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MODELLING AND ANALYSIS OF COMPLIMENT INVENTORY SYSTEM IN SUPPLY CHAINWITH PARTIAL BACKLOGGING

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Abstract:

This paper presents a continuous review two echelon inventory systems with two different items in stock, one is main product and other one is complement item for the main product. The operating policy at the lower echelon for the main product is (s, S) that is whenever the inventory level drops to '**s**' on order for Q = (S-s) items is placed, the ordered items are received after a random time which is distributed as exponential. We assume that the demands accruing during the stock-out period are partially backlogged. The retailer replenishes the stock of main product from the supplier which adopts (0, M) policy. The complement product is replenished instantaneously from local supplier. The joint probability disruption of the inventory levels of main product, complement item at retailer and the main product at supplier are obtained in the steady state case. Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of numerical examples, which provide insight into the behavior of the system, are presented.

Key Words: Continuous Review Inventory System, Two-Echelon, Positive Lead Time & Lost Sale

1. Introduction:

Study on multi-echelon systems are much less compared to those on single commodity systems. The determination of optimal policies and the problems related to a multi-echelon systems are, to some extent, dealt by Veinott and Wagner [17] and Veinott [18].Sivazlian [15] discussed the stationary characteristics of a multi commodity single period inventory system. The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain) when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw – materials, work-in process or finished goods. The main objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature. As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies Started with the work of Harris [9]. Clark and Scarf [7] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size. One of the oldest papers in the field of continuous review multi-echelon inventory system is written by Sherbrooke in 1968.Hadley, G and Whitin, T. M., [6], Naddor .E [14] analyses various inventory Systems. HP's (Hawlett Packard) Strategic Planning and Modeling (SPaM) group initiated this kind of research in 1977. Sivazlian and Stanfel [16] analyzed a two commodity single period inventory system. Kalpakam and Arivarignan [10] analyzed a multi-item inventory model with renewal demands under a joint replenishment policy. They assumed instantaneous supply of items and obtain various operational characteristics and also an expression for the long run total expected cost rate. Krishnamoorthyet.al., [11] analyzed a two commodity continuous review inventory system with zero lead time. A two commodity problem with Markov shift in demand for the type of commodity required, is considered by Krishnamoorthy and Varghese [12]. They obtain a characterization for limiting probability distribution to be uniform. Associated optimization problems were discussed in all these cases. However in all these cases zero lead time is assumed. In the literature of stochastic inventory models, there are two different assumptions about the excess demand unfilled from existing inventories: the backlog assumption and the lost sales assumption. The former is more popular in the literature partly because historically the inventory studies started with spare parts inventory management problems in military applications, where the backlog assumption is realistic. However in many other business situations, it is quite often that demand that cannot be satisfied on time is lost. This is particularly true in a competitive business environment. For example in many retail establishments, such as a supermarket or a department store, a customer chooses a competitive brand or goes to another store if his/her preferred brand is out of stock. All these papers deal with repairable items with batch ordering. A Complete review was provided by Benito M. Beamon [5]. Sven Axsater [1] proposed an approximate model of inventory structure in SC. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases. Anbazhagan and Arivarignan [2,3] have analyzed two commodity inventory system under various ordering policies. Yadavalliet. al., [19] have analyzed a model with joint ordering policy and varying order quantities. Yadavalliet. al., [20] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time. In a very recent paper, Anbazhaganet. al. [4] considered analysis of two commodity inventory system with compliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item is also assumed when the items are out of stock. The above model is studied only at single location (Lower echelon). We extend the same in to multi-echelon structure (Supply Chain). The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 provides Numerical examples and sensitivity analysis.

2. Model:

2.1 The Problem Description:

The inventory control system considered in this paper is defined as follows. A finished main product is supplied from manufacturer to supplier which adopts (0, M) replenishment policy then the product is supplied to retailer who adopts (s,S) policy. The retailer also maintain an inventory of the complement product which has instantaneous replenishment from local supplier. The demand at retailer node follows an independent Poisson distribution with rate λ_i (i = 1, 2) for main product and complement respectively. Demands accruing during the stock out periods of main product are backlogged up to some finite number b. The replacement of item in terms of product is made from supplier to retailer is administrated with exponential distribution having parameter $\mu > 0.$. The maximum inventory level at retailer node for main product is S, and the recorder point is s and the ordering quantity is Q(=S-s) items. The maximum inventory at supplier in M(=nQ) .

