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MORE RESULTS ON NON-ISOLATED RESOLVING NUMBER OF A GRAPH

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Abstract:

Let G be a connected graph. Let $W=\{w_1,w_2,...,w_k\}$ be a subset of V with an order imposed on it. For any $v\in V$, the vector $r(v|W)=(d(v,w_1),d(v,w_2),...,d(v,w_k))$ is called the metric representation of v with respect to W. If distinct vertices in V have distinct metric representation, then W is called a resolving set of G. The minimum cardinality of a resolving set of G is called the metric dimension of G and it is denoted by $dim(G)$. A resolving set W is called a non-isolated resolving set if the induced sub graph $\langle W\rangle$ has no isolated vertices. The minimum cardinality of a non-isolated resolving set of G is called the non-isolated resolving number of G and is denoted by $nr(G)$. In this paper, we determine the non-isolated resolving number for some standard graphs like double broom, the join of complete graphs and paths, etc. Further more, we discuss about the relationship of nr with other parameters.

Key Words : Resolving Set, Metric Dimension, Non-Isolated Resolving Set & Non-Isolated Resolving Number

1. Introduction:

Throughout this paper, we consider only finite, simple and undirected graphs. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The cardinality of the vertex set of a graph G is commonly denoted by n(G). For basic notations and terminology we refer [4]. The *distance* $d(u,v)$ between two vertices u and v is the length of a shortest path between them. For the graphs, $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ their *join* denoted by G_1+G_2 is the graph whose vertex set is $V_1\cup V_2$ and the edge set $E=E_1\cup E_2\cup\{uv:u\in V_1,v\in V_2\}$. For a subset S of V, let $\langle S\rangle$ denote the induced subgraph of G induced by S. A *clique* C is a subset of vertices such that $\langle C\rangle$ is complete. The *clique number* of a graph G denoted by $\omega(G)$, is the number of vertices in a maximum clique of G. An edge $uv\in E(G)$ is *subdivided* if the edge uv is deleted and a new vertex x (called a subdivision vertex) is added together with the new edges ux and vx. A subdivision graph $S_1(G)$ of a graph G is obtained from G by subdividing all edges of G exactly once. A *coloring* of a graph G is an assignment of colors to the vertices of G, one color to each vertex so that adjacent vertices are assigned different colors. A graph G is k*-colorable*, if there exists a coloring of G from a set of k colors. In other words, G is k-coloring of G. The minimum coloring positive integer k for which G is k-colorable is the *chromatic number* of G and is denoted by $\chi(G)$. The double broom $B(n,m,p)$ is a graph obtained by identifying the center vertex of a star $K_{1,m}$ at one pendant vertex of P_n and the center vertex of a star $K_{1,p}$ at the other pendant vertex of P_n. If $n=2$, then $B(2,m,p)$ is the bistar. Motivated by the problem of uniquely determining the locations of an intruder in a network, the concept of metric dimension of a graph was introduced by Slater in ([14] and [15]) and studied independently by Harary and Melter in [8]. Let $W=\{w_1,w_2,...,w_k\}$ be an ordered set of vertices of G and let v be a vertex of G. The *representation* $r(v|W)$ of v with respect to W is the k-tuple $(d(v,w_1),d(v,w_2),...,d(v,w_k))$. If distinct vertices of G have distinct representations with respect to W, then W is called a *resolving set* for G. A resolving set of minimum cardinality is called a *basis* for G and this cardinality is the *metric dimension* of G and it is denoted by $dim(G)$. For example, in the graph G shown in Figure 1.1, $W=\{v_1,v_5\}$ is a basis for G. Therefore, $dim(G)=2$.

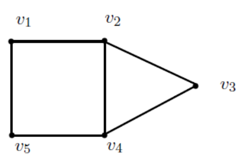

Figure 1.1

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Applications of resolving set arise in various areas including coin weighing problem [12], drug discovery [7] , robot navigation [10], network discovery and verification [2], connected joins in graphs [12] and strategies for master mind game [5]. For survey of results in metric dimension we refer to Chartrand and Zhang [5]. Several models of resolving set have been investigated by imposing conditions on the subgraph induced by a resolving set. Some of the well studied parameters of this type include connected resolving set [11] and independent resolving set [13]. A resolving set W of G is said to be an *independent resolving set* if no two vertices in W are adjacent. A resolving set W of G is said to be a *connected resolving set*, if the induced subgraph induced by W is a non-trivial connected subgraph of G. The minimum cardinality of a connected resolving set is the *connected resolving number* of G. It is denoted by $cr(G)$. In a similar line, a non-isolated resolving set was introduced in [9]. A resolving set W of G with at least two vertices is said to be a *non-isolated resolving set*, if the induced subgraph $\langle W \rangle$ induced by W has no isolated vertices. The minimum cardinality of a non-isolated resolving set in a graph G is the *non-isolated resolving number* of G and it is denoted by $nr(G)$. A non-isolated resolving set of cardinality $nr(G)$ is called an nr-set of G.

