \leq i \leq n\}$ be the vertex set and let $E(S(K_{1,n}) \circ K_1) = \{vv_i, v_iu_i, vw, v_ix_i, u_iy_i \; ; 1 \leq i \leq n\}$ be the edge set of $S(K_{1,n}) \circ K_1$.

Define $f : V \to \{1,2,\ldots,4n+2\}$ by $f(v) = 2$, $f(w) = 1$, $f(u_i) = 4i+2 \; ; 1 \leq i \leq n$

$f(v_i) = 4i \; ; 1 \leq i \leq n$

$f(x_i) = 4i-1 \; ; 1 \leq i \leq n$ and $f(y_i) = 4i+1 \; ; 1 \leq i \leq n$.

Then the induced edge labelings are

$$f^*(vw) = 1$$
$$f^*(vv_i) = 0 \; ; 1 \leq i \leq n$$
$$f^*(v_iu_i) = 0 \; ; 1 \leq i \leq n$$
$$f^*(v_ix_i) = 1 \; ; 1 \leq i \leq n$$
$$f^*(u_iy_i) = 1 \; ; 1 \leq i \leq n.$$

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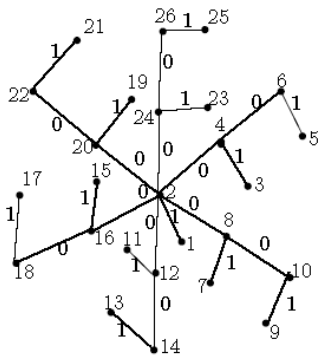
Thus, $e_f(0) = 2n$, $e_f(1) = 2n+1$. It follows that f is a prime cordial labeling. Thus the graph $S(K_{1,n}) \circ K_1$ is prime cordial. For example, a prime cordial labeling of $S(K_{1,6}) \circ K_1$ is shown in Figure 7.



A Prime Cordial Labeling of $S(K_{1,6}) \circ K_1$

Figure 7

Theorem 2.5: *The graph* $B_{n,n} \circ K_1$ *is prime cordial.*

Proof: Let $V(B_{n,n} \circ K_1) = \{u, v, u_i, v_i, x, y ; 1 \le i \le 2n\}$ be the vertex set and let $E(B_{n,n} \circ K_1) = \{ux, vy, uu_{2i-1}, u_{2i-1}u_{2i}, vv_{2i-1}, v_{2i-1}v_{2i}, uv ; 1 \le i \le n\}$ be the edge set of $B_{n,n} \circ K_1$.

Define $f : V \to \{1,2,\ldots,4n+4\}$ by $f(x)=3$, $f(y)=2$, $f(u)=1$, $f(v)=4$,

$f(u_{2i-1}) = 4i+1 ; 1 \le i \le n$

$f(u_{2i}) = 4i+3 ; 1 \le i \le n$

$f(v_{2i-1}) = 4i+2 ; 1 \le i \le n$ and $f(v_{2i}) = 4i+4 ; 1 \le i \le n$.

Then the induced edge labelings are

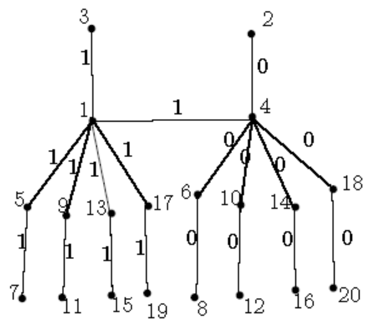
$f^*(ux) = 1$, $f^*(uv) = 1$, $f^*(vy) = 0$, $f^*(uu_{2i-1}) = 1 ; 1 \le i \le n$

$f^*(u_{2i-1}u_{2i}) = 1 ; 1 \le i \le n$

$f^*(vv_{2i-1}) = 0 ; 1 \le i \le n$ and $f^*(v_{2i-1}v_{2i}) = 0 ; 1 \le i \le n$.

Thus, $e_f(0) = 2n+1$, $e_f(1) = 2n+2$.

It follows that f is a prime cordial labeling. Thus the graph $B_{n,n} \circ K_1$ is prime cordial. For example, a prime cordial labeling of $B_{4,4} \circ K_1$ is shown in Figure 8.



A Prime Cordial Labeling of $B_{4,4} \circ K_1$

Figure 8

Let G be any graph. Then $G * K_{1,n}$ is the graph obtained by identifying a vertex of G with central vertex of $K_{1,n}$. For example, the graph $C_5 * K_{1,7}$ is shown in Figure 9.