2.2 Notations and Variables:

We use the following notations and variables for the analysis of the paper.

Notations /variables	Used for
$[C]_{ij}$	The element of sub matrix at (i,j)th position of ***C***
0	Zero matrix
$\lambda_1,\ \lambda_2$	Mean arrival rate for Main& Compliment product at retailer
μ	Mean replacement rate for main product at retailer
S, N	Minimum inventory level for main& Compliment product at retailer
s	reorder level for main product at retailer
b	Backlog capacity
M	Maximum inventory level for main product at supplier
H_m	Holding cost per item for main product at retailer
H_c	Holding cost per item for Compliment product at retailer
H_d	Holding cost per item for main product at distributor
O_r	Ordering cost per order for main product at retailer
O_c	Ordering cost per order for compliment product at retailer
O_m	Ordering cost per order for main product at retailer
I_m	Average inventory level for main product at retailer
I_c	Average inventory level for compliment product at retailer
I_d	Average inventory level for main product at retailer
R_d	Mean reorder rate for main productatsupplier.
R_c	Mean reorder rate for compliment product at retailer
R_m	Mean reorder rate for main product at retailer
S_m	Shortage rate for main product at retailer
T_m	Penalty rate for main product at retailer
$\sum_{i=Q}^{nQ} i$	Q + 2Q + 3Q + ... + nQ

3. Analysis:

Let $I_m(t)$ and $I_c(t)$ denote the on hand Inventory levels of Main product, Compliment product at retailer and $I_d(t)$denote the on hand inventory level of Main product at supplier at time t+. We define I (t) = $\{(I_m(t), I_c(t), I_d(t)): t \geq 0$ as Markov process with state space E = { (i, j,k) | i = S, S-1, ...s,..1, 0, -1, -2,..-b, j =1, 2, …, N, k=Q,2Q,.....nQ }. Since E is finite and all its states are aperiodic, recurrent, non- null and also irreducible. That is all the states are Ergodic. Hence the limiting distribution exists and is independent of the initial state. The infinitesimal generator matrix of this process C = (*a* (*i, j, k, :l, m, n*)) $_{(i,j,k)(l,m,n)\in E}$ can be obtained from the following arguments.

- ✓ The arrival of a demand for main product at retailer make a state transition in the Markov process from (i, j, k) to (i-1, j-1, k) with the intensity of transition $\lambda_1 > 0$.
- ✓ The arrival of a demand for compliment product at retailer make a state transition in the Markov process from (i, j, k) to (i, j-1, k) with the intensity of transition $\lambda_2 > 0$.
- ✓ The replacement of inventory atretailer make a state transition in the Markov process from (i, j, k) to (i+Q, j, k-Q) or (i, j, Q) to (i+Q, j, nQ) with the intensity of transition $\mu > 0$.

The infinitesimal generator C is given by

$$\mathbf{C}=\begin{bmatrix} A & B & 0 & \dots & 0 & 0 \\ 0 & A & B & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & A & B \\ B & 0 & 0 & \dots & 0 & A \end{bmatrix}$$

The entries of C are given by

$$[C]_{pq}=\begin{cases} A & p=q; \quad q=nQ, \quad (n-1)Q,\dots Q \\ B & p=q+Q; \quad q=(n-1)Q,\dots.Q \\ B & p=q-(n-1)Q \quad q=nQ \\ 0 & otherwise \end{cases}$$

