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A STUDY ON PROPERTIES OF LATTICE ORDERED SOFT GROUP

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Abstract:

In this paper, operations like basic union, basic intersection, conjunction and disjunction of soft sets have been introduced and also some properties for lattice ordered soft group have been derived.

Key Words: Soft Sets, Soft Groups, Lattice Ordered Soft Sets & Lattice Ordered Soft Groups

1. Introduction:

Soft set theory was developed by Molodtsov in 1999. This concept was mainly introduced to solve the complicated problems arise in economics,engineering,environmental science, medical science etc, that contains the data with uncertainty. Molodtsov's this new approach was solvable for these uncertainties. We have observed that P.K.Maji et.al defined some binary operations and the results were verified by M.I.Ali et al. Aslihan Sezgin,Akin Osman Atagun and D.Pei,D. Miao introduced some new operations and also discussed the De Morgan's laws for soft set theory. The notion of soft group was introduced by H.Aktas and N.Cagman. Some algebraic properties of soft subgroups were investigated by Xia Yin, Zuhua Liao. Lattice theory was developed by Garett Birkhoff. The Lattice structure of the soft group was outlined by Yingehao Shao,Keyun Qin. The concept of soft lattice was introduced by Faruk Karaasalan,Cagman.N and Enginoglu.S and the properties were given in. M.I Ali et al. introduced the concept of lattice ordered soft sets. Also Natarajan.R,and Vimala.J introduced the standard and distributive l-ideal in a commutative lattice ordered group.Fuzzy l-ideals in a commutative l-group was discussed by Bharathi.P and Vimala.J. In 2014, J.Vimala introduced Fuzzy lattice ordered group. Lattice ordered soft group was initiated by Vijayalakshmi.L and Vimala.J. This paper has been organized as follows: In sec 2, some preliminary definitions and examples are given. In sec 3, we study the properties of lattice ordered soft group and some related results.

2. Preliminaries:

In this section, we recall some basic definitions of soft sets and soft groups. Let U be an initial universe set and E a set of parameters with respect to U. Let $P(U)$ denote the power set of U and $A \subseteq E$. For the soft group,universe be a group.

Definition 2.1 *[11] A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set (F, A) over U is a parameterized family of subsets of U. For $e \in A$, $F(e)$ may be considered as the set of e-elements or e-approximate elements of the soft set (F, A).*

Definition 2.2 *[11] Let (F, A) and (H, B) be two soft sets over common universe U, we say that (H, B) is said to be a soft subset of (F, A) if*

1. $A \subseteq B$ and
2. $F(e) \subseteq H(e)$ for all $e \in A$.

We write $(H, B) \tilde{\subseteq} (F, A)$, and (H, B) is said to be a soft super set of (F, A), if (F, A) is a soft subset of (H, B). We denote it by $(H, B) \tilde{\supseteq} (F, A)$

Definition 2.3 *[11] Two soft sets (F, A) and (H, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).*

Definition 2.4 *[11] A soft set (F, A) over U is said to be a Null soft set denoted by Φ if for all $e \in A, F(e) = \varnothing$.*

Definition 2.5 *[11] A soft set (F, A) over U is said to be a Absolute soft set denoted by $\tilde{A}$ if for all $e \in A$, $F(e) = U$.*

Definition 2.6 *[10] Let (F, A) and (G, B) be two soft sets over a common universe U. Then the union of (F, A) and (G, B) denoted $(F, A) \tilde{\cup} (G, B)$ is a soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ F(e) \cup G(e), & if\ e \in A \cap B \end{cases}$$

Definition 2.7 *[10] Let (F, A) and (G, B) be two soft sets over a common universe U. Then the intersection of (F, A) and (G, B) denoted $(F, A) \tilde{\cap} (G, B)$ is a soft set (H, C), where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e)$ or $G(e)$ (as both are same set).*

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Definition 2.8 *[17] Let* (F,A) *and* (G,B) *be two soft sets over a common universe* U. *Then the basic union of* (F,A) *and* (G,B) *defined as the soft set* $(H,C)=(F,A)\vee(G,B)$ *where* $C=A\times B$, *and* $H(a,b)=F(a)\cup G(b)$, *for all* $(a,b)\in A\times B$.

Definition 2.9 *[17] Let* (F,A) *and* (G,B) *betwo soft sets over a common universe* U *.Then the basic intersection of* (F,A) *and* (G,B) *defined as the soft set* $(H,C)=(F,A)\wedge(G,B)$ *where* $C=A\times B$, *and* $H(a,b)=F(a)\cap G(b)$, *for all* $(a,b)\in A\times B$.