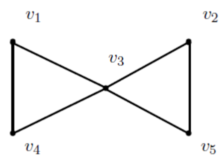

Figure 1.2

For example, consider the graph G given in Figure 1.2, $W = \{v_1, v_2\}$ is a basis for G and $W' = \{v_1, v_2, v_3\}$ is an nr-set. Hence, $dim(G) = 2$ and $nr(G) = 3$. Since, every non-trivial connected graph has no isolated vertices, $nr(G) \le cr(G)$. Also, it has been proved in [9] that, for any graph G, $nr(G) \le 2dim(G)$. In [9], nr-values of some families of graphs, cartesian product of some graphs and corona product of a graph G with $\overline{K_2}$ have been obtained. Further more, for any two positive integers k and n with $2 \le k \le n-1$, a graph G of order n with $nr(G) = k$ has been constructed. For more results on non-isolated resolving number one can refer [1]. In this paper, we determine the non-isolated resolving number for some standard graphs such as double broom, bistar and for the join of complete graphs and paths, etc. Further more, we discuss about the relationship of nr with the parameters $\chi(G)$ and $\Delta(G)$.

2. nr-Value of Some Graphs:

In this section, we find the nr-value for some graphs.

Theorem 2.1: *Let G be the double broom $B(n,m,p)$. Then $nr(G) = m + p$.*

Proof: Let $V(G) = \{w_1, w_2, ..., w_m; v_1, v_2, ..., v_n; u_1, u_2, ..., u_p\}$ and $E(G) = \{w_j v_1, v_i v_{i+1}, v_n u_k : 1 \le j \le m, 1 \le i \le n-1, 1 \le k \le p\}$ where G is the double broom $B(n,m,p)$. Take $W = \{w_1, w_2, ..., w_{m-1}; v_1, v_n; u_1, u_2, ..., u_{p-1}\}$. Then $|W| = m + p$. Now, $r(w_m | W) = (2,2,...,2,1,n,n+1,n+1,...,n+1)$ where 1 appears at the m^{th} place, $r(v_i | W) = (i,i,...,i,i-1,n-i,n-i+1,n-i+1,...,n-i+1)$ where $i-1$ appears at the m^{th} place, $2 \le i \le n-1$ and $r(u_p | W) = (n+1,n+1,...,n+1,n,1,2,2,...,2)$ where n appears at the m^{th} place. Therefore, W is a non-isolated resolving set for G. Hence, $nr(G) \le m + p$. Let W_1 be a non-isolated resolving set for G. For $i \ne j$ and $1 \le i, j \le m$, if $w_i, w_j \notin W_1$, then $r(w_i | W_1) = r(w_j | W)$, a contradiction. Therefore, there can be at least $m-1$ values of i, such that $w_i \in W_1$. This forces that $v_1 \in W_1$, since W_1 is a non-isolated resolving set for G. Similarly we can prove that, there can be at least $p-1$ value of k such that $u_k \in W_1$, $1 \le k \le p$. This again implies that $v_n \in W_1$. Hence $|W_1| \ge m + p$. That is, $nr(G) \ge m + p$. Thus $nr(G) = m + p$. Next, we evaluate the non-isolated resolving number of join of path and complete graph as follows. When $m = 1$ or 2, $nr(P_m + K_n) = nr(K_{m+n}) = m + n - 1$. For the remaining values of m, the next theorem gives the nr-value.

Theorem 2.2: *For positive integers $m \ge 3$, $n \ge 1$,*

$$nr(P_m + K_n) = \left\lceil \frac{2m}{5} \right\rceil + n - 1, \text{ if } m \equiv 0,2,4 (\text{mod } 5) \text{ and } nr(P_m + K_n) = \left\lfloor \frac{2m}{5} \right\rfloor + n - 1, \text{ if } m \equiv 1,3 (\text{mod } 5).$$

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Proof Let $G = P_m + K_n$ and $V(P_m) = \{v_1, v_2, ..., v_m\}$ and $V(K_n) = \{u_1, u_2, ..., u_n\}$.