Where the entries of the matrices are given by

$$[A]_{ij}=\begin{cases} A_1 & i=j; \quad i=1,2,\dots N \\ A_2 & i+1=j; \quad i=1,2,\dots(N-1) \\ A_2 & i-(N-1)=j; \quad i=N \\ 0 & otherwise \end{cases}$$

$$[B]_{ij}=\begin{cases} B_1 & i=j; i=1,2,\dots,N \\ 0 & otherwise \end{cases}$$

The elements in the sub matrices of A and B are

$$\begin{cases} [A_1]_{ij} = -(\lambda_1+\lambda_2) & if\ i=j\ : i=S,S-1,\dots\dots.(s+1) \\ -(\lambda_1+\lambda_2+\mu) & if\ i=j\ : i=s,s-1,\dots\dots.1,0,-1,..-(b+1) \\ -(\lambda_2+\mu) & if\ i=j\ : i=0 \\ \text{Otherwise} & \end{cases}$$

$$[A_2]_{ij} = \begin{cases} \lambda_2\ , & if\ i=j \quad : i=S,S-1,\dots\dots.1. \\ \lambda_1\ , & if\ i+1=j\ : i=S,S-1,\dots\dots.1,0,-1,\dots-(b+1) \\ 0, & \text{Otherwise} \end{cases}$$

$$[B_1]_{ij} = \begin{cases} \mu & if\ i+Q=j\ : i=s,s-1,\dots\dots.1,0,-1,-2,\dots-b. \\ 0 & \text{Otherwise} \end{cases}$$

3.1 Steady State Analysis:

The structure of the infinitesimal matrix C, reveals that the state space E of the Markov process $\{ I(t) : t \geq 0 \}$ is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$$\prod_{i,j}^{k} = \lim_{t\to\infty} Pr\{(I_m(t), I_c(t), I_d(t) = (i,j,k))\}$$

Where $\prod_{i,j}^{k}$ *is* the steady state probability that the system be in state *(i, j, k)*.

Let $\prod = \{\prod_{i,j}^{nQ}, \prod_{i,j}^{(n-1)Q}, \dots\dots\dots.\prod_{i,j}^{Q}.\}$denote the steady state probability distribution. For each $((i,j,k)$,$\prod_{i,j}^{k}$ *can* be obtained by solving the matrix equation $\prod$ C = 0. The system of equations may be written as follows

i. $\prod_j^{nQ} A_1 + \prod_i^{nQ} A_2 + \prod_j^{Q} B_1 = 0$ *i = 1; j = S*

ii. $\prod_j^{nQ} B_1 + \prod_j^{(n-1)Q} A_1 + \prod_i^{(n-1)Q} A_2 = 0$ *i = 1; j = S*

iii. $\prod_j^{(n-1)Q} B_1 + \prod_j^{Q} A_1 + \prod_i^{Q} A_2 = 0$ *i = 1; j = S*

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iv. $\prod_{j}^{nQ} A_2 + \prod_{i-1}^{nQ} A_1 + \prod_{i-1}^{Q} B_1 = 0$ $\quad i = S, S-1, \ldots\ldots, 2, 1, 0, -1, -2 \ldots -(b+2)$

v. $\prod_{i-1}^{nQ} B_1 + \prod_{i}^{(n-1)Q} A_2 + \prod_{i-1}^{(n-1)Q} A_1 = 0$ $\quad i = S, S-1, \ldots\ldots, 2, 1, 0, -1, -2 \ldots -(b+2)$

vi. $\prod_{i-1}^{(n-1)Q} B_1 + \prod_{i}^{Q} A_2 + \prod_{i-1}^{Q} A_1 = 0$ $\quad i = S, S-1, \ldots\ldots, 2, 1, 0, -1, -2 \ldots -(b+2)$

By solving the above system of equations, together with normalizing condition $\sum_{(i,j,k)\in E} \prod_{i,j}^{k} = 1$, the steady probability of all the system states are obtained.

4. Operating Characteristic:

In this section we derive some important system performance measures.