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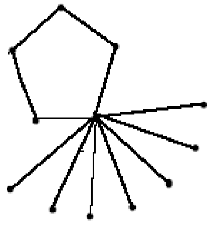


The graph $C_5 * K_{1,7}$

Figure 9

Theorem 2.6: *The graph* $C_n * K_{1,n+m}$ *is prime cordial.*

Proof Let $V(C_n * K_{1,n+m}) = \{ u_i , v_i , v_j ; 1 \le i \le n ; n+1 \le j \le n+m \}$ be the vertex set and let $E(C_n * K_{1,n+m}) = \{ u_i u_{i+1}, u_1 v_i , u_1 v_j ; 1 \le i \le n ; n+1 \le j \le n+m \}$ be the edge set of $C_n * K_{1,n+m}$.

Define $f : V \to \{1,2,...,3n\}$ by $f(u_i) = 2i ; 1 \le i \le n$

$f(v_i) = 2i-1 ; 1 \le i \le n$ and $f(v_j) = n+j ; n+1 \le j \le n+m$.

Then the induced edge labelings are $f^*(u_i u_{i+1}) = 0 ; 1 \le i \le n-1$, $f^*(u_n u_1) = 0$

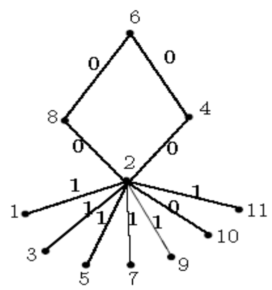
$f^*(u_1 v_i) = 1 ; 1 \le i \le n$

$f^*(u_1 v_{n+2i}) = 0 ; n+1 \le j \le n+m$ and $f^*(u_1 v_{n+2i-1}) = 1 ; n+1 \le j \le n+m$.

We note that $e_f(0) = e_f(1) = n+2$ if $m \equiv 0 (mod 2)$

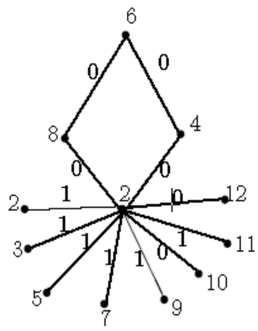
$e_f(1) = n+3$, $e_f(0) = n+2$ if $m \equiv 1 (mod 2)$

It follows that f is a prime cordial labeling. Thus the graph $C_n * K_{1,n+m}$ is prime cordial. The cases when $m = 3$ and $m = 4$ are illustrated in Figure 10 and 11 respectively



A Prime Cordial Labeling of $C_4 * K_{1,4+3}$

Figure 10



A Prime Cordial Labeling of $C_4 * K_{1,4+4}$

Figure 11

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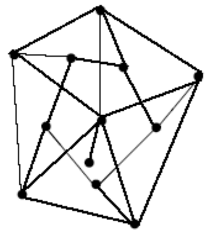
Let J_n denote the graph with vertex set $V(J_n) = \{v, w, v_i, u_i \; ; 1 \le i \le n\}$ and the edge set $E(J_n) = \{u_n u_1, v_n v_1, vw, vu_i, u_i v_i \; ; 1 \le i \le n\} \bigcup \{u_i u_{i+1}, v_i v_{i+1} \; ; 1 \le i \le n-1\}$. For example, the graph J_5 is shown in Figure 12.



The graph J_5

Figure 12

Theorem 2.7: *The graph* J_n *is prime cordial.*

Proof: Let $V(J_n) = \{v, w, v_i, u_i \; ; 1 \le i \le n\}$ be the vertex set and let $E(J_n) = \{u_n u_1, v_n v_1, vw, vu_i, u_i v_i \; ; 1 \le i \le n\} \bigcup \{u_i u_{i+1}, v_i v_{i+1} \; ; 1 \le i \le n-1\}$ be the edge set of J_n.

Define $f : V \to \{1, 2, \ldots, 2n+2\}$ by $f(v) = 2$, $f(w) = 3$, $f(v_1) = 1$,

$f(u_i) = 2i + 2 \; ; 1 \le i \le n$ and $f(v_i) = 2i + 1 \; ; 1 \le i \le n$.

Then the induced edge labelings are $f^*(vw) = 1$,

$f^*(vu_i) = 0 \; ; 1 \le i \le n$

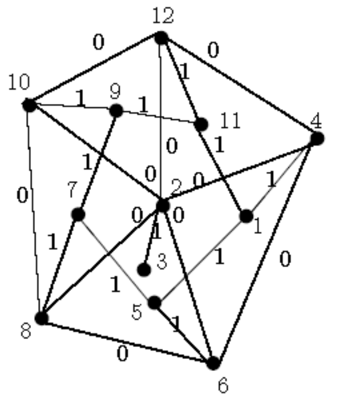
$f^*(u_i u_{i+1}) = 0 \; ; 1 \le i \le n-1; \; f^*(u_n u_1) = 0$

$f^*(u_i v_i) = 1 \; ; 1 \le i \le n$

$f^*(v_i v_{i+1}) = 1 \; ; 1 \le i \le n-1$; and $f^*(v_n v_1) = 1$.