If $m \equiv 0,2,3,4(mod\ 5)$, take $W = \{u_1, u_2, ..., u_{n-1}, v_i, v_m : 3 \le i \le m-1$ and $i \equiv 0,3(mod\ 5)\}$. Then for $m \equiv 0,2,4(mod\ 5)$, $|W| = \left\lceil \frac{2m}{5} \right\rceil + n - 1$ and for $m \equiv 3(mod\ 5)$, $|W| = \left\lfloor \frac{2m}{5} \right\rfloor + n - 1$. Now, $r(u_n | W) = (1,1,...,1)$, $r(v_1 | W) = (1,1,...,1,2,2,...,2)$ where 1 appears at the first $n-1$ places, $r(v_2 | W) = (1,1,...,1,2,2,...,2)$ where 1 appears at the first n places. If $k = 5r+1$, $r \ge 1$, then $r(v_k | W) = (1,1,...,1,2,2,...,2,1,2,2,...,2)$ where 1 appears at the first $n-1$ places and at the $\left(n + 2\left\lfloor \frac{k}{5} \right\rfloor - 1\right)^{th}$ place. If $k = 5r+2$, $r \ge 1$, then $r(v_k | W) = (1,1,...,1,2,2,...,2,1,2,2,...,2)$ where 1 appears at the first $n-1$ places and at the $\left(n + 2\left\lfloor \frac{k}{5} \right\rfloor\right)^{th}$ place. If $k = 5r+4$, $r \ge 1$ then $r(v_k | W) = (1,1,...,1,2,2,...,2,1,1,2,2,...,2)$ where 1 appears at the first $n-1$ places and at the $\left(n + 2\left\lfloor \frac{k}{5} \right\rfloor\right)^{th}$ and $\left(n + 2\left\lfloor \frac{k}{5} \right\rfloor + 1\right)^{th}$ places respectively. Therefore, W is a non-isolated resolving set for G. Hence, $nr(G) \le \left\lceil \frac{2m}{5} \right\rceil + n - 1$ if $m \equiv 0,2,4(mod\ 5)$ and $nr(G) \le \left\lfloor \frac{2m}{5} \right\rfloor + n - 1$ if $m \equiv 3(mod\ 5)$.

If $m \equiv 1(mod\ 5)$, take $W = \{u_1, u_2, ..., u_{n-1}, v_i : 3 \le i \le m$ and $i \equiv 0,3(mod\ 5)\}$, then $r(u_n | W) = (1,1,...,1)$, $r(v_1 | W) = (1,1,...,1,2,2,...,2)$ where 1 appears at the first $n-1$ places, $r(v_2 | W) = (1,1,...,1,2,2,...,2)$ where 1 appears at the first n places. If $k = 5r+1$, $r \ge 1$, then $r(v_k | W) = (1,1,...,1,2,2,...,2,1,2,2,...,2)$ where 1 appears at the first $n-1$ places and at the $\left(n + 2\left\lfloor \frac{k}{5} \right\rfloor - 1\right)^{th}$ place. If $k = 5r+2$, $r \ge 1$, then $r(v_k | W) = (1,1,...,1,2,2,...,2,1,2,2,...,2)$ where 1 appears at the first $n-1$ places and at the $\left(n + 2\left\lfloor \frac{k}{5} \right\rfloor\right)^{th}$ place. If $k = 5r+4$, $r \ge 1$ then $r(v_k | W) = (1,1,...,1,2,2,...,2,1,1,2,2,...,2)$ where 1 appears at the first $n-1$ places and at the $\left(n + 2\left\lfloor \frac{k}{5} \right\rfloor\right)^{th}$ and $\left(n + 2\left\lfloor \frac{k}{5} \right\rfloor + 1\right)^{th}$ places respectively. Therefore, W is a non-isolated resolving set for G. Hence, $nr(G) \le \left\lfloor \frac{2m}{5} \right\rfloor + n - 1$.