4.1 Average Inventory Level:

The event I_m, I_c *and* I_d denote the average inventory level for main product, complement product at retailer and main product at distributor respectively,

(i) $$I_m = \sum_{k=Q}^{nQ} \sum_{j=1}^{N} \sum_{i=0}^{S} i.\prod_{i,j}^{k}$$

(ii) $$I_c = \sum_{k=Q}^{nQ} \sum_{i=0}^{S} \sum_{j=1}^{N} j.\prod_{i,j}^{k}$$

(iii) $$I_d = \sum_{i=0}^{S} \sum_{j=1}^{N} \sum_{k=Q}^{nQ} k.\prod_{i,j}^{k}$$

4.2 Mean Reorder Rate:

Let R_m, R_c, *and* R_d be the mean reorder rate for main product, complement product at retailer and main product at distributor respectively,

(i) $$R_m = \lambda_1 \sum_{k=Q}^{nQ} \sum_{j=1}^{N} \prod_{s+1,j}^{k}$$

(ii) $$R_c = (\lambda_1 + \lambda_2) \sum_{k=Q}^{nQ} \sum_{i=0}^{S} \prod_{i,1}^{k}$$

(iii) $$R_d = \mu \sum_{i=0}^{S} \sum_{j=1}^{N} \prod_{i,j}^{Q}$$

4.3 Shortage Rate:

Shortage occur at retailer only for main product. Let S_m be the shortage rate at retailer for main product, then

(i) $$S_m = \lambda_1 \sum_{k=Q}^{nQ} \sum_{j=1}^{N} \prod_{-b,j}^{k}$$

5. Cost Analysis:

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time. The long run expected cost rate *TC(s, Q)* is given by

$$TC(s, Q) = I_m.H_m + I_c.H_c + I_d.H_d + R_m.O_m + R_c.O_c + R_d.O_d + S_m.T_m$$

Although we have a not proved analytically the convexity of the cost function *TC(s, Q)* our experience with considerable number of numerical examples indicate that *TC(s, Q)* for fixed '*S*' appears to be convex in s. In some cases it turned out to be increasing function of s. For large number case of *TC(s, Q)* revealed a locally convex structure. Hence we adopted the numerical search procedure to determine the optimal value of '*s*'

6. Numerical Example and Sensitivity Analysis:

6.1 Numerical Example:

In this section we discuss the problem of minimizing the structure. We assume $H_c \le H_m \le H_d$, i.e, the holding cost for compliment product is at retailer node is less than that of main product at retailer node and the holding cost of main product is less than that of main product at distributor node. Also $O_c \le O_m \le O_d$ the ordering cost at retailer node for compliment product is less than that of main product. Also the ordering cost at the distributor is greater than that of compliment product at retailer node.

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The results we obtained in the steady state case may be illustrated through the following numerical example,

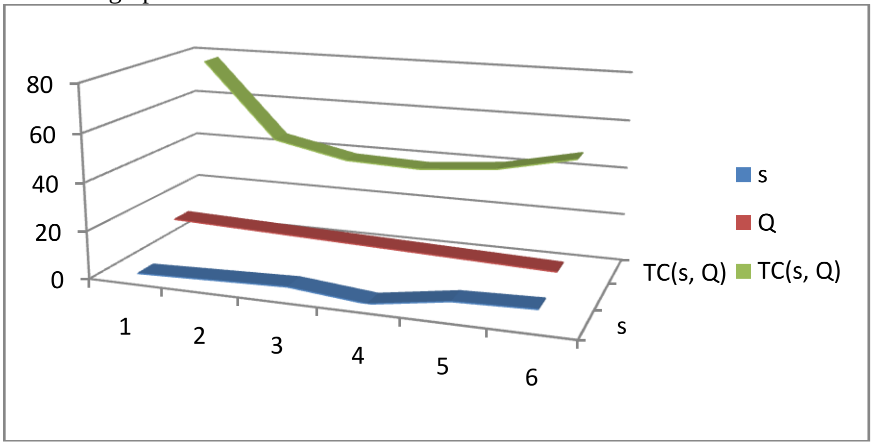
S =16, N=15, M = 80, $\lambda_1 = 3$, $\lambda_2 = 2$, b = 3 $\mu = 3$, $H_c = 1.1$, $H_m = 1.2$, $H_d = 1.3$ $O_c = 2.1$, $O_m = 2.2$, $O_d = 2.3$ $T_m = 3.1$

The cost for different reorder level are given by

s	1	2	3	4*	5	6
Q	15	14	13	12	11	10
TC(s, Q)	77.98279	45.35418	38.8074	37.67178	40.40994	47.31084

Table 1: Total expected cost rate as a function s and Q

For the inventory capacity S, the optimal reorder level s* and optimal cost *TC(s, Q)* are indicated by the symbol *. The Convexity of the cost function is given in the graph.