Thus, $e_f(0) = 2n$, $e_f(1) = 2n+1$.

It follows that f is a prime cordial labeling. Thus the graph J_n is prime cordial. For example, a prime cordial labeling of J_5 is shown in Figure 13.



A Prime Cordial Labeling of J_5

Figure 13

Let $V(J_n^2) = \{v, w, u_i, v_i, x_i, y_i \; ; 1 \le i \le n\}$ be the vertex set and let $E(J_n^2) = \{u_n u_1, v_n v_1, x_n x_1, y_n y_1, vw, vu_i, u_i v_i, x_i y_i, u_i x_i \; ; 1 \le i \le n\} \bigcup \{u_i u_{i+1}, v_i v_{i+1}, x_i x_{i+1}, y_i y_{i+1} \; ; 1 \le i \le n-1\}$ be the edge set of J_n^2. For example, the graph J_5^2 is shown in Figure 14.

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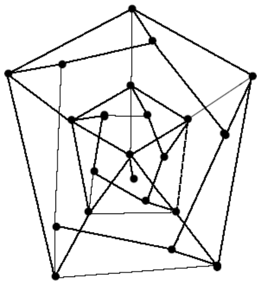


The graph J_5^2

Figure 14

Theorem 2.8: *The graph* J_n^2 *is prime cordial.*

Proof: Let $V(J_n^2) = \{v, w, u_i, v_i, x_i, y_i \; ; 1 \le i \le n\}$ be the vertex set and let $E(J_n^2) = \{u_n u_1, v_n v_1, x_n x_1, y_n y_1, vw, vu_i, u_i v_i, x_i y_i, u_i x_i \; ; 1 \le i \le n\} \bigcup \{u_i u_{i+1}, v_i v_{i+1}, x_i x_{i+1}, y_i y_{i+1} \; ; 1 \le i \le n-1\}$ be the edge set of J_n^2.

Define $f : V \to \{1,2,\dots,4n+2\}$ by $f(v) = 2,\; f(w) = 3,\; f(v_1) = 1,\; f(u_i) = 2i+2 \quad ; 1 \le i \le n$

$f(v_i) = 2i+1 \quad ; 2 \le i \le n$

$f(x_i) = 2n+2+2i \; ; 1 \le i \le n$

$f(u_i) = 2n+1+2i \; ; 1 \le i \le n$.

Then the induced edge labeling are $f^*(vw) = 1,\; f^*(vu_i) = 0 \quad ; 1 \le i \le n$

$f^*(u_i u_{i+1}) = 0 \; ; 1 \le i \le n \; ; f^*(u_n u_1) = 0$

$f^*(u_i v_i) = 1 \; ; 1 \le i \le n$

$f^*(v_i v_{i+1}) = 1 \; ; 1 \le i \le n-1 \; ; f^*(v_n v_1) = 1$

$f^*(x_i x_{i+1}) = 0 \; ; 1 \le i \le n-1 \; ; f^*(x_n x_1) = 0$

$f^*(u_i x_i) = 0 \; ; 1 \le i \le n$

$f^*(x_i y_i) = 1 \; ; 1 \le i \le n$

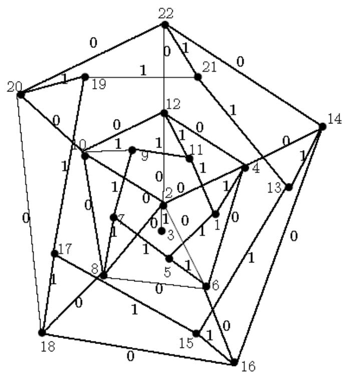
$f^*(y_i y_{i+1}) = 1 \; ; 1 \le i \le n-1$ and $f^*(y_n y_1) = 0$ if gcd $(2n+3, 4n+1) > 1$

$= 1$ otherwise.

if $f^*(y_n y_1) = 0$ then $e_f(0) = 4n+1,\; e_f(1) = 4n$

else $e_f(0) = 4n,\; e_f(1) = 4n+1$

It follows that f is a prime cordial labeling. Thus the graph J_n^2 is prime cordial. For example, a prime cordial labeling of J_5^2 is shown in Figure 15.



A Prime Cordial Labeling of J_5^2 (Figure 15)

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