Let W_1 be a non-isolated resolving set for G. For $i \ne j$, $1 \le i, j \le m$, if both u_i and u_j are not in W_1. Then $r(u_i | W_1) = r(u_j | W_1) = (1,1,...,1)$, a contradiction. Therefore, there can be at least $m-1$ values of i, such that $u_i \in W_1$. Let $m = 5s+t$, $0 \le t \le 4$. We first consider the vertices v_1, v_2, v_3, v_4 and v_5. If v_1, v_2 and v_3 are not in W_1, then v_1 and v_2 have the same representation. Hence v_1 or v_2 or v_3 must belong to W_1. If $v_1 \in W_1$, then v_4 must belong to W_1, otherwise $r(v_3 | W_1) = r(v_4 | W_1)$. If $v_2 \in W_1$, then v_4 must belong to W_1, otherwise $r(v_1 | W_1) = r(v_3 | W_1)$. If $v_3 \in W_1$, then v_5 must belong to W_1, otherwise $r(v_2 | W_1) = r(v_4 | W_1)$. Therefore, without loss of generality, we can assume that v_3 and v_5 are in W_1. Similarly, for every 5 vertices from v_{5r+1} to $v_{5(r+1)}$, we choose v_{5r+3} and $v_{5(r+1)}$, $1 \le r \le s-1$. Hence v_{5s} is the last chosen vertex. If $t = 2$ or 3, then $v_{5s+t} \in W_1$. If $t = 4$, then $\{v_{5s+t-2}, v_{5s+t-1}\} \subseteq W_1$ or $\{v_{5s+t-2}, v_{5s+t}\} \subseteq W_1$ or $\{v_{5s+t-1}, v_{5s+t}\} \subseteq W_1$. Therefore, any non-isolated resolving set must contain $\left\lceil \frac{2m}{5} \right\rceil + n - 1$ vertices for $m \equiv 0,2,4(mod\ 5)$ and $\left\lfloor \frac{2m}{5} \right\rfloor + n - 1$ vertices for $m \equiv 1,3(mod\ 5)$. Hence $|W_1| \ge \left\lceil \frac{2m}{5} \right\rceil + n - 1$ for $m \equiv 0,2,4(mod\ 5)$ and $|W_1| \ge \left\lfloor \frac{2m}{5} \right\rfloor + n - 1$ for $m \equiv 1,3(mod\ 5)$. Thus we conclude that $nr(G) = \left\lceil \frac{2m}{5} \right\rceil + n - 1$ for $m \equiv 0,2,4(mod\ 5)$ and $nr(G) = \left\lfloor \frac{2m}{5} \right\rfloor + n - 1$ for $m \equiv 1,3(mod\ 5)$.

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3. Relation with Other Parameters:

In this section, we compare the nr value of graphs with the chromatic number $\chi(G)$ and the maximum degree $\Delta(G)$ .We note that the parameters $nr(G)$ and $\chi(G)$ are independent. For example, consider the graphs G_1, G_2 and G_3 given in Figure 3.1. Here, $\chi(G_1) < nr(G_1)$, $\chi(G_2) = nr(G_2)$ and $\chi(G_3) > nr(G_3)$.

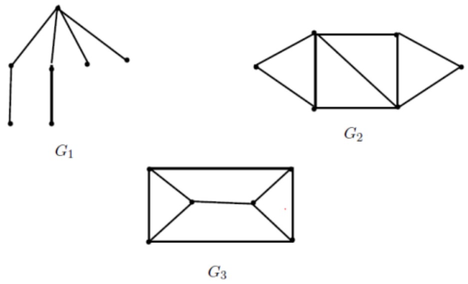

Figure 3.1

Now we classify the graphs into three families. A graph G is said to be a χ_{nr}^{-}-graph if $\chi(G) < nr(G)$, χ_{nr}^{*}-graph if $\chi(G) = nr(G)$ and χ_{nr}^{+}-graph if $\chi(G) > nr(G)$.

Theorem 3.1: *For given two positive integers m and n, $m > n \geq 3$, there exists a χ_{nr}^{-}-graph G with $\chi(G) = n = \omega(G)$ and $nr(G) = m$.*

Proof: Consider $G = \overline{K_p} + K_{n-1}$, where $p = m + 3 - n$, $V(\overline{K_p}) = \{v_1, v_2, ..., v_p\}$ and $V(K_{n-1}) = \{u_1, u_2, ..., u_{n-1}\}$. Take $W = \{v_1, v_2, ..., v_{p-1}, u_1, u_2, ..., u_{n-2}\}$. Then $r(v_p | W) = (2,2,...,2,1,1,...,1)$ where 2 appears at the first $(p-1)$ places and $r(u_{n-1} | W) = (1,1,...,1)$. Therefore, W is a non-isolated resolving set for G. Hence, $nr(G) \leq p + n - 3$.

Let W_1 be a non-isolated resolving set for G. If $v_i, v_j \notin W_1$, for any $i \neq j$ such that $1 \leq i, j \leq p$, then $r(v_i | W_1) = r(v_j | W_1)$, which is a contradiction. Therefore, there can be at least $p-1$ values of i, such that $v_i \in W_1$. Similar argument shows that $n-2$ vertices of u_i, $1 \leq i \leq n-1$ must belong to W_1. Hence, $nr(G) \geq p + n - 3$. Thus $nr(G) = p + n - 3 = m$. Also, in G, the maximum induced complete subgraph is K_n, which implies that $\omega(G) = n$. And it is easy to verify that $\chi(G) = n$.