6.2 Sensitivity Analysis:

Below tables are represented a numerical study to exhibit the sensitivity of the system on the effect of varying different parameters. $\lambda_1 \,\&\, \mu$, $\lambda_2 \,\&\, \mu$, $H_c \,\&\, H_m$; $H_m \,\&\, H_d$, $O_m \,\&\, O_c$, $O_m \,\&\, P_d$;

For the following cost structure S =16,s = 4, N=15, M = 80, $\lambda_1 = 3$, $\lambda_2 = 2$, $\mu = 3$ $H_c = 1.1$, $H_m = 1.2$, $H_d = 1.3$, $O_c = 2.1$, $O_m = 2.2$, $O_d = 2.3$ $T_m = 3.1$.

Table 2: Effect on Replenishment rate& Demand rates $\mu \setminus \lambda_1$

$\mu \setminus \lambda_1$	**1**	**2**	**3**	**4**	**5**
1	44.9562	105.736	183.073	262.016	341.248
2	64.192	80.6682	132.526	204.329	281.382
3	83.6582	100.861	120.95	167.608	234.24
4	99.8989	127.775	141.663	165.368	210.307
5	114.624	151.974	171.738	187.085	213.553

Table 3: Effect on Replenishment rate& Demand rates $\mu \setminus \lambda_{2,}$

$\mu \setminus \lambda_{2,}$	**1**	**3**	**5**	**7**	**9**
1	264.4122	186.988	225.608	308.413	388.178
3	261.1814	161.193	171.85	229.599	285.558
5	260.5082	155.149	158.612	209.956	259.907
7	260.2163	152.461	152.627	201.05	248.266
9	260.0576	150.94	149.217	195.96	241.605

Table 4: Effect on Holding cost ($H_m \setminus H_{d,}$)

$H_m \setminus H_{d,}$	**1.1**	**2.1**	**3.1**	**4.1**	**5.1**
1.1	167.7286	169.917	171.935	173.955	175.974
2.1	171.927	173.947	175.959	177.987	180.004
3.1	175.959	177.979	179.996	182.016	184.036
4.1	179.9885	182.001	184.028	186.045	188.065
5.1	184.0205	186.038	188.058	190.077	192.097

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Table 5: Effect on Ordering Cost ($H_c \backslash H_{m,}$)

$H_c \backslash H_{m,}$	2.1	2.2	2.3	2.4	2.5
2.1	167.7286	168.2995	168.778	169.306	169.83
2.2	168.2995	168.8269	169.352	159.887	170.404
2.3	168.873	169.4003	169.925	170.45	170.977
2.4	169.4464	169.4003	170.499	171.023	171.551
2.5	170.0198	170.5446	171.072	171.597	172.122

It is observed that from the table, the total expect cost *TC(s, Q)* is increases with the cost increases.

7. Conclusion:

This paper deals with a two echelon Inventory system with two products namely maim and compliment product. The demand at retailer node follows independent Poisson with rate λ_1 for main product λ_2 for compliment product. If the demand occur for the main product then it is also the demand for the compliment product. But the compliment product demand do not disturb the main product. The structure of the chain allows vertical movement of goods from to supplier to Retailer. If there is no stock for main product at retailer the demand is backlogged. The model is analyzed within the framework of Markov processes. Joint probability distribution of inventory levels at DC and Retailer for both products are computed in the steady state. Various system performance measures are derived and the long-run expected cost is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values on the total expected cost rate. It would be interesting to analyze the problem discussed in this paper by relaxing the assumption of exponentially distributed lead-times to a class of arbitrarily distributed lead-times using techniques from renewal theory and semi-regenerative processes. Once this is done, the general model can be used to generate various special eases.

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