Definition 2.10 *[4] Let E be a set of parameters and* $A,B\subseteq E$. *For* $(a,b)\in A\times B$*,(a and b) is called the conjunction parameter of* (a,b), *and (a or b) is called the disjunction parameter of ordered pair* (a,b). *These denoted by* $(a\wedge b)$ *and* $(a\vee b)$ *respectively. we denote* $A\otimes B=\{(a\wedge b):(a,b)\in A\times B\}$. *Also* $A\oplus B=\{(a\vee b):(a,b)\in A\times B\}$.

Definition 2.11 *[4] Let* (F,A) *and* (G,B) *be two soft sets over a common universe* U *.Then (a)* $(F,A)\cap_{\wedge}(G,B)=(H,A\otimes B)$ *is the soft set defined as* $H(a\wedge b)=F(e)\cap G(e)$, *for all* $a\wedge b\in A\otimes B$. *(b)* $(F,A)\cup_{\vee}(G,B)=(H,A\oplus B)$ *is the soft set defined as* $H(a\vee b)=F(e)\cup G(e)$, *for all* $a\vee b\in A\oplus B$.

Definition 2.12 *[1] Let* G *be a group and* E *be a set of parameters. For* $A\subseteq E$, *the pair* (F,A) *is called a soft group over* G *if and only if* $F(a)$ *is a subgroup of* G *for all* $a\in A$, *where* F *is a mapping of* A *into the set of all subsets of* G.

Definition 2.13 *[4] If* U *is an initial universe, then* $P(U)$ *is a bounded lattice and the set of parameters* E, *is also a lattice with respect to certain binary operations (or) partial order and* $A\subseteq E$. *A soft set* (F,A) *is called a lattice ordered soft set if for the mapping* $F:A\to P(U)$, $x\le y$ *implies* $F(x)\subseteq F(y)$, *for all* $x,y\in A\subseteq E$.

3. Properties of Lattice Ordered Soft Group:

Definition 3.1 *[18] Let* G *be a group and* $P(G)$ *be the power set of* G. *Let* E *be the set of parameters(lattice),* $A\subseteq E$. *Then a soft set* (F,A) *is said to be a lattice ordered soft group if for the mapping* $F:A\to P(G)$,

1. $F(a)$ is a subgroup of G, for all $a\in A$.[1]
2. For all $a,b\in A$, $a\vee b, a\wedge b\in A$ then $Sup\{F(a),F(b)\}$ and $Inf\{F(a),F(b)\}$ exist in (F,A).

Note: Throughout this section, $F(a)\vee F(b)$ and $F(a)\wedge F(b)$ are used for $Sup\{F(a),F(b)\}$ and $Inf\{F(a),F(b)\}$ respectively. Also $\vee$ denotes $\cup$, $\wedge$ denotes $\cap$ and L denotes the set of all lattice ordered soft groups.
The following are few properties of lattice ordered soft group.

Proposition 3.2 *Basic union of two lattice ordered soft groups is a lattice ordered soft group.*

Proof: Let (F,A) and (G,B) be two lattice ordered soft groups, then for all $a_1,a_2\in A, F(a_1)\vee F(a_2), F(a_1)\wedge F(a_2)$ exist in (F,A) and for all $b_1,b_2\in B, G(b_1)\vee G(b_2), G(b_1)\wedge G(b_2)$ exist in (G,B).

Case(i):

Suppose $F(a_1)\vee F(a_2)=F(a_2)$ and $G(b_1)\vee G(b_2)=G(b_2)$.

$\Rightarrow F(a_1)\subseteq F(a_2)$ and $G(b_1)\subseteq G(b_2)$.

Now for all (a_1,b_1) and $(a_2,b_2)\in A\times B, F(a_1)\vee G(b_1)\subseteq F(a_2)\vee G(b_2)$,

$\Rightarrow Sup\{F(a_1),G(b_1)\}\subseteq Sup\{F(a_2),G(b_2)\}$

$\Rightarrow F(a_1)\cup G(b_1)\subseteq F(a_2)\cup G(b_2)$. $\Rightarrow H(a_1,b_1)\subseteq H(a_2,b_2)$.

$\Rightarrow H(a_1,b_1)\vee H(a_2,b_2)=H(a_2,b_2)$ and $H(a_1,b_1)\wedge H(a_2,b_2)=H(a_1,b_1)$ exist in (H,C).

Case(ii):

Suppose $F(a_1)\vee F(a_2)=F(a_1)$ and $G(b_1)\vee G(b_2)=G(b_1)$.