Note that $K_{1,n}$, $n \geq 3$ are χ_{nr}^{-}-graphs. In addition, the path P_n and the even cycles C_{2n} prove the existence of the χ_{nr}^{*}-graphs.

Theorem 3.2: *For a given positive integer n, there exists χ_{nr}^{+} graphs with $\chi(G) = n$ and $nr(G) = n - 1$.*

Proof: The complete graph K_n is the required χ_{nr}^{+}-graph.

Next we discuss the relationship between $nr(G)$ and $\Delta(G)$. We note that the parameters $nr(G)$ and $\Delta(G)$ are independent. For example, consider the graphs H_1, H_2 and H_3 given in Figure 3.2. Here, $\Delta(H_1) < nr(H_1)$, $\Delta(H_2) = nr(H_2)$ and $\Delta(H_3) > nr(H_3)$.

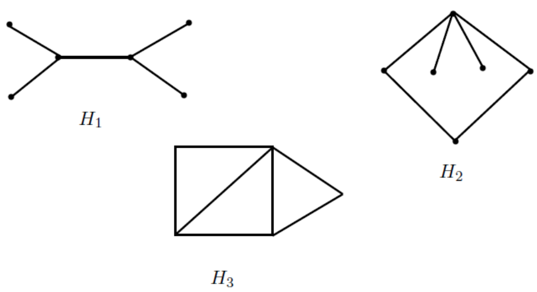

Figure 3.2

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Now we classify the graphs into three families. A graph G is said to be a Δ_{nr}^{-}-graph if $\Delta(G) < nr(G)$, Δ_{nr}^{*}-graph if $\Delta(G) = nr(G)$ and Δ_{nr}^{+}-graph if $\Delta(G) > nr(G)$.

Theorem 3.3: *For given positive integers* $m \geq 1$, $n \geq 2$, *there exists a* Δ_{nr}^{+}*-graph* G *with order* $m+2n$.

Proof: Consider the graph $G = K_m + K_{n,n}$, where $V(K_m) = \{u_1, u_2, ..., u_m\}$ and $V(K_{n,n}) = \{v_1, v_2, ..., v_n, v_1', v_2', ..., v_n'\}$. Therefore $n(G) = m+2n$. Let $W = \{u_1, u_2, ..., u_{m-1}; v_1, v_2, ..., v_{n-1}; v_1', v_2', ..., v_{n-1}'\}$. Then $r(u_m | W) = (1,1,...,1)$, $r(v_n | W) = (1,1,...,1,2,2,...,2,1,1,...,1)$ where 1 appears at the first $m-1$ places and the last $n-1$ places and $r(v_n' | W) = (1,1,...,1,2,2,...,2)$ where 1 appears at the first $m+n-2$ places. Therefore, W is a non-isolated resolving set for G. Hence $nr(G) \leq m+2n-3$. Let W_1 be a non-isolated resolving set for G. For $p \neq q$, $1 \leq p,q \leq m$, if both u_p and u_q are not in W_1, then $r(u_p | W_1) = r(u_q | W_1) = (1,1,...,1)$, a contradiction. Therefore, there can be at least $m-1$ values of p, such that $u_p \in W_1$. By similar argument, there can be at least $n-1$ values of i, such that $v_i \in W_1$, $1 \leq i \leq n$ and $v_i' \in W_1$. Hence $|W_1| \geq m+2n-3$. Thus, $nr(G) = m+2n-3$. Now, $\Delta(G) = m-1+2n = nr(G)+2$.

In the above theorem, when $n=1$, the constructed graph is isomorphic to the complete graph K_{m+2} for which $nr(G) = \Delta(G)$. Even more, one can easily note that in this family of graphs, the two parameters $\Delta(G)$ and $nr(G)$ are of opposite parity to m. That is, the above theorem can be restated as: For any given $2k$, there exists a graph G with two consecutive numbers $2k-1$ and $2k-3$ to be $\Delta(G)$ and $nr(G)$ respectively.

Note that the paths, cycles, complete graphs and star graphs prove the existence of Δ_{nr}^{*}-graphs and the double broom B(n,m,p) and the subdivision of $K_{m,n}$ prove the existence of Δ_{nr}^{-}-graphs.

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