$\Rightarrow F(a_2)\subseteq F(a_1)$ and $G(b_2)\subseteq G(b_1)$.

Now for all (a_1,b_1) and $(a_2,b_2)\in A\times B, F(a_2)\vee G(b_2)\subseteq F(a_1)\vee G(b_1)$,

$\Rightarrow Sup\{F(a_2),G(b_2)\}\subseteq Sup\{F(a_1),G(b_1)\}$

$\Rightarrow F(a_2)\cup G(b_2)\subseteq F(a_1)\cup G(b_1)$. $\Rightarrow H(a_2,b_2)\subseteq H(a_1,b_1)$.

$\Rightarrow H(a_2,b_2)\vee H(a_1,b_1)=H(a_1,b_1)$ and $H(a_2,b_2)\wedge H(a_1,b_1)=H(a_2,b_2)$ exist in (H,C).

Case(iii):

Suppose $F(a_1)\vee F(a_2)=F(a_2)$ and $G(b_1)\vee G(b_2)=G(b_1)$.

$\Rightarrow F(a_1)\subseteq F(a_2)$ and $G(b_2)\subseteq G(b_1)$.

Now for all (a_1,b_1) and $(a_2,b_2)\in A\times B, F(a_1)\vee G(b_2)\subseteq F(a_2)\vee G(b_1)$,

$\Rightarrow Sup\{F(a_1),G(b_2)\}\subseteq Sup\{F(a_2),G(b_1)\}$

$\Rightarrow F(a_1)\cup G(b_2)\subseteq F(a_2)\cup G(b_1)$. $\Rightarrow H(a_1,b_2)\subseteq H(a_2,b_1)$.

$\Rightarrow H(a_1,b_2)\vee H(a_2,b_1)=H(a_2,b_1)$ and $H(a_1,b_2)\wedge H(a_2,b_1)=H(a_1,b_2)$ exist in (H,C).

Case(iv):

Suppose $F(a_1)\vee F(a_2)=F(a_1)$ and $G(b_1)\vee G(b_2)=G(b_2)$.

$\Rightarrow F(a_2)\subseteq F(a_1)$ and $G(b_1)\subseteq G(b_2)$.

Now for all (a_1,b_1) and $(a_2,b_2)\in A\times B, F(a_2)\vee G(b_1)\subseteq F(a_1)\vee G(b_2)$,

$\Rightarrow Sup\{F(a_2),G(b_1)\}\subseteq Sup\{F(a_1),G(b_2)\}$

$\Rightarrow F(a_2)\cup G(b_1)\subseteq F(a_1)\cup G(b_2)$. $\Rightarrow H(a_2,b_1)\subseteq H(a_1,b_2)$.

$\Rightarrow H(a_2,b_1)\vee H(a_1,b_2)=H(a_1,b_2)$ and $H(a_2,b_1)\wedge H(a_1,b_2)=H(a_2,b_1)$ exist in (H,C).

Thus in all the cases $(H,C)=(F,A)\bigvee(G,B)$ is a lattice ordered soft group.

Proposition 3.3 *Basic intersection of two lattice ordered soft groups is a lattice ordered soft group.*

Proof is similar to the above Proposition.

Proposition 3.4 *Let* (F,A) *and* (G,B) *be two lattice ordered soft groups.Then (i)* $(F,A)\cup_{\vee}(G,B)$ *and (ii)* $(F,A)\cap_{\wedge}(G,B)$ *are lattice ordered soft groups.*

Proof: (i) Let $(F,A)\cup_{\vee}(G,B)=(H,A\oplus B)$ and is defined as

$H(a\vee b)=F(a)\cup G(b)$ for all $(a\vee b)\in A\oplus B$

$=F(a)\vee G(b)$

$=Sup\{F(a),G(b)\}$ eixst in either (F,A) (or) (G,B).Since (F,A) and (G,B) are lattice ordered soft groups, $(H,A\oplus B)$ is also a lattice ordered soft group.

(ii)Let $(F,A)\cap_{\wedge}(G,B)=(H,A\otimes B)$ and is defined as

$H(a\wedge b)=F(a)\cap G(b)$ for all $(a\wedge b)\in A\otimes B$

$=F(a)\wedge G(b)$

$=Inf\{F(a),G(b)\}$ eixst in either (F,A) (or) (G,B).Since (F,A) and (G,B) are lattice ordered soft groups, $(H,A\otimes B)$ is also a lattice ordered soft group.

4. Conclusion:

In this paper, few properties of lattice ordered soft group have been studied using soft set operations.Furthermore one can discuss the other algebraic properties of lattice ordered soft group.

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